Government Polytechnic Nanakpur

DEPARTMENT OF MECHANICAL ENGINEERING

E - NOTES

THERMODYNAMICS - I

HOD/O.I. (Mechanical) : **ER.SHALANDER MOR**

Subject Teacher: Er.Sanjay Kumar

Semester: 3rd Sem

By: Er. Amit Kumar
Designed, Mechanical Editing & Developmental Editing

LEARNING OUTCOMES

After undergoing this subject, the students will be able to:

- Apply thermodynamic laws.
- Solve basic problems of gas equation using perfect gas laws.
- Determine enthalpy, specific heat capacity and P-V-T surface of an ideal and real gas.
- Determine various properties of Steam
- Explain the working, construction and applications of steam boilers and steam generators
- Explain the working, construction and application of air compressor.

1

Fundamental Concepts

1. Basic Concepts of Thermodynamics

1.1 Thermodynamic system

A specified collection of matter is called **a system**, which is defined by the mass and the composition.

a. **Open system**: mass is exchanged with its surroundings;

b. **Closed system**: NO mass is exchanged with its surroundings.

What type of system does atmospheric thermodynamics deal with?

The systems that atmospheric thermodynamics deal with include

1) an air parcel;
2) a cloud;
3) the atmosphere;
4) an air mass etc.

Precisely speaking, they are open systems because mass can be changed by the entrainment and mixing processes.

But, we will treat them as a closed system in this course.

Assumptions:

1) the volume is large that mixing at the edges is negligible; or

2) the system is imbedded in a much larger mass which has the same properties.

1.2 Thermodynamic properties

The properties define the thermodynamic **state** of a system.

a. **Intensive property**: does not depend on the mass (m) or does not change with subdivision of the system, denoted by lowercase letters, e.g., z.

b. **Extensive property**: does depend on the mass (m) or does change with subdivision of the system, denoted by uppercase letters, e.g., Z.

Exception to the convention: T for temperature and m for mass

* An intensive property is also called a **specific property** if

$$z = \frac{Z}{m}$$

For example, volume V is an extensive property, so v=V/m (i.e., volume per unit mass) is a specific property and an intensive property.

* Homogeneous vs heterogeneous

a. A system is considered to be **homogeneous** if every intensive property has the same value for every point of the system.

$$Z = mz$$

b. A system is said to be **heterogeneous** if the intensive property of one portion is different from the property of another portion.

$$Z = \sum_i m_i z_i$$

* A system can exchange energy with its surroundings through two mechanisms:

1) Mechanical exchange (Expansion work)

   performing work on the surroundings

2) Thermal exchange (Heat transfer)

   transferring heat across the boundary

* A system is in thermodynamic equilibrium if it is in mechanical and thermal equilibrium.

Mechanical equilibrium: the pressure difference between the system and its surroundings is infinitesimal;

Thermal equilibrium: the temperature difference between the system and its surroundings is infinitesimal.

1.3 Expansion work

If a system is not in mechanical equilibrium with its surrounding it will expand or contract.

The incremental expansion work:

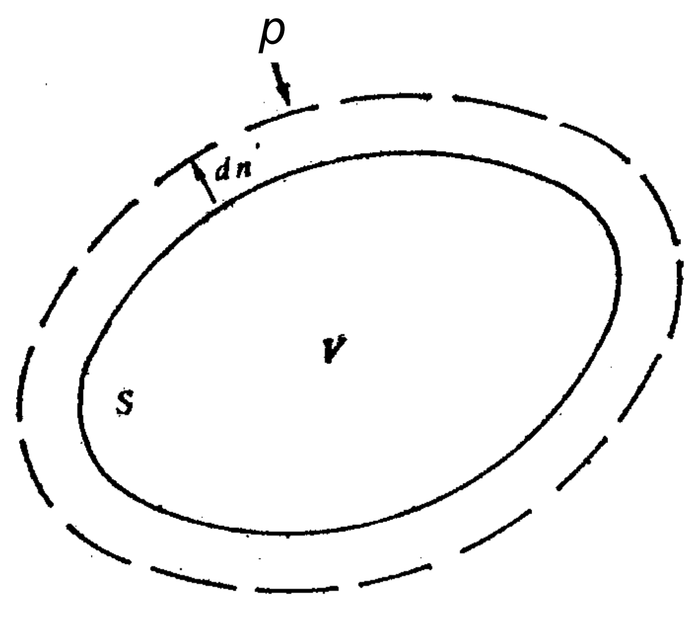
$$\delta W = pdSdn = pdV$$

p: the pressure exerted by the surroundings over the system

dV: the incremental volume

dS: the displaced section of surface

dn: the normal distance between original and expanded surface



1.4 Heat transfer

Adiabatic process: no heat is exchanged between the system and the environment.

Diabatic process: heat is exchanged between the system and the environment.

Which one will we use the most? Why?

1.5 State variables and equation of state

* A system, if its thermodynamic state is uniquely determined by any two intensive properties, is defined as a **pure substance**. The two properties are referred to as ***state variables***.

* From any two state variables, a third can be determined by an ***equation of state***,

$$f(z_1, z_2, z_3) = 0.$$

A pure substance only has two degrees of freedom. Any two state variables fix the thermodynamic state,

* Any third state variable as a function of the two independent state variables forms a **state surface** of the thermodynamic states, i.e.,

$$z_3 = g(z_1, z_2).$$

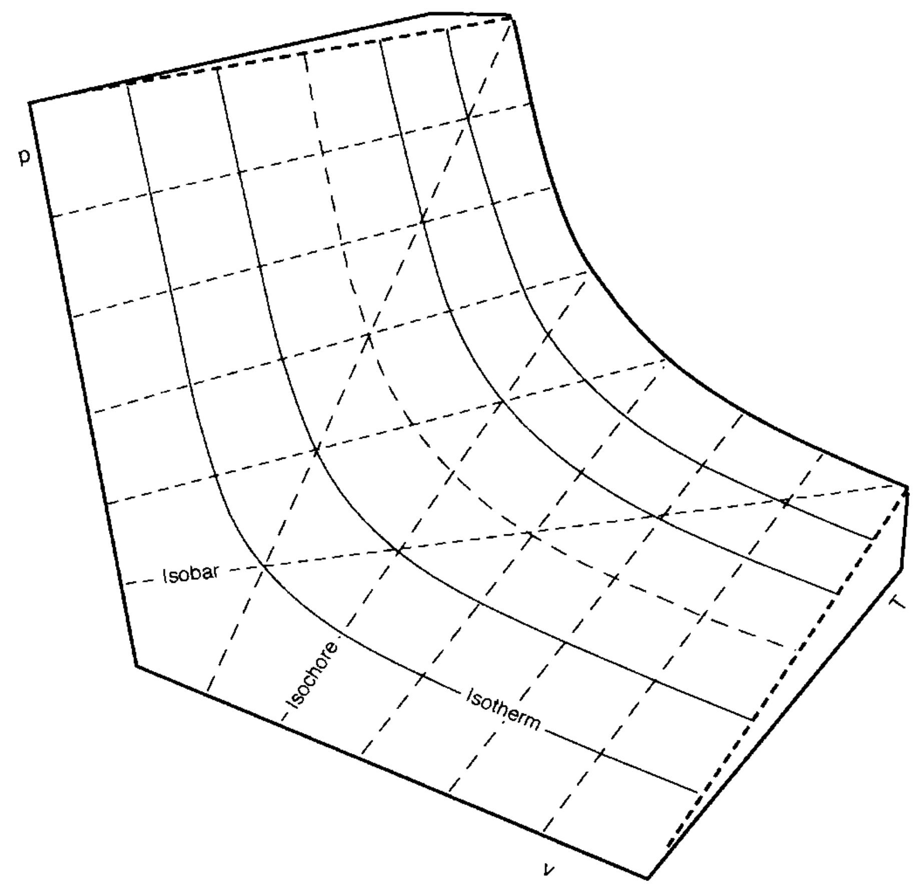


Figure 2.3 State space of an ideal gas, illustrated in terms of p as a function of the state variables v and T. Superposed are contours of constant pressure (*isobars*), constant temperature (*isotherms*), and constant volume (*isochores*).

1.6 Thermodynamic process

* The transformation of a system between two states describes a path, which is called a **thermodynamic process**.

* There are infinite paths to connect two states.

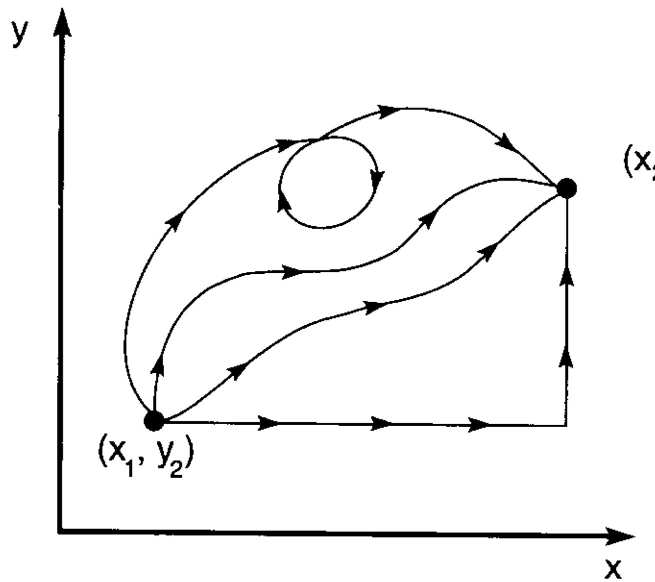


Figure 2.4 Possible thermodynamic processes between two states: (x_1, y_1) and (x_2, y_2).

1.7 Equation of state for ideal gases

1.7.1 How to obtain the ideal gas equation?

The most common way to deduce fundamental equations is to observe controlled experiments.

* Based on Boyle's observation, if the temperature of a fixed mass of gas is constant, the volume of the gas (V) is inversely proportional to its pressure (p), i.e.,

$$pV = const \quad (1)$$

* From Charles' observation, for a fixed mass of gas at constant pressure, the volume of the gas is directly proportional to its absolute temperature (T), i.e.,

$$\frac{V}{T} = const \quad (2)$$

* For a fixed mass of gas, consider three different equilibrium states that have $(p, V, T),\ (p_s, V_s, T),\ and\ (p_s, V_{0s}, T_0)$, respectively.

* From (1) and (2), we have

$$pV = p_s V_s \ , \quad \frac{V_s}{T} = \frac{V_{0s}}{T_0}$$

Combine them,

$$\frac{pV}{T} = \frac{p_s V_{0s}}{T_0} \qquad (3)$$

Divide (3) by the **molar abundance (or number of moles)**

$$n = \frac{m}{M}$$

which is constant since the **mass (m)** and **molecular weight (M)** are constant, we have

$$\frac{pV}{Tn} = \frac{p_s V_{0s}}{T_0 n} \qquad (4)$$

* For a standard condition,

$$T_0 = 273.15K,\ p_s = 1atm = 1013.25mb = 1.01325\times10^5\, kgm^{-1}s^{-2}$$

$$V_{0s}/n = 22.4\times10^{-3}m^3/mol$$

$$R^* = \frac{p_s V_{0s}}{T_0 n} = 8.3143\, Jmol^{-1}K^{-1}$$

is called the **universal gas constant**.

Now, (4) can be rearranged to get the equation of state for the ideal gas

$$pV = nR^*T \qquad (5)$$

1.7.2 Equivalent forms of ideal gas equation

Ideal gas equation (5) can be written in several forms,

$$pV = \frac{m}{M} R^* T = mRT \qquad (6)$$

$R = \frac{R^*}{M}$ is the **specific gas constant**.

Since the specific volume

$v = \frac{V}{m} = \frac{1}{\rho}$, ρ is the density,

(6) can be also written as

$$pv = RT, \ or \ \ p = \rho RT \qquad (7)$$

1.7.3 Equation of state for mixture of ideal gases

Each gas obeys its own state equation, for the i th gas

$$p_i V_i = m_i R_i T_i \qquad (8)$$

Since in a mixture of gases,

* The partial pressure p_i is:
  the pressure the i th gas would have if the same mass existed alone at the same temperature and occupied the same volume as the mixture;

* The partial volume V_i is:
  the volume the i th gas would occupy if the same mass existed alone at the same temperature and pressure.

(8) can be written in form,

$$p_i V = m_i R_i T \tag{9}$$

Sum (9) over all gases in the mixture, and apply Dalton's law,

$$p = \sum_i p_i$$

we get the equation of state for the mixture,

$$pV = m\overline{R}T \tag{10}$$

which is similar to the ideal gas equation (6).

$$m = \sum_i m_i$$

$$\overline{R} = \frac{\sum_i m_i R_i}{m}$$ is the **mean specific gas constant**

The **mean molecular weight** of the mixture is defined by

$$\overline{M} = \frac{\sum_i n_i M_i}{n} = \frac{\sum_i m_i}{n} = \frac{m}{n} \qquad (11)$$

Since

$$n = \sum_i n_i = \sum_i \frac{m_i}{M_i}$$

(11) can be written as

$$\overline{M} = \frac{R^* m}{\sum_i m_i (R^* / M_i)} = \frac{R^* m}{\sum_i m_i R_i} = \frac{R^*}{\overline{R}}$$

The *absolute concentration* of the i th gas is measured by its **density** ρ_i .

The **molar fraction** is used to measure the *relative concentration* of the i th gas over the *total abundance air* in the mixture,

$$N_i = \frac{n_i}{n}$$

Using the state equations for the i th gas and the mixture of gases, we can also have

$$N_i = \frac{n_i}{n} = \frac{p_i}{p} = \frac{V_i}{V} \qquad (12)$$

The **mass fraction** is also used to measure the relative concentration.

Using $n = m / \overline{M}$, *and* $n_i = m_i / M_i$ in (12), we can get

$$\frac{m_i}{m} = \frac{n_i}{n} \frac{M_i}{\overline{M}} = N_i \frac{M_i}{\overline{M}}$$

The **mixing ratio** is used to measure the relative concentration of the i th gas over *dry air*, e.g., the **mass mixing ratio** is defined in form,

$$r_i = \frac{m_i}{m_d} \tag{13}$$

m_d is the mass of dry air; r_i is dimensionless and expressed in $g\,kg^{-1}$ for tropospheric water vapor.

Since the mass of air in the presence of *water vapor and ozone* is virtually identical to the mass of dry air, (13) can be related to the molar fraction,

$$r_i = \frac{m_i}{m_d} = \frac{n_i M_i}{n_d M_d} = \frac{n_i}{n}\frac{n}{n_d}\frac{M_i}{M_d} \cong N_i \varepsilon_i \tag{14}$$

where $\varepsilon_i = \dfrac{M_i}{M_d}, \qquad \dfrac{n}{n_d} \cong 1$

We can also have the **volume mixing ratio** related to the molar fraction,

$$\frac{V_i}{V_d} = \frac{V_i}{V - V_i} = \frac{N_i}{1 - N_i} \tag{15}$$

1.8 Atmospheric composition

Atmospheric air is composed of

1) A mixture of gases (**Nitrogen, Oxygen**, Argon and Carbon dioxide etc.)

   * Remarkably constant up to 100 km height (except for CO2);

   * These four gases are the main components of **dry air**.

   The specific gas constant: $R_d = 287.05\ J\,kg^{-1}\,K^{-1}$
   The mean molecular weight: $M_d = 28.96\, g\, mol^{-1}$

2) Water substance in any of its three physical states (**vapor**, **droplets** and **ice particles**)

   * very important in radiative processes, cloud formation and interaction with the oceans, and highly variable.

3) Solid or liquid particles of very small size (atmospheric aerosols)

2

Laws of Perfect Gases

I. Characteristics of a Gas

A) Gases assume the shape and volume of a container.

B) Gases are the most compressible of all the states of matter.

C) Gases will mix evenly and completely when confined to the same container.

D) Gases have lower densities than liquids or solids.

The Gas Laws

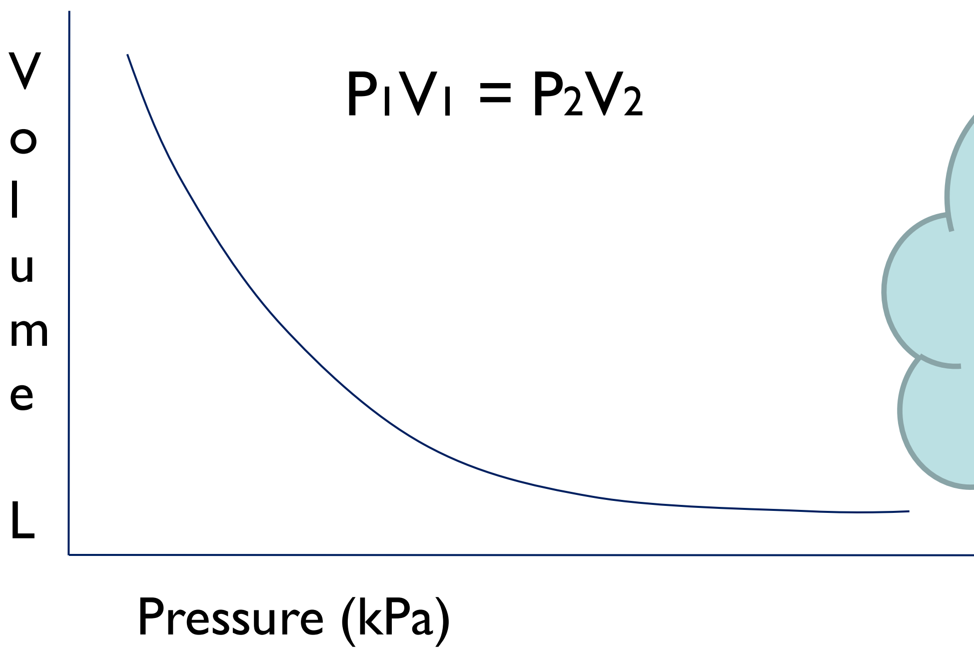
1. Boyle’s Law
2. Charles’ Law
3. Gay-Lussac’s Law
4. Avogadro’s Law

Boyle's Law

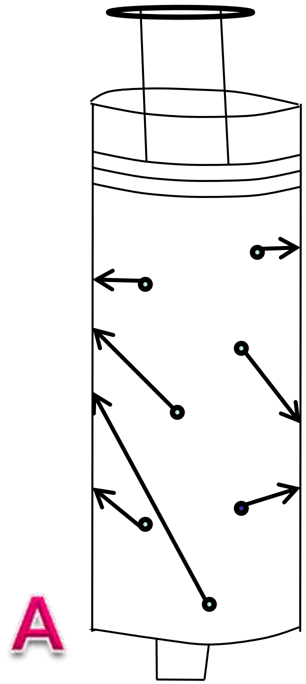
Boyle's Law – at **constant temperature**, the volume of the gas increases as the pressure decreases. The volume of the gas decreases and the pressure increases.

V↑ P↓

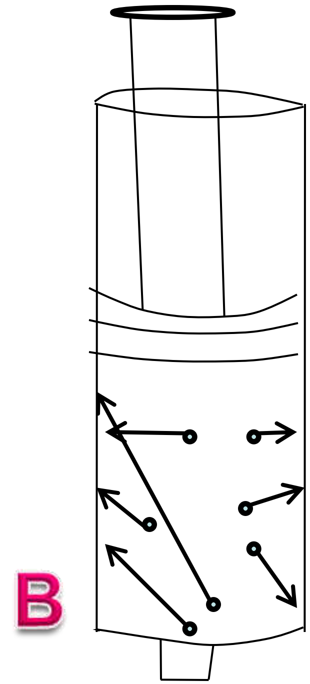
$$P_1V_1 = P_2V_2$$



If you squeeze a gas sample, you make its volume smaller.



Moveable piston



Now . . . a container where the volume can change (syringe)

Same temperature

Volume is 100 mL at 25°C

Volume is 50 mL at 25°C

In which system is the pressure higher? (Which has the greater number of collisions with the walls and each other?)

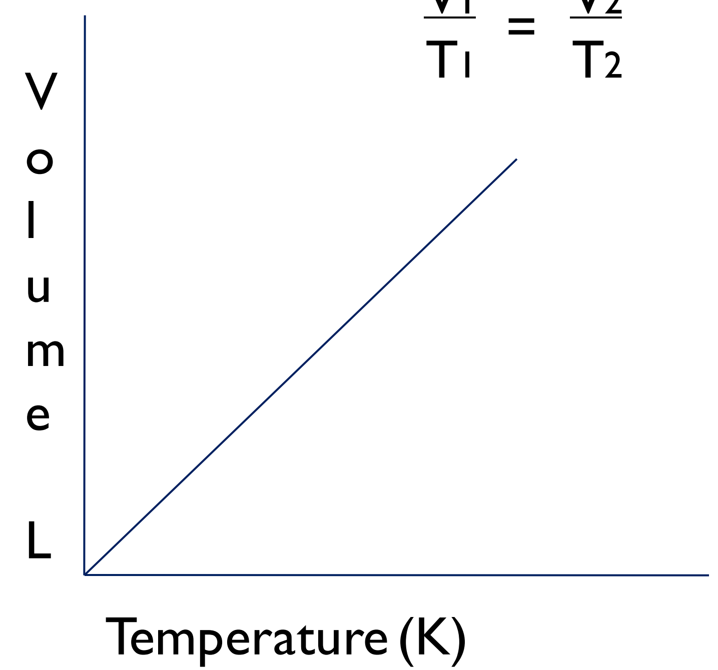
B

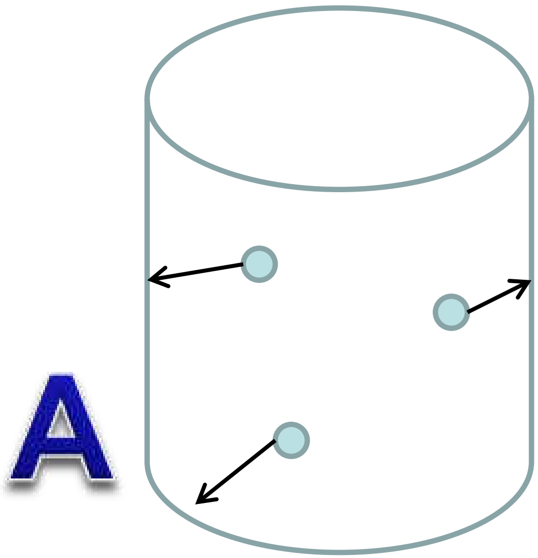
Charles' Law

Charles' Law – at a **constant pressure**, the volume of a gas increases as the temperature of the gas increases and the volume decreases when the temperature decreases.

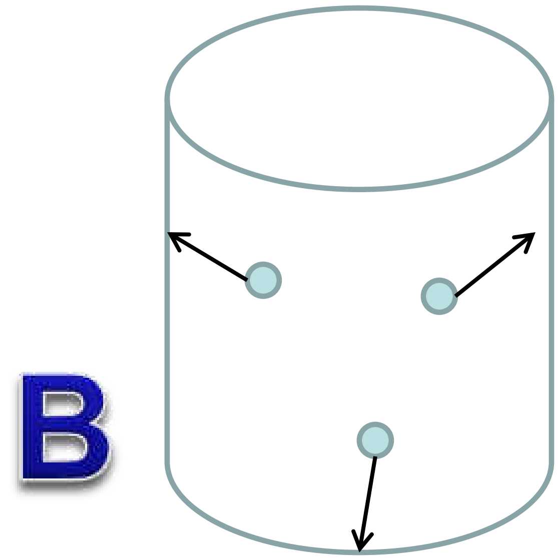
- increase AKE
- increase the speed of the particles
- the walls of a flexible container expand – think of hot air balloons!

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$





Steel cylinder (2L) contains 500 molecules of O_2 at 400 K



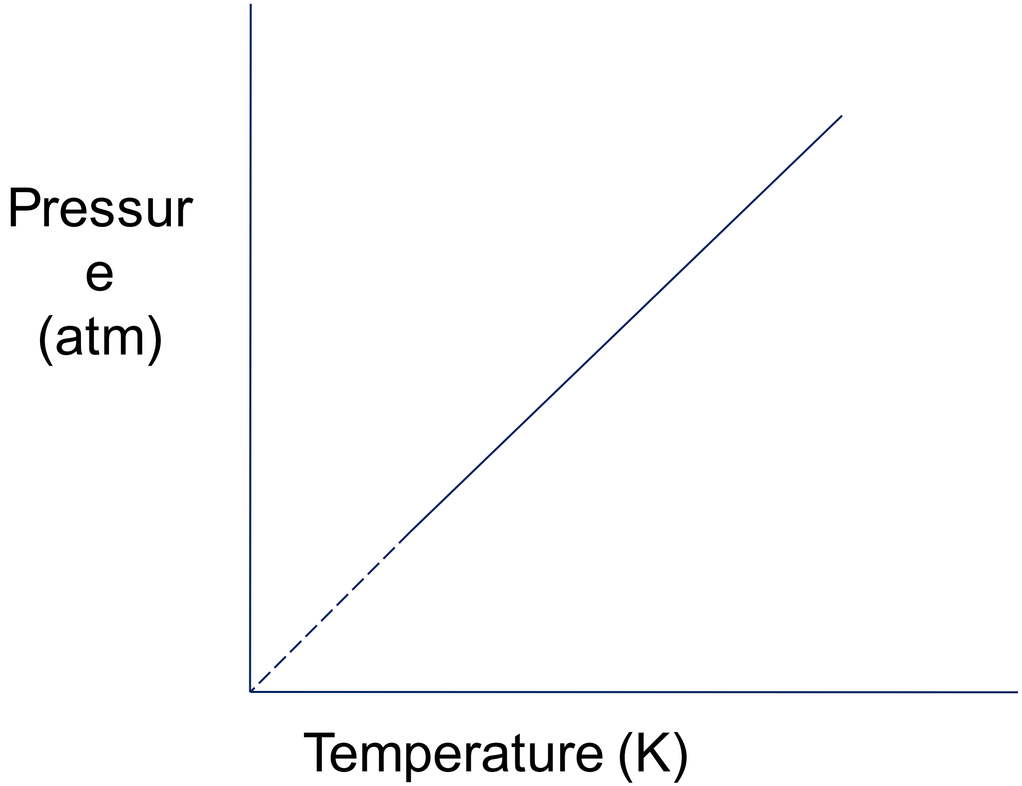
Steel cylinder (2L) contains 500 molecules of O_2 at 800 K

1. In which system do the O_2 molecules have the highest average kinetic energy? B
2. In which system will the particles collide with the container walls with the greatest force? B
3. In which system is the pressure higher? B

Gay-Lussac's Law

Gay-Lussac's Law – the pressure of a gas is directly proportional to its absolute temperature at a constant volume.

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$



To remember which constants go with which law . . .

Boyle's **L**aw – **T**emperature is constant **<u>BLT</u>**

Charles' Law – **P**ressure is constant **<u>C</u>heese <u>P</u>izza**

Gay-Lussac's Law – **V**olume is constant **<u>G</u>reen <u>V</u>eggies**

Combined Gas Law

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

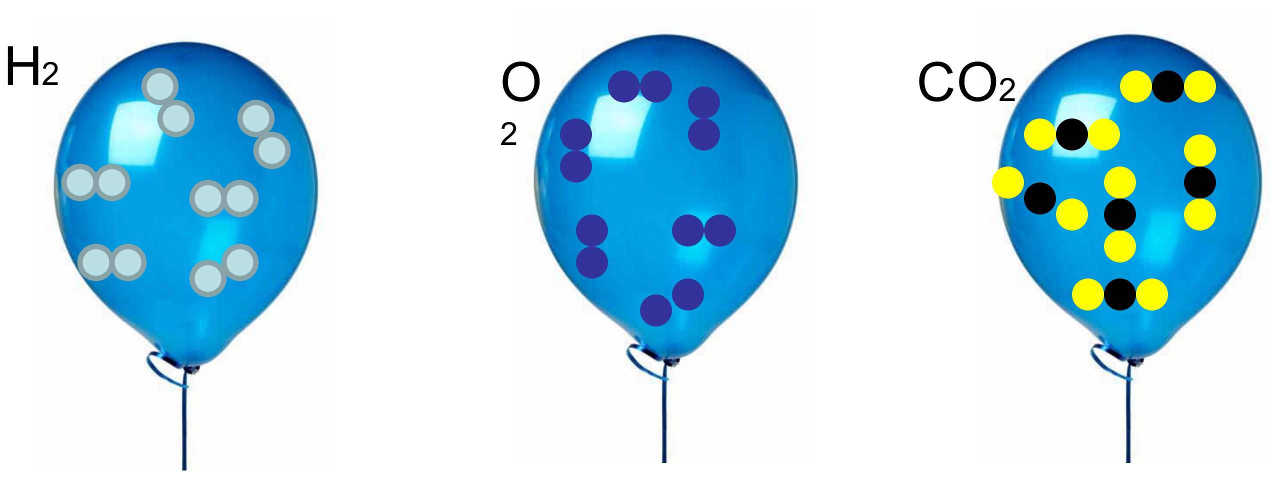
The equation is found on **Table T**.
Note that all temperatures must be in Kelvin!

Units used to describe gas samples:

Volume	Temperature	Pressure
Liter (L)	Kelvin **ONLY**	Atmosphere (atm)
Milliliter (mL)		Kilopascale (kPa)
1000 mL = 1L		1 atm = 101.3 kPa
		1 atm = 760 mm Hg
		1 atm = 760 torr

Avogadro's Law

Avogadro's Law – equal volumes of gases at the same temperature and pressure contain equal numbers of molecules.



1 mole of ANY gas takes up a volume of 22.4 L at STP.

Ideal Gases

- Gases whose behavior can be predicted by the kinetic molecular theory are called ideal, or perfect, gases. No gases are truly ideal because no gas totally obeys all of the gas laws.

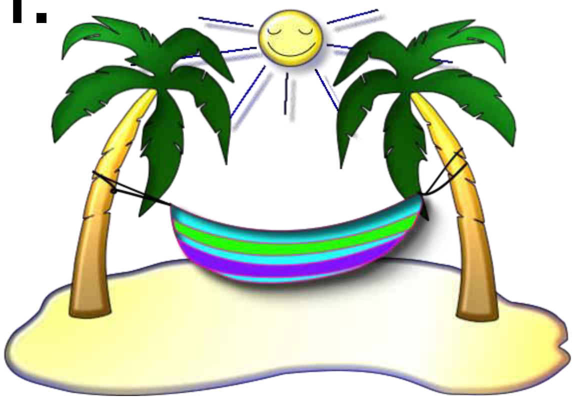
- An ideal gas is an imaginary gas that is perfect and does follow everything perfectly.

Ideal Gases, continued

- An ideal gas does not condense to a liquid at low temperatures
- An ideal gas does not have forces of attraction or repulsion between particles
- An ideal gas is composed of particles that have no volume.

Real Gas Vs. Ideal Gas

- A real gas is most like an ideal gas when the real gas is at a low pressure and a high temperature.
- The gases that act most like ideal gases are the small mass ones – hydrogen and helium.



Diffusion

- movement of particles from areas of **high** concentrations to areas of **low** concentration.

- Gases diffuse and mix with other gases very rapidly due to their rapid motion. (Think ammonia, tuna or skunk smell.)

- It eventually reaches equilibrium and the mixture is homogeneous.

Entropy is the randomness of particles.

Effusion – the passage of gas under pressure through a small opening. (Gases effuse through a hole in your tire!)

Two More Laws!!

Graham's Law – Particles of low molar mass travel faster than heavier particles.

Hydrogen effuses 4 times faster than oxygen.

Dalton's Law of Partial Pressure -

In a mixture of gases, each gas exerts a certain pressure as if it were alone. The pressure of each one of these gases is called the partial pressure. The total pressure of a mixture of gases is the sum of all of the partial pressures.

$$P_{total} = P_A + P_B + P_C$$

Example:

A closed cylinder contains 3L of He, 1L of H_2 and the total pressure in the system is 800 torr. What is the partial pressure of the He?

$$P_{total} = P_A + P_B + P_C$$

3L + 1L = 4L **AND** 4L = 800 torr

$$\frac{4L}{4} = \frac{800\ torr}{4} \rightarrow 1L = 200\ torr$$

1L H2	200 torr
<u>3L He</u>	<u>600 torr</u>
4L gas	800 torr

3

Thermodynamic Processes

Thermodynamic Processes

• States of a thermodynamic system can be changed by interacting with its surrounding through work and heat. When this change occurs in a system, it is said that the system is undergoing a **process**.

• A thermodynamic **cycle** is a sequence of different processes that begins and ends at the same thermodynamic state.

• Some sample processes:
- ➔ Isothermal process: temperature is constant T=C
- ➔ Isobaric process: pressure is constant, P=C
- ➔ Isentropic process: entropy is constant, s=C
- ➔ Constant-volume process, v=C
- ➔ Adiabatic process: no heat transfer, Q=0

Process-1

- Use ideal gas assumption (closed system):

$\longrightarrow$ Isothermal process: T=constant

Energy balance ΔU=Q-W, for ideal gas ΔU=ΔH=0

since both are functions of temperature only

$$\text{Q=W,}\quad \text{W=}\int \text{P}dV = \int \frac{mRT}{V}dV = mRT\int_1^2 \frac{dV}{V}$$

$$= mRT\ln\left(\frac{V_2}{V_1}\right) = mRT\ln\left(\frac{P_1}{P_2}\right)$$

$\longrightarrow$ Isobaric process: P=constant

$$\Delta\text{U=Q-W, W=}\int \text{PdV=P}\int_1^2 \text{dV=P(V}_2 - V_1)$$

$$Q = \Delta U + P(V_2 - V_1) = (U_2 - U_1) + P(V_2 - V_1)$$

$$= (U_2 + PV_2) - (U_1 + PV_1) = H_2 - H_1 = \Delta H$$

Process-2

$\longrightarrow$ Constant volume process: V=constant

Q-W=ΔU, W=$\int PdV = 0$, no work done

Q=ΔU=mΔu=m$\int c_v dT$

$\longrightarrow$ Adiabatic process: Q=0

Q-W=ΔU, -W=ΔU

-δW=dU (infinitesimal increment of work and energy)

dU+PdV=0, $mc_v dT + \left(\frac{mRT}{V}\right) dV = 0$

$c_v dT + \left(\frac{RT}{V}\right) dV = 0, \ \frac{c_v}{R}\frac{dT}{T} = -\frac{dV}{V}$, integrate and assume

c_v=constant

$$\frac{c_v}{R}\ln\left(\frac{T_2}{T_1}\right) = -\ln\left(\frac{V_2}{V_1}\right), \ \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{R/c_v} = \left(\frac{V_1}{V_2}\right)^{k-1}$$

Process-3

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1} \text{, from ideal gas relation}$$

$$\text{PV=RT, } \left(\frac{V_1}{V_2}\right) = \left(\frac{T_1}{T_2}\right)\frac{P_2}{P_1}\text{, substitute}$$

$$\frac{T_2}{T_1} = \left(\left(\frac{T_1}{T_2}\right)\left(\frac{P_2}{P_1}\right)\right)^{k-1} \text{, multiply } \left(\frac{T_2}{T_1}\right)^{k-1} \text{ from both sides}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k}, \; and \; \left(\frac{P_2}{P_1}\right) = \left(\frac{V_1}{V_2}\right)^{k}$$

Also $P_1 V_1^{\mathrm{k}} = P_2 V_2^{k}$ and $pV^{k} = cons\tan t$

For an ideal gas undergoing adiabatic process

Process-4

Polytropic Process: its P-V relation can be expressed as
PV^n = constant, where n is a constant for a specific process

- Isothermal, T=constant, if the gas is an ideal gas then PV=RT=constant, n=1
- Isobaric, P=constant, n=0 (for all substances)
- Constant-volume, V=constant, V=constant$(P)^{(1/n)}$, n=∞ (for all substances)
- Adiabatic process, n=k for an ideal gas

$$P_1V_1^n = P_2V_2^n = PV^n$$

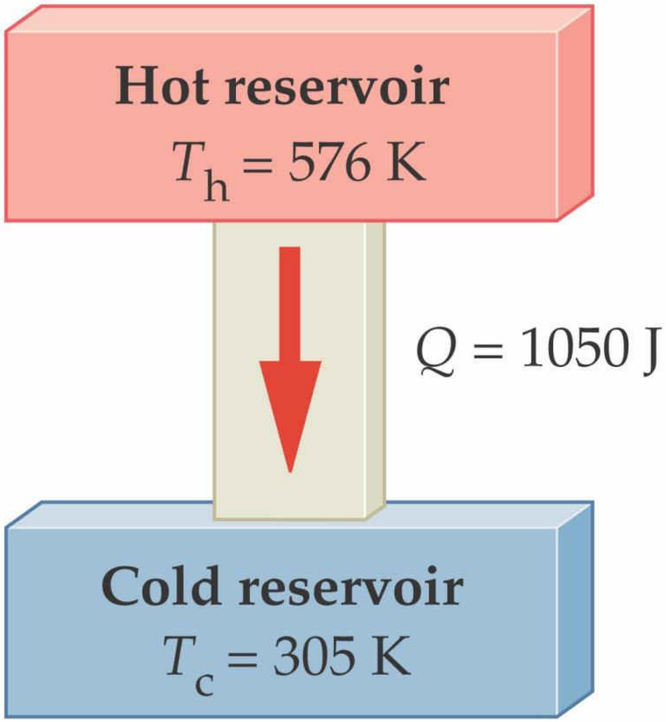
$$W = \int_1^2 PdV = \int_1^2 (P_1V_1^n)V^{-n}dV$$

$$= (P_1V_1^n)\int_1^2 V^{-n}dV = \frac{(P_1V_1^n)}{1-n}(V_2^{1-n} - V_1^{1-n}) = \frac{P_2V_2 - P_1V_1}{1-n}$$

4

Laws of Thermodynamics

The Laws of Thermodynamics



- The Zeroth Law of Thermodynamics
- The First Law of Thermodynamics
- Thermal Processes
- Specific Heats for an Ideal Gas: Constant Pressure, Constant Volume
- The Second Law of Thermodynamics
- Heat Engines and the Carnot Cycle

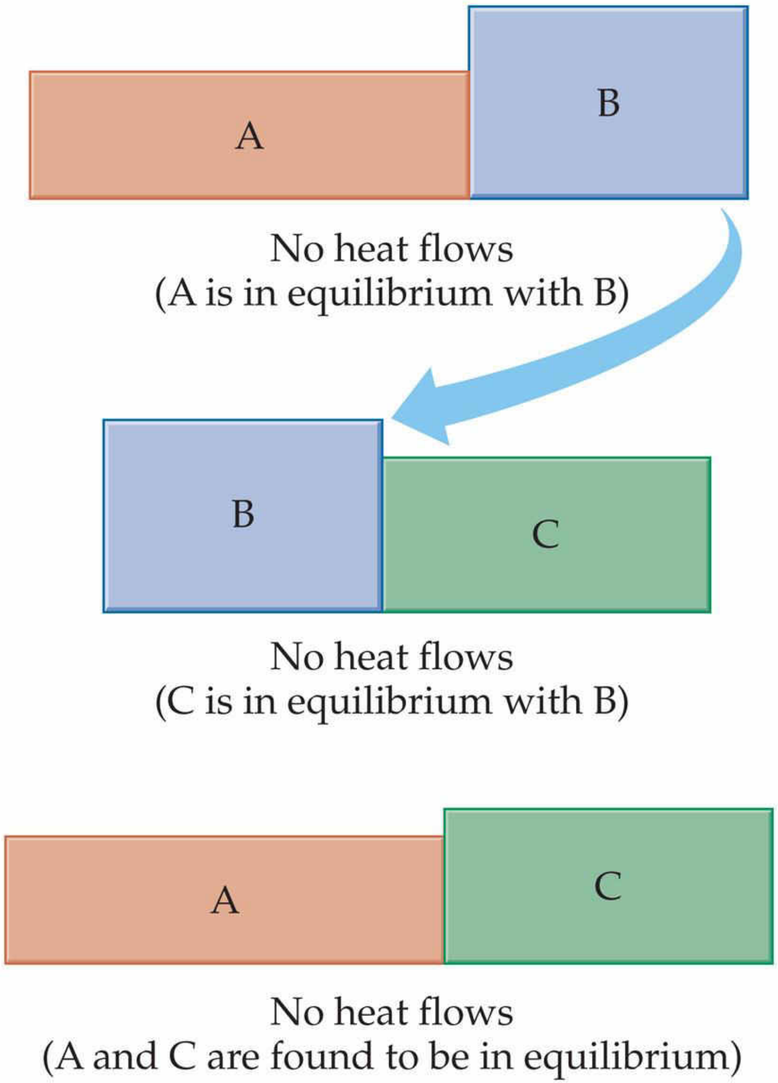
Units of Chapter 18

- Refrigerators, Air Conditioners, and Heat Pumps
- Entropy
- Order, Disorder, and Entropy
- The Third Law of Thermodynamics

The Zeroth Law of Thermodynamics

We have already discussed the zeroth law, and include it here for completeness:

If object A is in thermal equilibrium with object C, and object B is separately in thermal equilibrium with object C, then objects A and B will be in thermal equilibrium if they are placed in thermal contact.

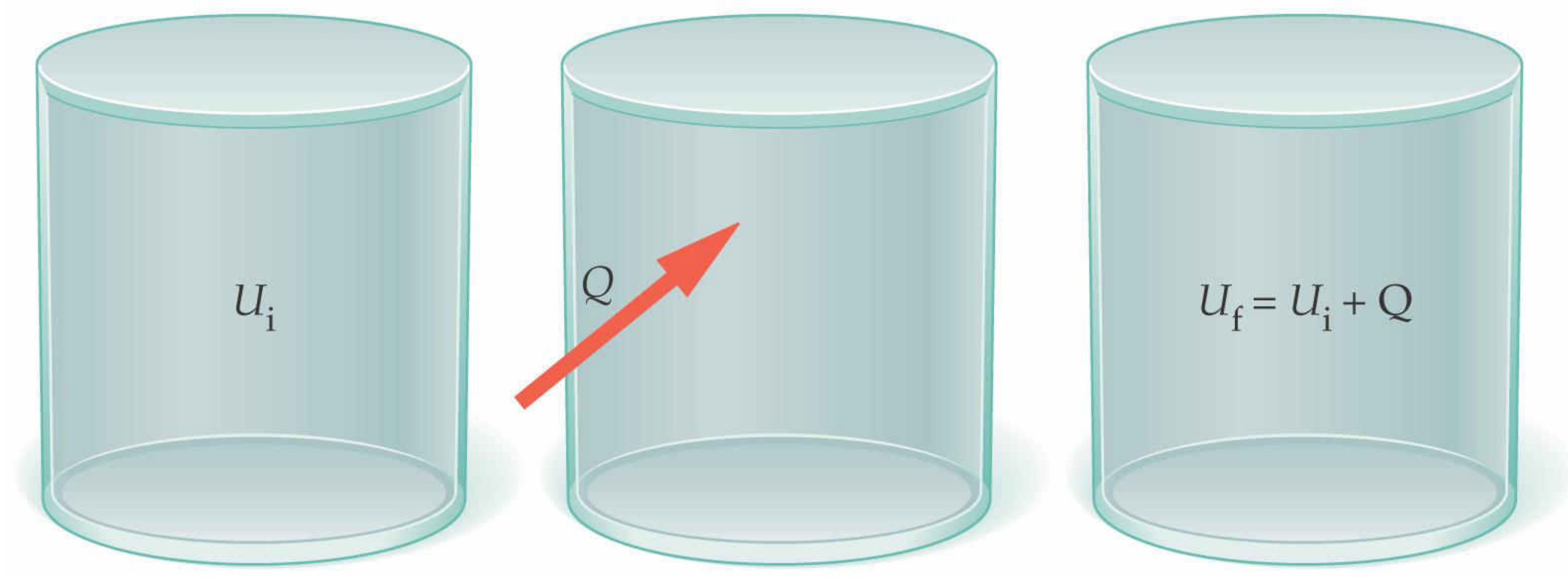


The First Law of Thermodynamics

The first law of thermodynamics is a statement of the conservation of energy.

If a system's volume is constant, and heat is added, its internal energy increases.

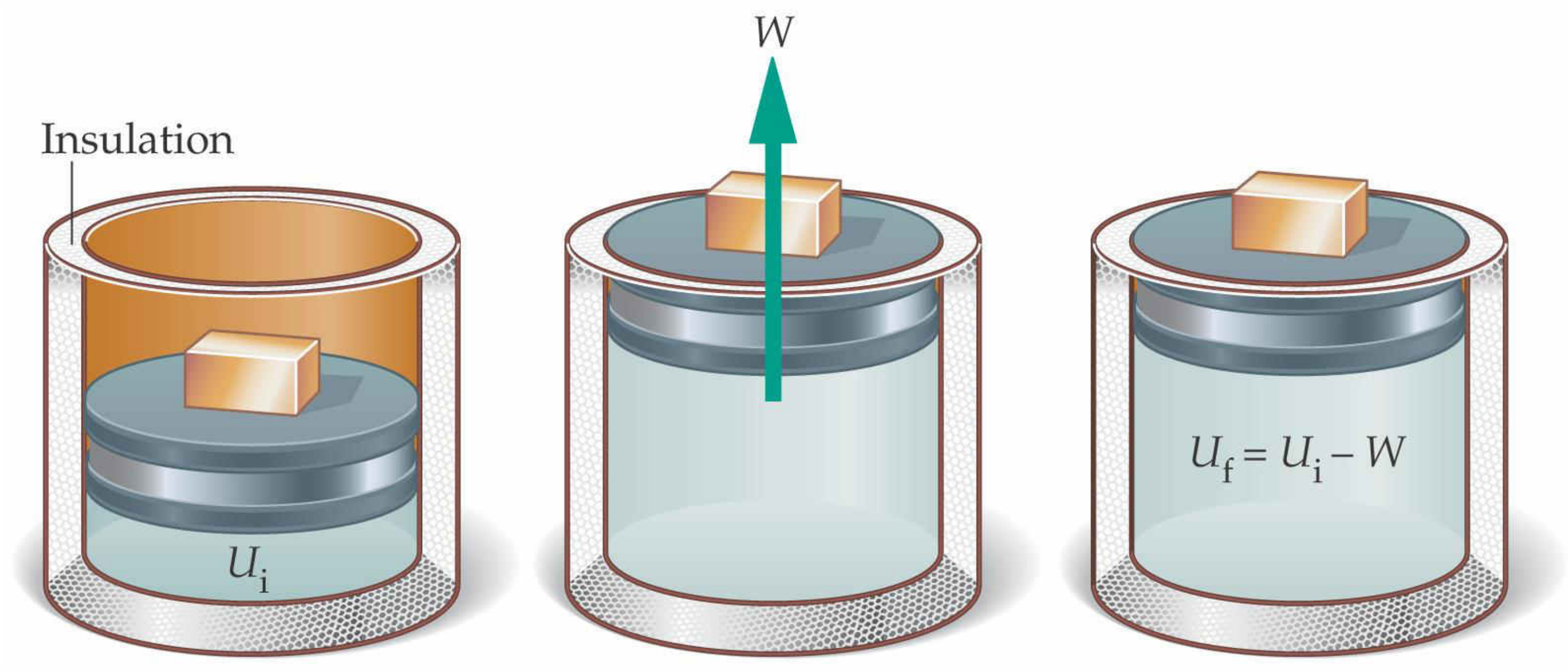
$$\Delta U = U_f - U_i = Q$$



The First Law of Thermodynamics

If a system does work on the external world, and no heat is added, its internal energy decreases.

$$\Delta U = U_f - U_i = -W$$



18-2 The First Law of Thermodynamics

Combining these gives the first law of thermodynamics. The change in a system's internal energy is related to the heat Q and the work W as follows:

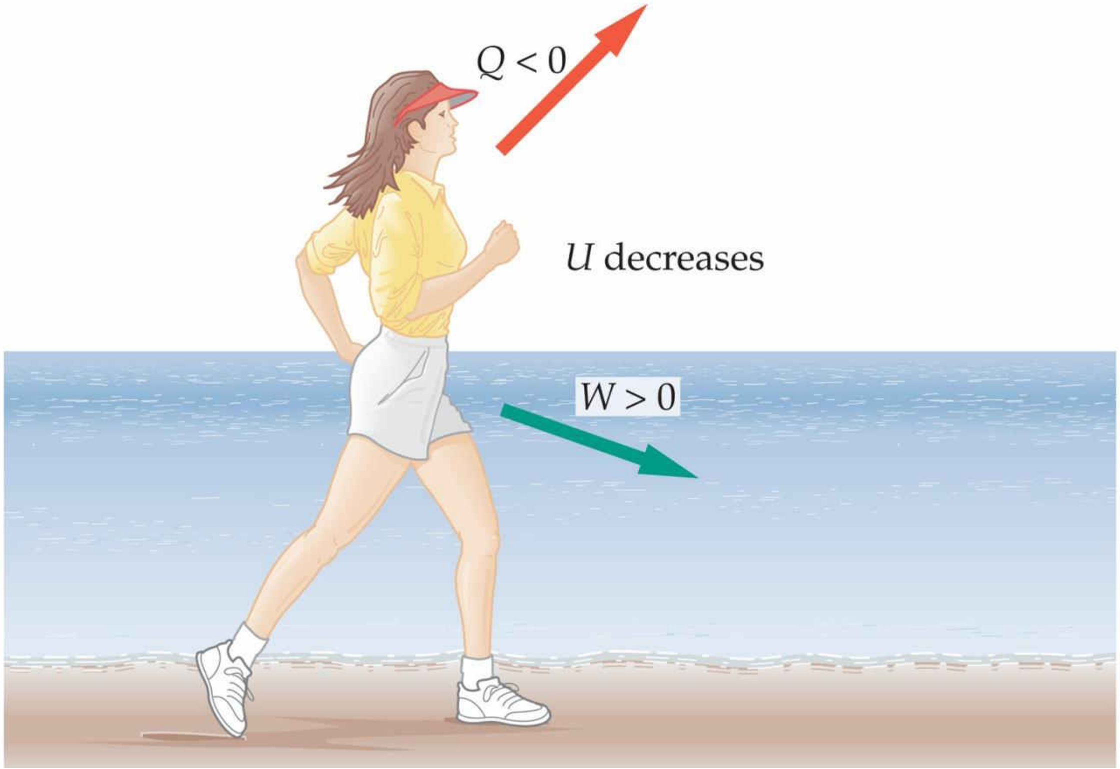
$$\Delta U = Q - W$$

It is vital to keep track of the signs of Q and W.

TABLE 18–1 *Signs of Q and W*	
Q positive	System *gains* heat
Q negative	System *loses* heat
W positive	Work done *by* system
W negative	Work done *on* system

The First Law of Thermodynamics

The internal energy of the system depends only on its temperature. The work done and the heat added, however, depend on the details of the process involved.



Thermal Processes

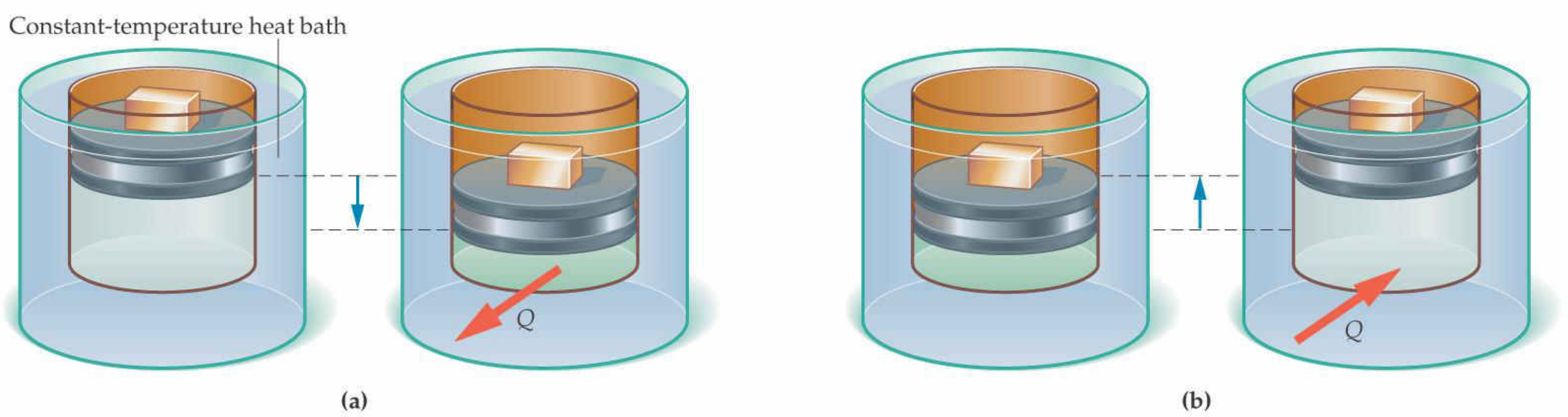
We will assume that all processes we discuss are quasi-static – they are slow enough that the system is always in equilibrium.

We also assume they are reversible:

For a process to be reversible, it must be possible to return both the system and its surroundings to exactly the same states they were in before the process began.

Thermal Processes

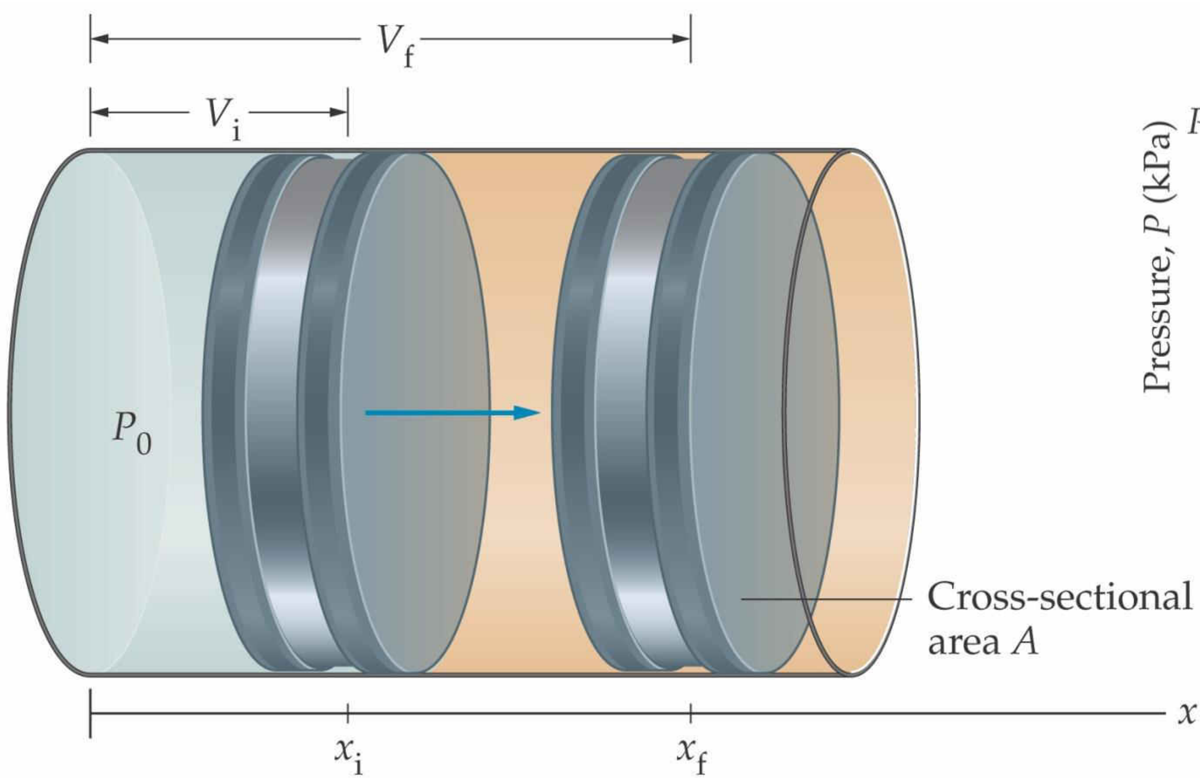
This is an idealized reversible process. The gas is compressed; the temperature is constant, so heat leaves the gas. As the gas expands, it draws heat from the reservoir, returning the gas and the reservoir to their initial states. The piston is assumed frictionless.

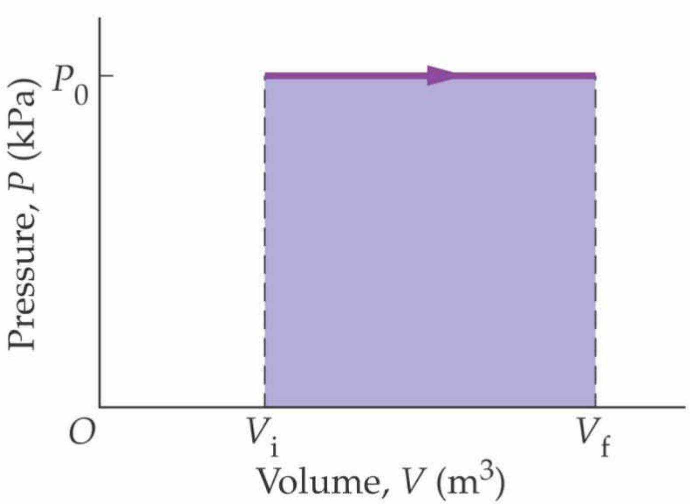


Thermal Processes

Work done by an expanding gas, constant pressure:

$$W = P\Delta V \qquad (constant\ pressure)$$

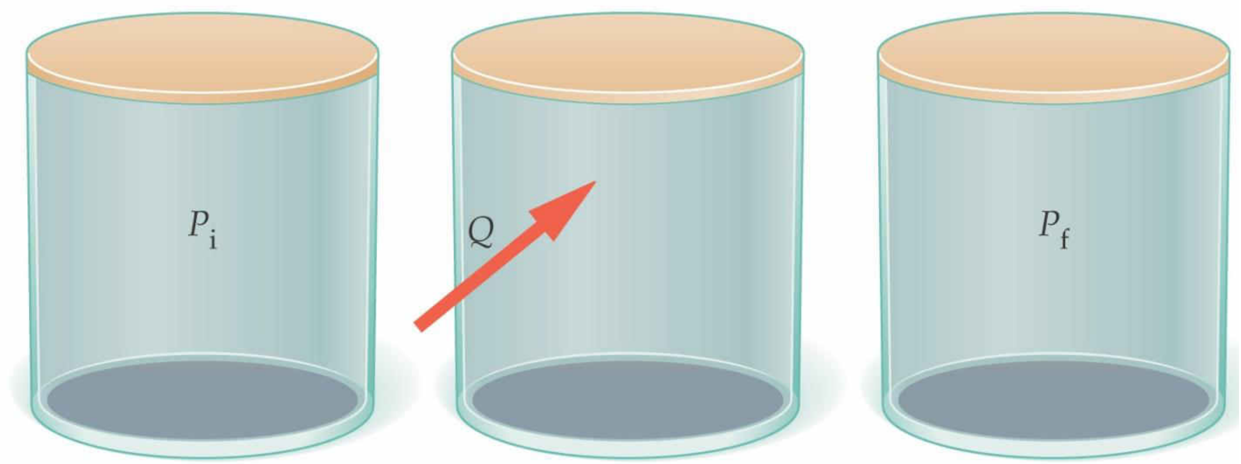


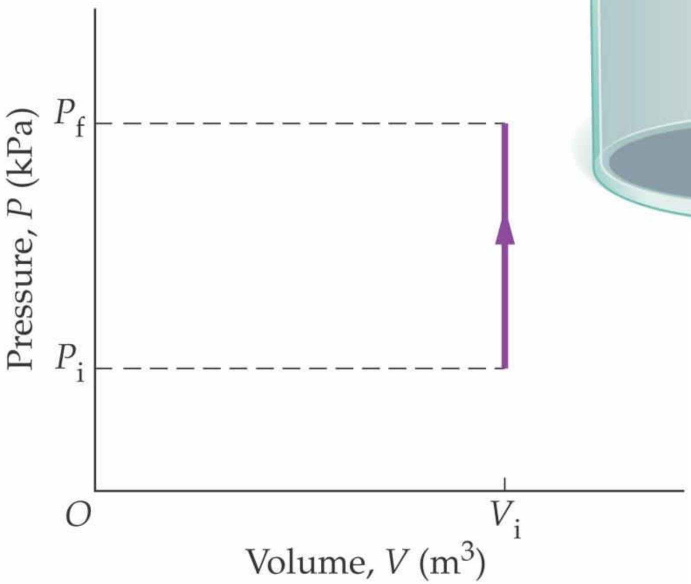


Thermal Processes

If the volume stays constant, nothing moves and no work is done.

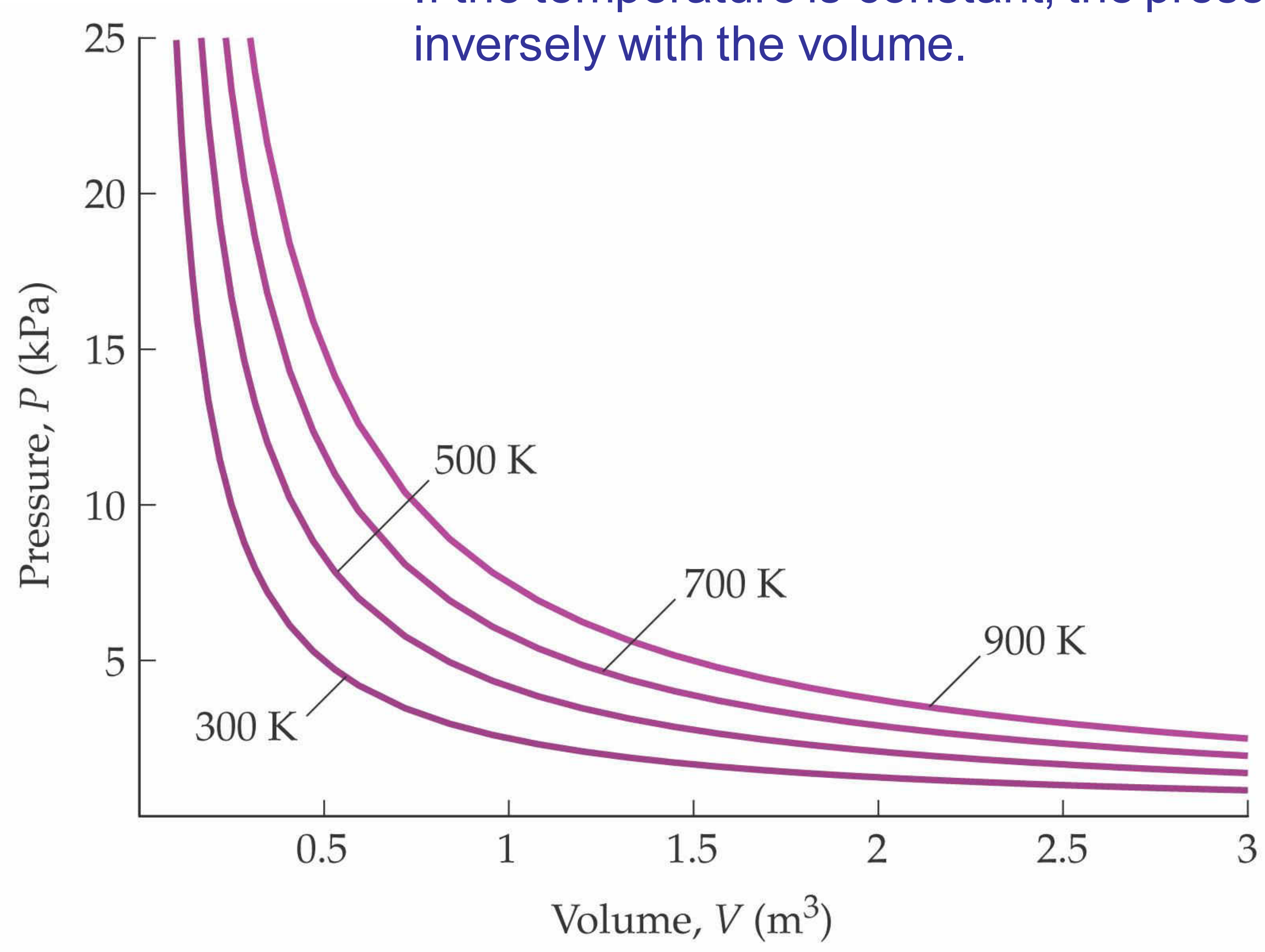
$$W = 0 \qquad (\textit{constant volume})$$





Thermal Processes

If the temperature is constant, the pressure varies inversely with the volume.

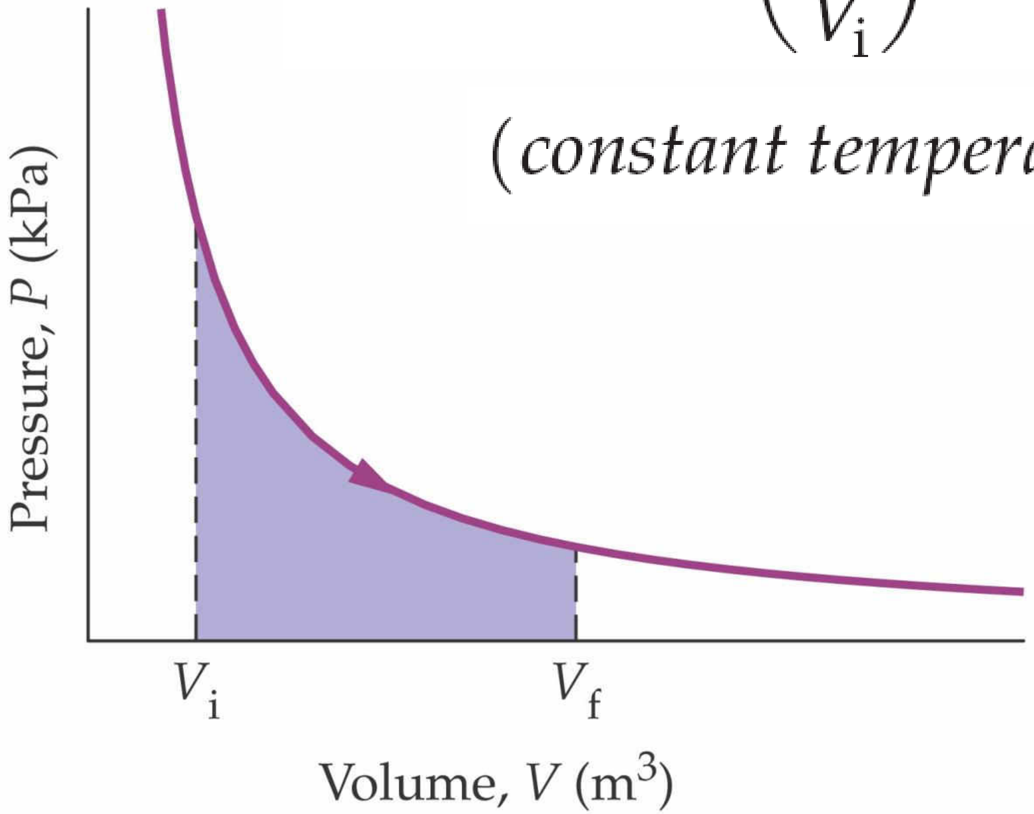


Thermal Processes

The work done is the area under the curve:

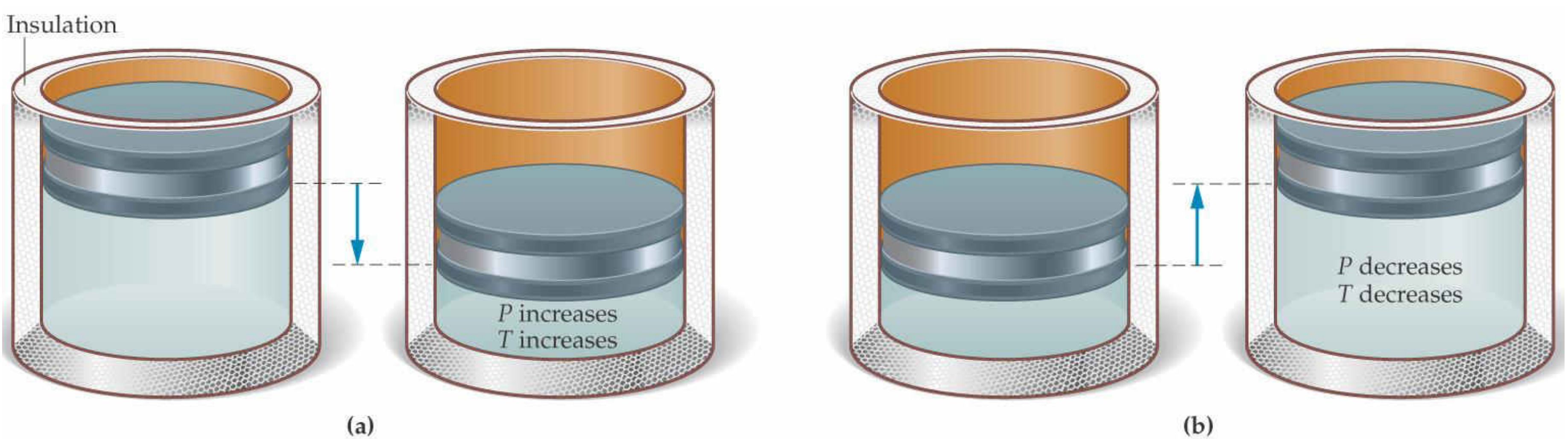
$$W = NkT \ln\left(\frac{V_f}{V_i}\right) = nRT \ln\left(\frac{V_f}{V_i}\right)$$

$$(constant\ temperature)$$



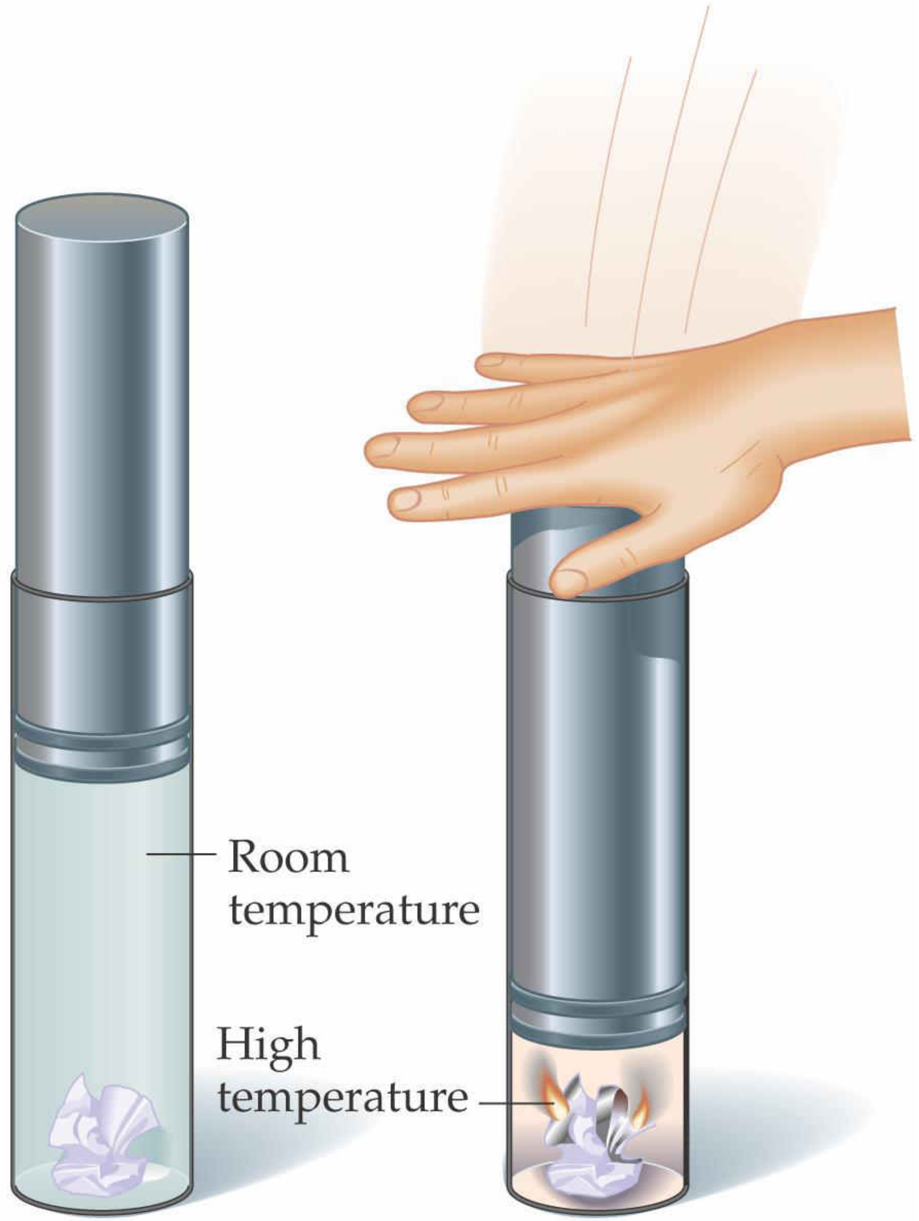
18-3 Thermal Processes

An adiabatic process is one in which no heat flows into or out of the system. The adiabatic P-V curve is similar to the isothermal one, but is steeper. One way to ensure that a process is adiabatic is to insulate the system.



Thermal Processes

Another way to ensure that a process is effectively adiabatic is to have the volume change occur very quickly. In this case, heat has no time to flow in or out of the system.



Thermal Processes

Here is a summary of the different types of thermal processes:

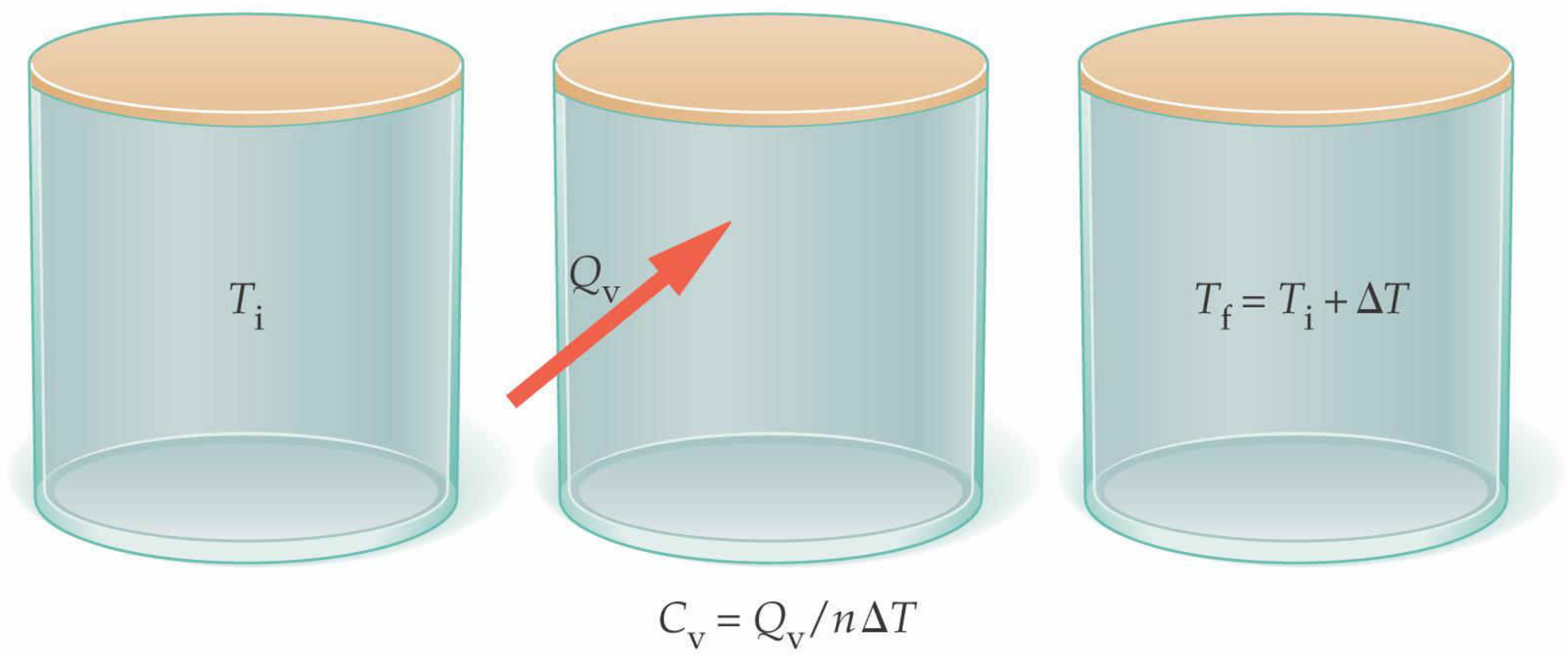
TABLE 18–2 Thermodynamic Processes and Their Characteristics

Constant pressure	$W = P\Delta V$	$Q = \Delta U + P\Delta V$
Constant volume	$W = 0$	$Q = \Delta U$
Isothermal (constant temperature)	$W = Q$	$\Delta U = 0$
Adiabatic (no heat flow)	$W = -\Delta U$	$Q = 0$

18-4 Specific Heats for an Ideal Gas: Constant Pressure, Constant Volume

Specific heats for ideal gases must be quoted either at constant pressure or at constant volume. For a constant-volume process,

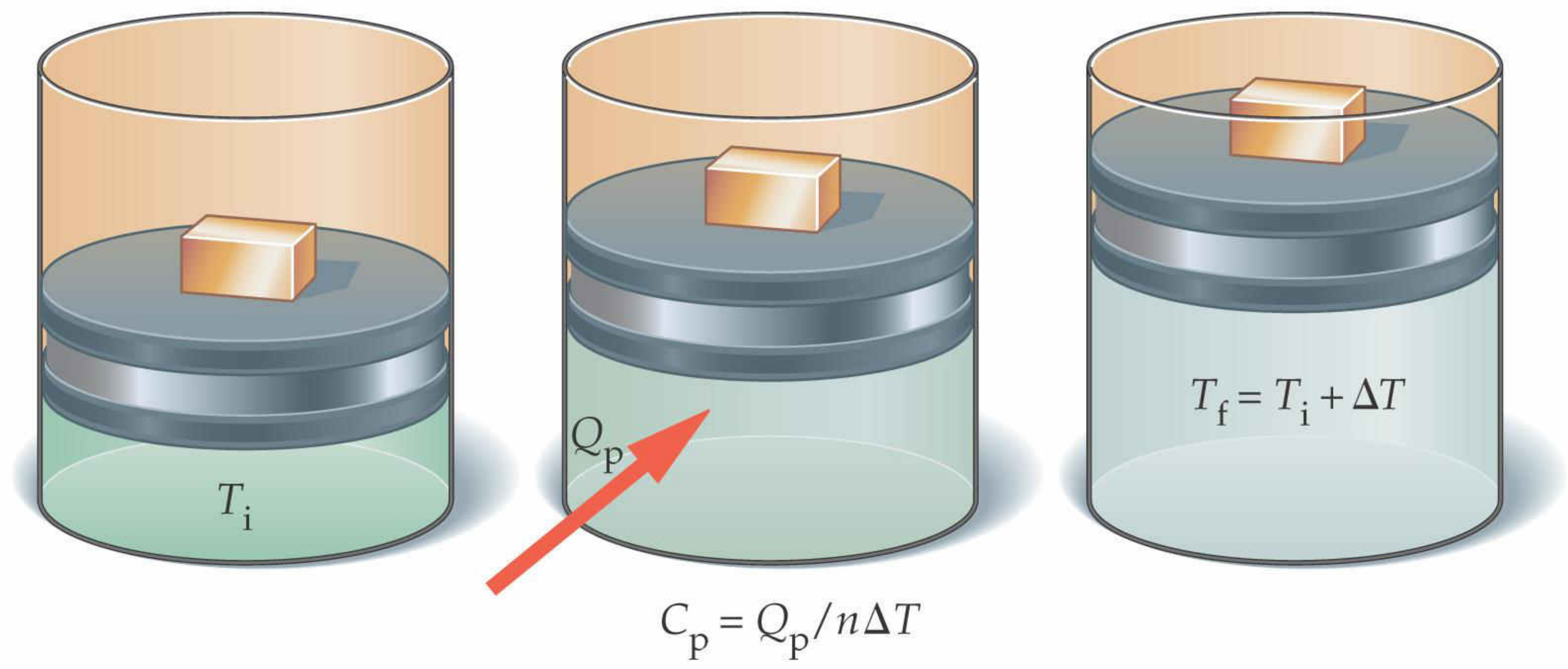
$$Q_V = nC_V \Delta T$$



Specific Heats for an Ideal Gas: Constant Pressure, Constant Volume

At constant pressure,

$$Q_p = nC_p \Delta T$$



Specific Heats for an Ideal Gas: Constant Pressure, Constant Volume

Both C_V and C_P can be calculated for a monatomic ideal gas using the first law of thermodynamics.

$$C_V = \frac{3}{2}R$$

$$C_p = \frac{5}{2}R$$

$$C_p - C_v = R$$

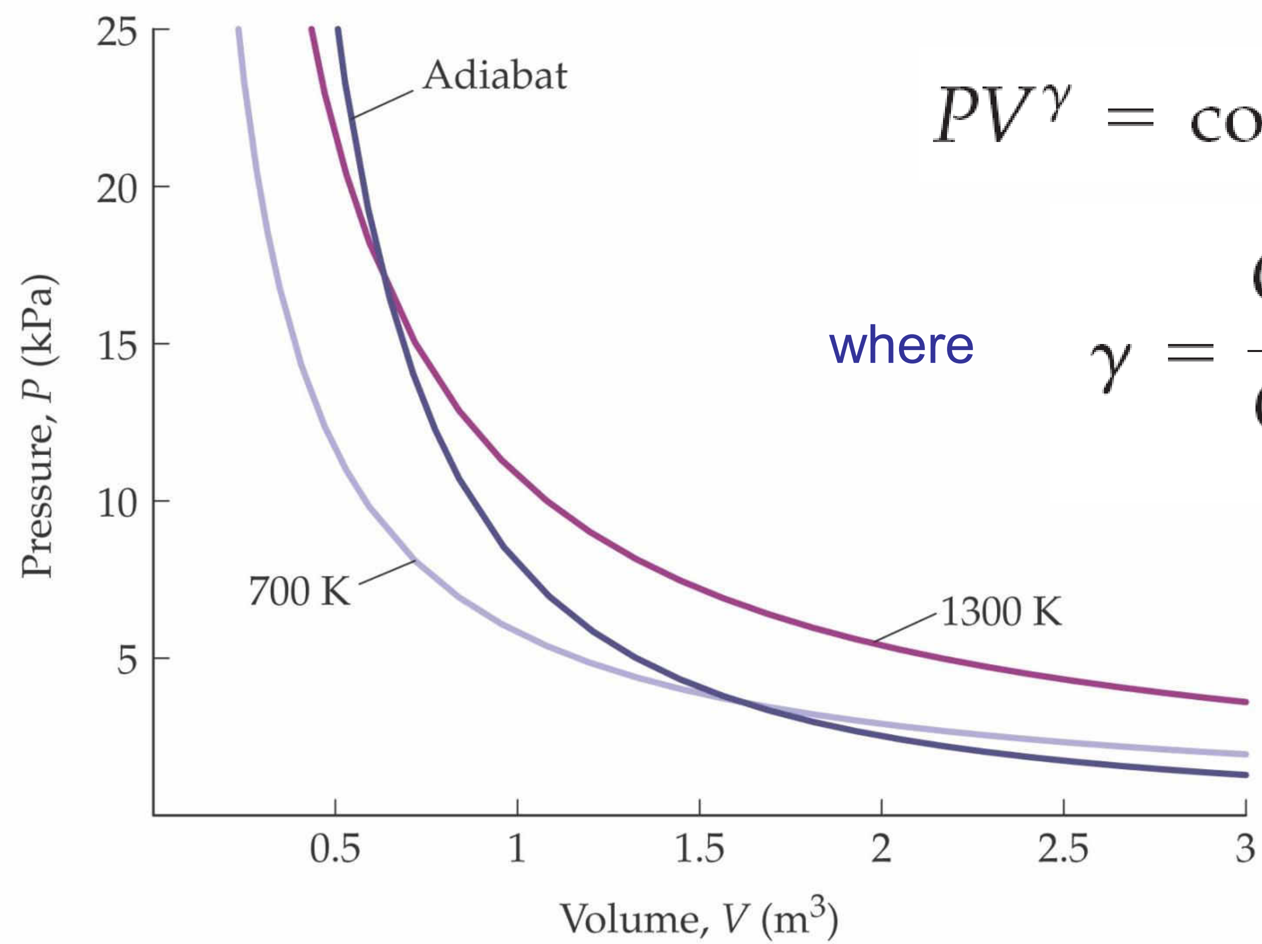
Specific Heats for an Ideal Gas: Constant Pressure, Constant Volume

Although this calculation was done for an ideal, monatomic gas, it works well for real gases.

TABLE 18–3 $C_p - C_v$ for Various Gases	
Helium	0.995 R
Nitrogen	1.00 R
Oxygen	1.00 R
Argon	1.01 R
Carbon dioxide	1.01 R
Methane	1.01 R

Specific Heats for an Ideal Gas: Constant Pressure, Constant Volume

The P-V curve for an adiabat is given by



$$PV^{\gamma} = \text{constant}$$

where $$\gamma = \frac{C_p}{C_v} = \frac{\frac{5}{2}R}{\frac{3}{2}R} = \frac{5}{3}$$

The Second Law of Thermodynamics

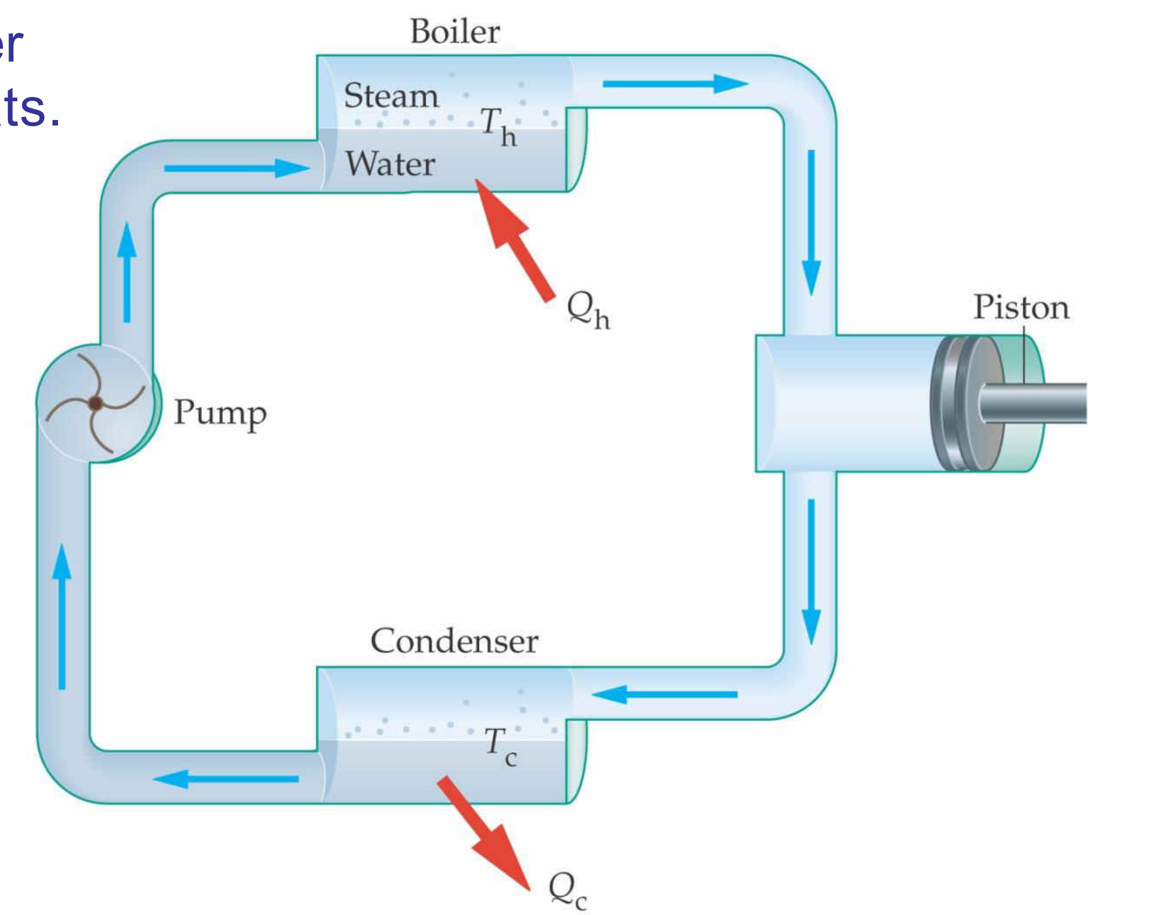
We observe that heat always flows spontaneously from a warmer object to a cooler one, although the opposite would not violate the conservation of energy. This direction of heat flow is one of the ways of expressing the second law of thermodynamics:

When objects of different temperatures are brought into thermal contact, the spontaneous flow of heat that results is always from the high temperature object to the low temperature object. Spontaneous heat flow never proceeds in the reverse direction.

18-6 Heat Engines and the Carnot Cycle

A heat engine is a device that converts heat into work. A classic example is the steam engine. Fuel heats the water; the vapor expands and does work against the piston; the vapor

condenses back into water again and the cycle repeats.

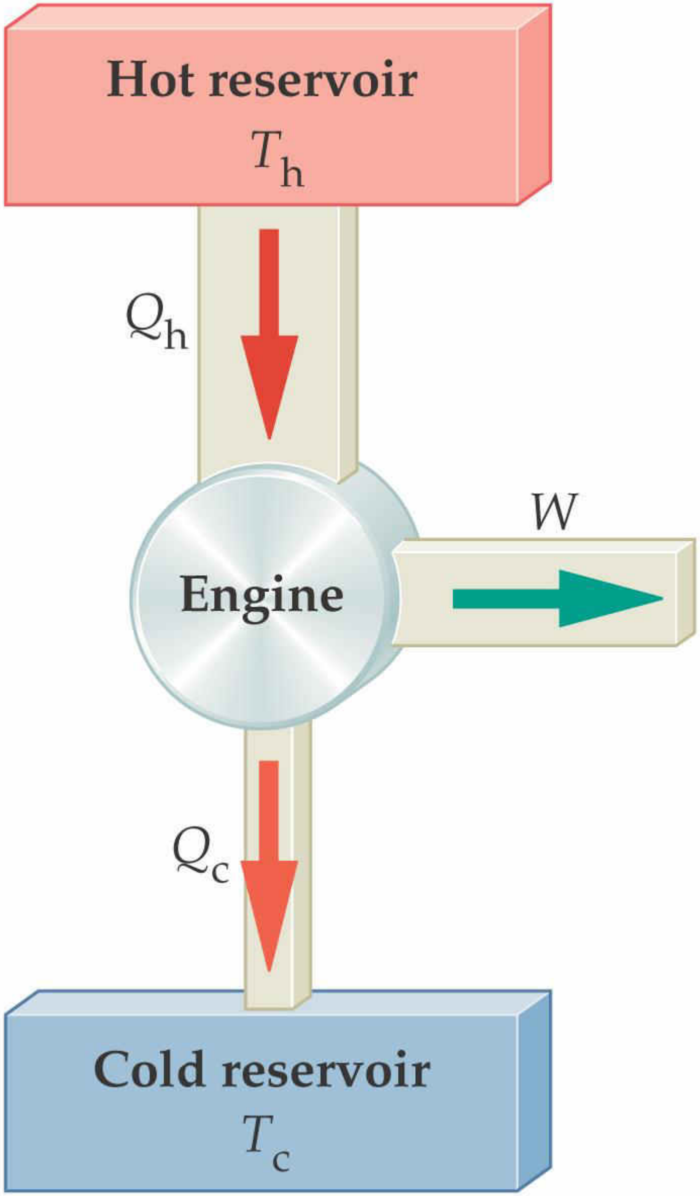


Heat Engines and the Carnot Cycle

All heat engines have:

- a high-temperature reservoir
- a low-temperature reservoir
- a cyclical engine

These are illustrated schematically here.



Heat Engines and the Carnot Cycle

An amount of heat Q_h is supplied from the hot reservoir to the engine during each cycle. Of that heat, some appears as work, and the rest, Q_c, is given off as waste heat to the cold reservoir.

$$W = Q_h - Q_c$$

The efficiency is the fraction of the heat supplied to the engine that appears as work.

$$e = \frac{W}{Q_h}$$

18-6 Heat Engines and the Carnot Cycle

The efficiency can also be written:

Efficiency of a Heat Engine, *e*

$$e = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h}$$

SI unit: dimensionless

In order for the engine to run, there must be a temperature difference; otherwise heat will not be transferred.

Heat Engines and the Carnot Cycle

The maximum-efficiency heat engine is described in Carnot's theorem:

If an engine operating between two constant-temperature reservoirs is to have maximum efficiency, it must be an engine in which all processes are reversible. In addition, all reversible engines operating between the same two temperatures, T_c and T_h, have the same efficiency.

This is an idealization; no real engine can be perfectly reversible.

Heat Engines and the Carnot Cycle

If the efficiency depends only on the two temperatures, the ratio of the temperatures must be the same as the ratio of the transferred heats. Therefore, the maximum efficiency of a heat engine can be written:

Maximum Efficiency of a Heat Engine

$$e_{\mathrm{max}} = 1 - \frac{T_{\mathrm{c}}}{T_{\mathrm{h}}}$$

18-6 Heat Engines and the Carnot Cycle

The maximum work a heat engine can do is then:

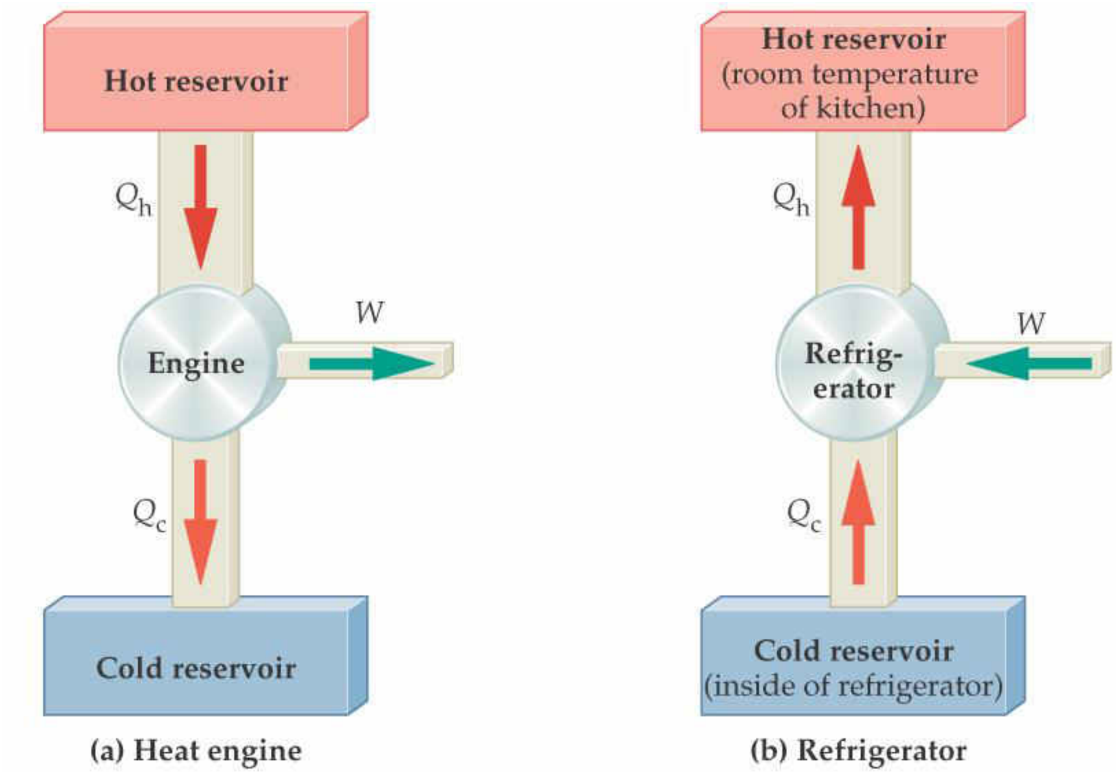
$$W_{\mathrm{max}} = e_{\mathrm{max}} Q_{\mathrm{h}} = \left(1 - \frac{T_{\mathrm{c}}}{T_{\mathrm{h}}}\right) Q_{\mathrm{h}}$$

If the two reservoirs are at the same temperature, the efficiency is zero; the smaller the ratio of the cold temperature to the hot temperature, the closer the efficiency will be to 1.

Refrigerators, Air Conditioners, and Heat Pumps

While heat will flow spontaneously only from a higher temperature to a lower one, it can be made to flow the other way if work is done on the system. Refrigerators, air conditioners, and heat pumps all use work to transfer heat from a cold object to a hot object.

Refrigerators, Air Conditioners, and Heat Pumps



If we compare the heat engine and the refrigerator, we see that the refrigerator is basically a heat engine running backwards – it uses work to extract heat from the cold reservoir (the inside of the refrigerator) and exhausts to the kitchen. Note that

$$Q_h = Q_c + W$$

- more heat is exhausted to the kitchen than is removed from the refrigerator.

Refrigerators, Air Conditioners, and Heat Pumps

An ideal refrigerator would remove the most heat from the interior while requiring the smallest amount of work. This ratio is called the coefficient of performance, COP:

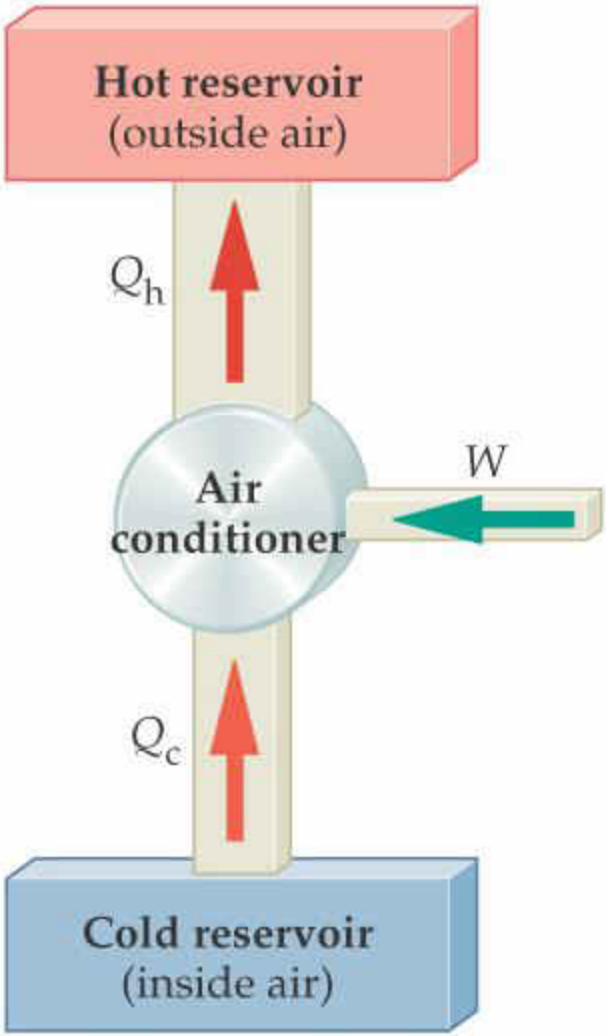
Coefficient of Performance for a Refrigerator, COP

$$\mathrm{COP} = \frac{Q_c}{W}$$

SI unit: dimensionless

Typical refrigerators have COP values between 2 and 6. Bigger is better!

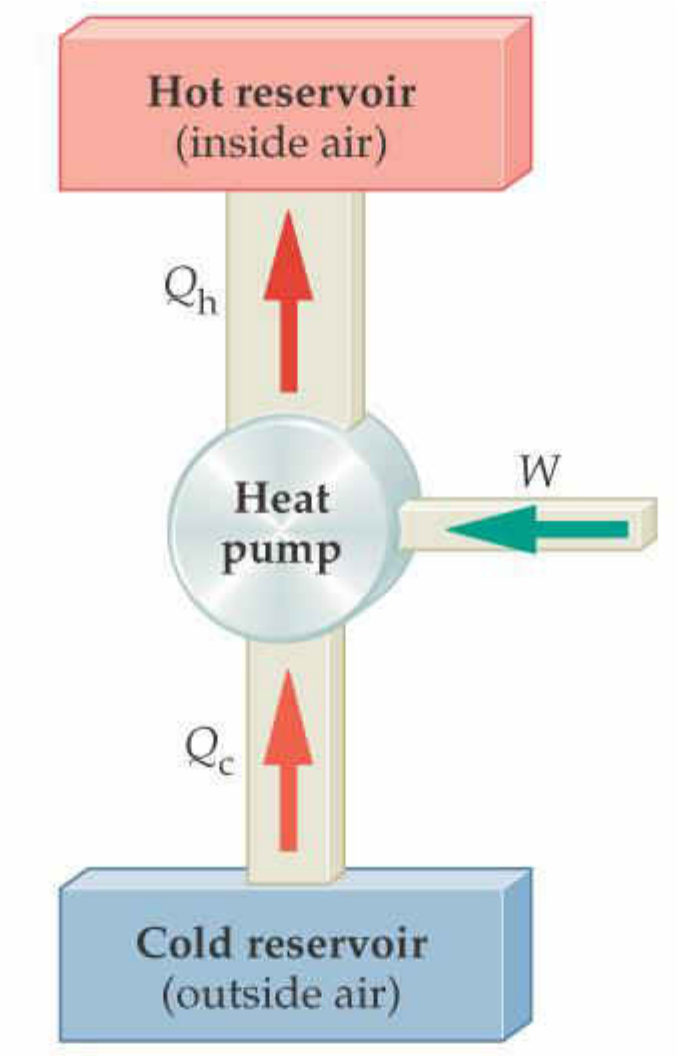
Refrigerators, Air Conditioners, and Heat Pumps



(c) Air conditioner

An air conditioner is essentially identical to a refrigerator; the cold reservoir is the interior of the house or other space being cooled, and the hot reservoir is outdoors. Exhausting an air conditioner within the house will result in the house becoming warmer, just as keeping the refrigerator door open will result in the kitchen becoming warmer.

Refrigerators, Air Conditioners, and Heat Pumps



(d) Heat pump

Finally, a heat pump is the same as an air conditioner, except with the reservoirs reversed. Heat is removed from the cold reservoir outside, and exhausted into the house, keeping it warm. Note that the work the pump does actually contributes to the desired result (a warmer house) in this case.

Refrigerators, Air Conditioners, and Heat Pumps

In an ideal heat pump with two operating temperatures (cold and hot), the Carnot relationship holds; the work needed to add heat Q_h to a room is:

$$W = Q_h - Q_c = Q_h\left(1 - \frac{Q_c}{Q_h}\right) = Q_h\left(1 - \frac{T_c}{T_h}\right)$$

The COP for a heat pump:

Coefficient of Performance for a Heat Pump, COP

$$\text{COP} = \frac{Q_h}{W}$$

SI unit: dimensionless

18-8 Entropy

A reversible engine has the following relation between the heat transferred and the reservoir temperatures:

$$\frac{Q_c}{Q_h} = \frac{T_c}{T_h}$$

Rewriting,

$$\frac{Q_c}{T_c} = \frac{Q_h}{T_h}$$

This quantity, Q/T, is the same for both reservoirs, and is defined as the change in entropy.

Entropy

Definition of Entropy Change, ΔS

$$\Delta S = \frac{Q}{T}$$

SI unit: J/K

For this definition to be valid, the heat transfer must be reversible.

In a reversible heat engine, it can be shown that the entropy does not change.

Entropy

A real engine will operate at a lower efficiency than a reversible engine; this means that less heat is converted to work. Therefore,

$$\Delta S_{\text{total}} = -\frac{Q_{\text{h}}}{T_{\text{h}}} + \frac{Q_{\text{c}}}{T_{\text{c}}} > 0$$

Any irreversible process results in an increase of entropy.

Entropy

To generalize:

- The total entropy of the universe increases whenever an irreversible process occurs.
- The total entropy of the universe is unchanged whenever a reversible process occurs.

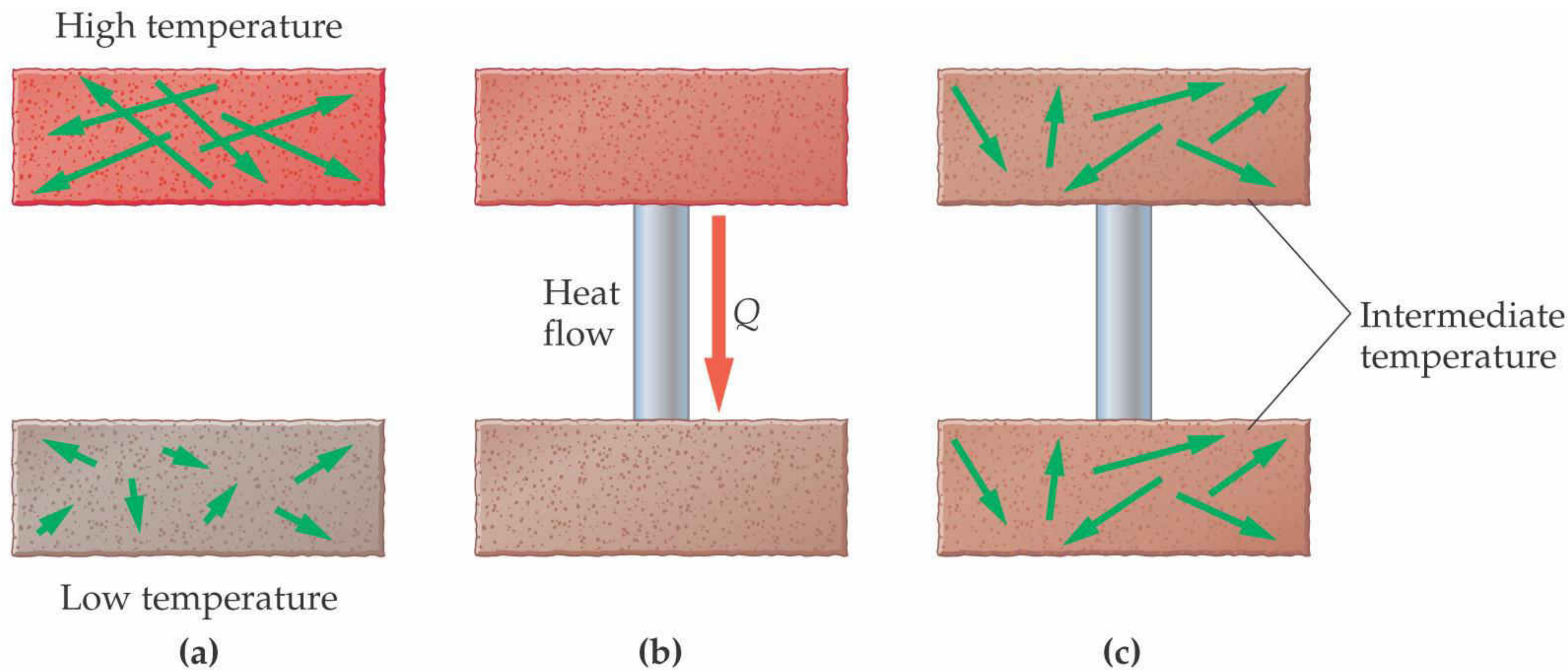
Since all real processes are irreversible, the entropy of the universe continually increases. If entropy decreases in a system due to work being done on it, a greater increase in entropy occurs outside the system.

18-8 Entropy

As the total entropy of the universe increases, its ability to do work decreases. The excess heat exhausted during an irreversible process cannot be recovered; doing that would require a decrease in entropy, which is not possible.

Order, Disorder, and Entropy

Entropy can be thought of as the increase in disorder in the universe. In this diagram, the end state is less ordered than the initial state – the separation between low and high temperature areas has been lost.



18-9 Order, Disorder, and Entropy

If we look at the ultimate fate of the universe in light of the continual increase in entropy, we might envision a future in which the entire universe would have come to the same temperature. At this point, it would no longer be possible to do any work, nor would any type of life be possible. This is referred to as the “heat death” of the universe.

Order, Disorder, and Entropy

So if entropy is continually increasing, how is life possible? How is it that species can evolve into ever more complex forms? Doesn’t this violate the second law of thermodynamics?

No – life and increasing complexity can exist because they use energy to drive their functioning. The overall entropy of the universe is still increasing. When a living entity stops using energy, it dies, and its entropy can increase rather quickly.

5

Ideal and Real Gases

Ideal Gas Law

$$PV = nRT$$

Brings together gas properties.

Can be derived from *experiment* and *theory*.

Ideal Gas Equation

Volume

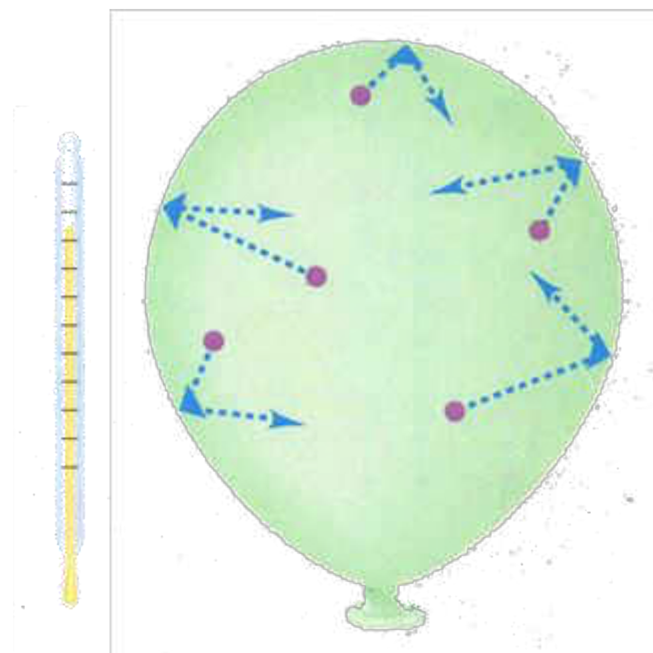
Universal Gas Constant

Pressure

$$PV = nRT$$

Temperature

No. of moles

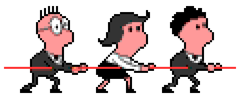


R = 0.0821 atm L / mol K

R = 8.314 kPa L / mol K

Kelter, Carr, Scott, Chemistry A Wolrd of Choices 1999, page 366

$PV = nRT$



P = pressure
V = volume
T = temperature (Kelvin)
n = number of moles
R = gas constant

Solve for constant (R)

$$\frac{PV}{nT}$$

Substitute values:

$$\frac{(1\ \text{atm})\ (22.4\ \text{L})}{(1\ \text{mole})(273\ \text{K})} = R$$

Standard Temperature and Pressure (STP)

T = 0 °C or 273 K

P = 1 atm = 101.3 kPa = 760 mm Hg

1 mol = 22.4 L @ STP

Recall: 1 atm = 101.3 kPa

$$R = 0.0821\ \frac{\text{atm L}}{\text{mol K}}\ \frac{(101.3\ \text{kPa})}{(1\ \text{atm})} = 8.31\ \frac{\text{kPa L}}{\text{mol K}}$$

R = 0.0821 atm L / mol K or R = 8.31 kPa L / mol K

Ideal Gas Law

What is the volume that 500 g of iodine will occupy under the conditions: Temp = 300°C and Pressure = 740 mm Hg?

Step 1) Write down given information

mass = 500 g iodine

T = 300°C

P = 740 mm Hg

R = 0.0821 atm · L / mol · K

Step 2) Equation: $PV = nRT$

Step 3) Solve for variable

$$V = \frac{nRT}{P}$$

Step 4) Substitute in numbers and solve

$$V = \frac{(500\text{ g})(0.0821\text{ atm} \cdot \text{L} / \text{mol} \cdot \text{K})(300^{o}\text{C})}{740\text{ mm Hg}}$$

V =

What MISTAKES did we make in this problem?

What mistakes did we make in this problem?

What is the volume that 500 g of iodine will occupy under the conditions: Temp = 300°C and Pressure = 740 mm Hg?

Step 1) Write down given information.

mass = 500 g iodine → Convert mass to gram;
recall iodine is diatomic (I_2)
x mol I_2 = 500 g I_2(1mol I_2 / 254 g I_2)

n = 1.9685 mol I_2

T = 300°C → Temperature must be converted to Kelvin
T = 300°C + 273

T = 573 K

P = 740 mm Hg → Pressure needs to have same unit as R; therefore, convert pressure from mm Hg to atm.

x atm = 740 mm Hg (1 atm / 760 mm Hg)

P = 0.8 atm

Ideal Gas Law

What is the volume that 500 g of iodine will occupy under the conditions:
Temp = 300^{o}C and Pressure = 740 mm Hg?

Step 1) Write down given information

mass = 500 g iodine

n = 1.9685 mol I_2

T = 573 K (300^{o}C)

P = 0.9737 atm (740 mm Hg)

R = 0.0821 atm · L / mol · K

V = ? L

Step 2) Equation: PV = nRT

Step 3) Solve for variable

$$V = \frac{nRT}{P}$$

Step 4) Substitute in numbers and solve

$$V = \frac{(1.9685\ \text{mol})(0.0821\ \text{atm} \cdot \text{L / mol} \cdot \text{K})(573\ \text{K})}{0.9737\ \text{atm}}$$

$$V = 95.1\ \text{L}\ I_2$$

6

Properties of Steam

“PROPERTIES OF STEAM”

Contents

- Basic terms
- Steam Thermodynamics
- P–V Diagram, T–s Diagram, h–s Diagram and P–s Diagram
- Steam Generation
- Steam types
- Measurement of dryness fraction of steam
- Mollar’s diagram (h-s chart)
- References

Basic terms

•**Steam Enthalpy (h)**
Enthalpy is a general measure of the internal stored energy per mass unit of a flowing stream. It is expressed in kJ/Kg (SI units). It is usually represented with letter h.

•**Steam Entropy (S)**
Entropy, is a measure of the thermodynamic potential of a flowing stream in the units of energy per mass unit and absolute temperature. It is expressed in kJ/kgK (SI units). It is usually represented with letter S.

Specific Volume (v)
As its name implies, **specific volume** is a measure of the volume of a flowing stream per mass unit. It is expressed in m^3/kg (SI units). It is usually represented with letter v.

- **Sensible Heat:**
  The amount of heat required to increase the temperature of 1 kg of water from 0°C to saturation temperature at a given pressure is known a s sensible heat of water.

- **Latent Heat**
  The amount of heat required to convert 1 kg of water at saturation temperature for a given pressure into dry saturated steam at that temperature and pressure is known as latent heat of vaporization.

- **Dryness Fraction**: Dryness fraction is defined as the mass of dry steam per kg of wet steam. It is represented by x.

  x = ms / (ms + mw),

  where: ms is the mass of steam and
  mw is the mass of water

Steam Thermodynamics

- Steam is the gaseous phase of water. It utilizes
- heat during the process and carries large
- quantities of heat later.

- Hence, it could be used as a working substance for steam engines. Steam is generated in boilers at constant pressure.

- Generally, steam may be obtained starting from ice or straight away from the water by adding heat to it.

Under superheated or subcooled conditions, properties such as enthalpy, entropy and specific volume are dependent on temperature and pressure. However, at saturated conditions where a mixture of steam and water coexist, an additional parameter, steam quality, needs to be defined.

Steam quality (i.e. dryness fraction), usually represented with letter x.

Steam quality is frequently recorded as a percentage of steam by weight after being multiplied by 100%

The mixture enthalpy, entropy and specific volume of a steam-water mixture can then be easily defined as follows:

$$h = hf + x * hfg \qquad \text{where,} \quad hfg = (hg-hf)$$

$$S = sf + x * sfg \qquad sfg = (sg-sf)$$

$$v = vf + x*vg$$

where subscripts f (from fluid) and g (from gas) refer to properties at saturated liquid and vapour conditions respectively.

P–V Diagram, T–s Diagram,

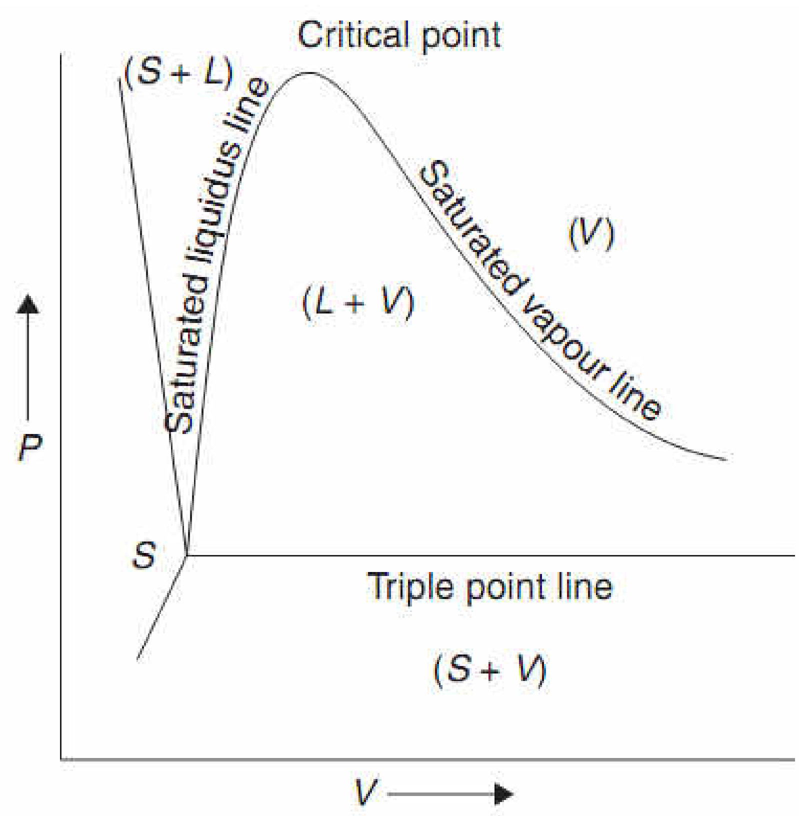


Figure 4.3(a) *P–V* Diagram

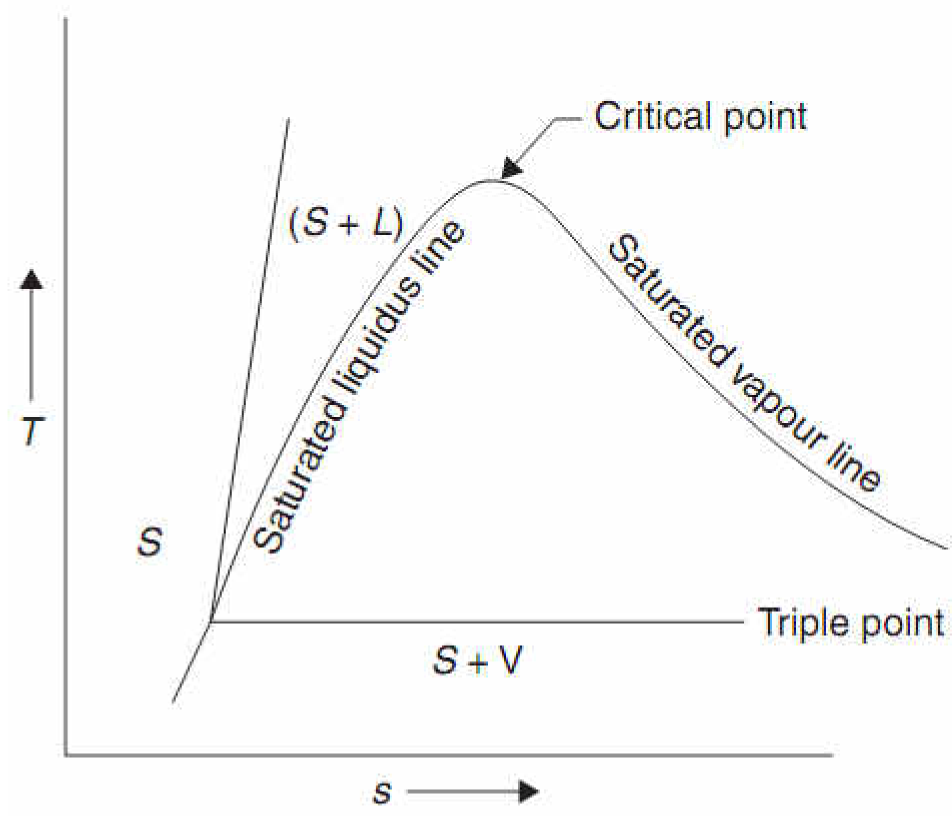


Figure 4.3(b) *T–s* Diagram

h–s Diagram and P–s Diagram

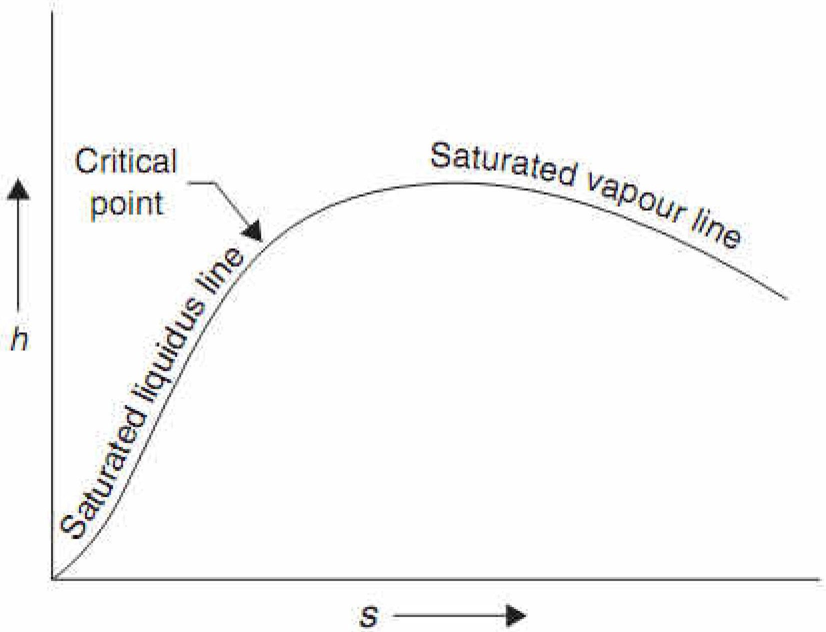


Figure 4.3(c) *h*–*s* Diagram

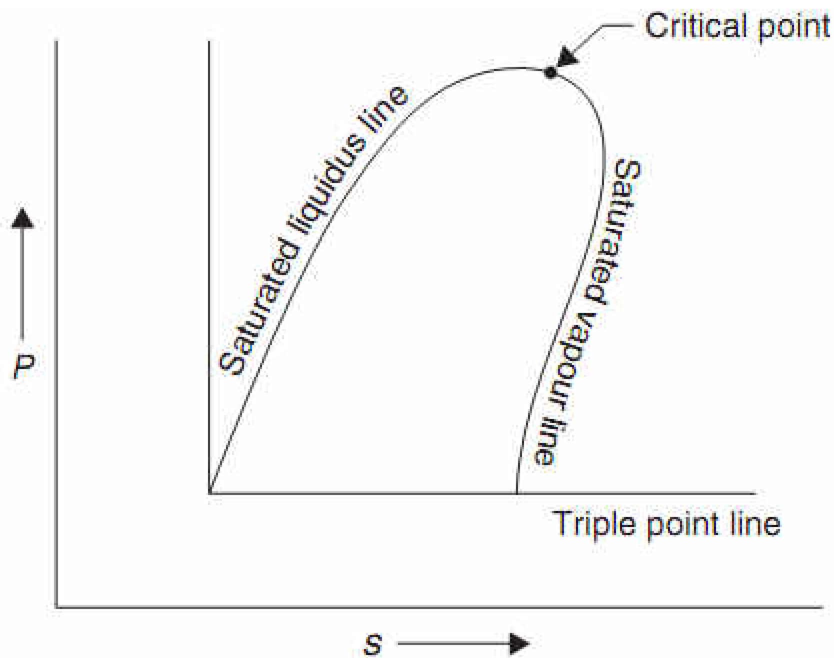


Figure 4.3(d) *P*–*s* Diagram

Steam Generation

- Enthalpy Change in Generation of Steam from 0°C at 0°C

$$h_0 = u_0 + PV_0;\ \text{at } 0°\text{C},\ u_0 = 0$$

$$h_0 = PV_0; \qquad \text{where } V_0 \text{ is specific volume at } 0°\text{C}.$$

- 0°C to Saturation Temperature

$$h_f = u_f + PV_f \quad \text{where } V_f \text{ is specific volume at saturation temperature.}$$

$$h_0 = PV_0$$

$$h_f - h_0 = h = u_f + P\,(V_f - V_0)$$

- **Entropy of Steam**

$$S = S_f + S_{fg} = C_p \log_e \frac{T_S}{T_0} + \frac{h_{fg}}{T_S} \text{ for dry saturated steam at constant pressure } (x = 1)$$

$$S = S_f + xS_{fg} = C_p \log_e \frac{T_S}{T_0} + \frac{xh_{fg}}{T_S} \text{ for wet steam at constant pressure}$$

$$S = S_f + xS_{fg} + S_g = C_{pw} \log_e \frac{T_S}{T_0} + \frac{xh_{fg}}{T_S} + C_p \log_e \frac{T_{Sup}}{T_S} \text{ for superheated steam at constant pressure}$$

where $C_{pw} \approx 1$, specific heat of water.

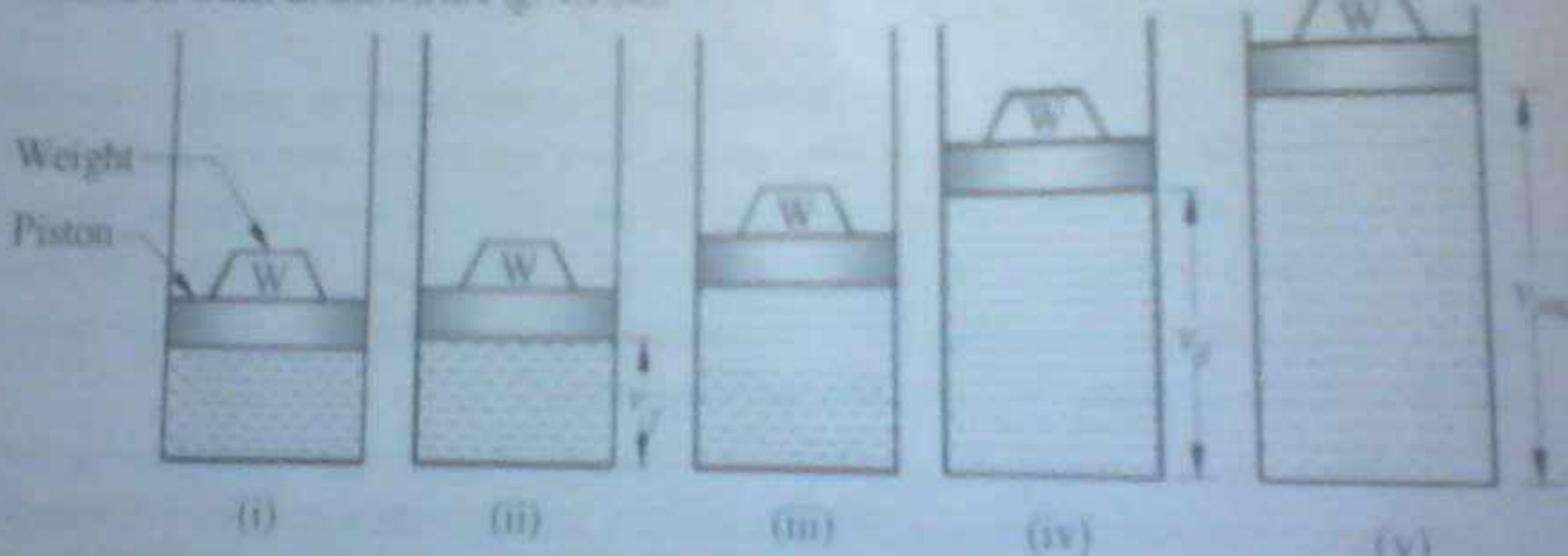


Fig. 4.1 Formation of Steam.

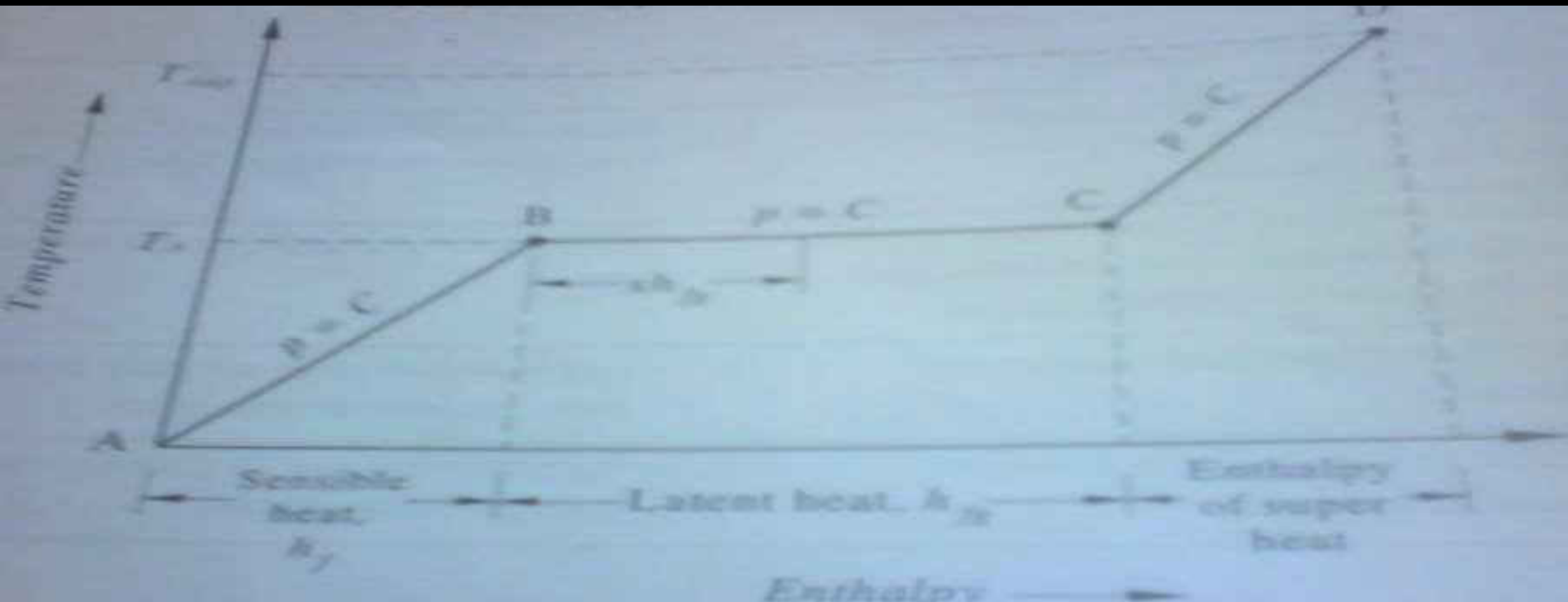


Fig. 4.2 Temperature - Enthalpy diagram

Steam Types

Wet Steam: Wet steam contains partly water as suspended in it and partly steam

Dry saturated Steam A steam at the saturation temprature corresponding to a given pressure and having no water molecules in it is known as dry saturated steam or dry steam.
Since the dry saturated steam does not contain any water
molecules in it, its dryness fraction will be unity.

Superheated steam:
When a dry saturated steam is heated further at the given constant pressure, its temperature rise beyond its
saturation temperature. The steam in this state is said to
be superheated.

Enthalpy of Steam

1. Enthalpy of liquid:
   $hf=Cpw\ (tf-0)$

2. Enthalpy of Dry saturated steam:
   $hg=hf + hfg$

3. Enthalpy of Wet steam:
   $h=hf + x.hfg$

4. Enthalpy of Superheated steam:
   $hsup= hg+ Cps(Tsup - Tsat)$

Specific Volume of Steam (v)

1. Specific volume of saturated water:

   $v = v_f$

2. Specific volume of dry saturated steam:

   $v = v_g$

3. Specific volume of wet steam:

   $v = x.v_g$

4. Specific volume of superheated steam:

   $v = V_g (T_{sup}/T_{sat})$

Entropy of Steam (S)

1. Entropy of liquid:

   $S = S_f$

2. Entropy of Dry saturated steam:

   $S = S_g$

3. Enthalpy of Wet steam:

   $S = S_f + x.\ S_{fg}$

4. Enthalpy of Superheated steam:

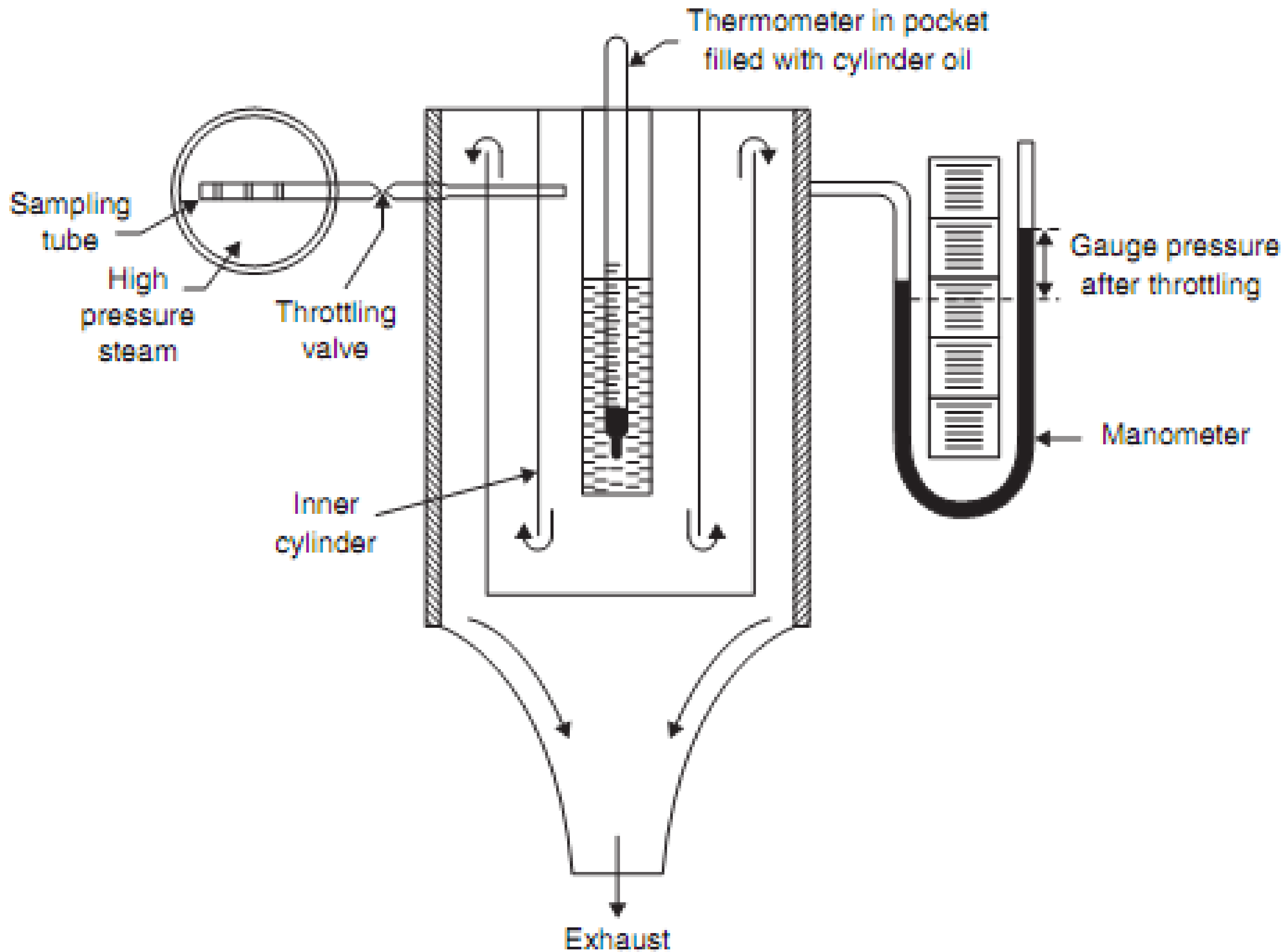
   $S = S_g + C_{psup} \log (T_{sup}/T_{sat})$

Measurement of dryness fraction (x) of steam by:

1. Throttling calorimeter

2. Separating and Throttling Calorimeter

1. Throttling Calorimeter



$$x_1 = \frac{h_{g2} + C_p(t_{sup} - t_{S2}) - h_{f1}}{h_{fg}}, \text{ where } C_p = 0.48$$

P_1 = Initial pressure of steam

P_2 = Final pressure = Atmospheric pressure + Manometer reading

h_{f1} = Enthalpy of water at pressure, P_1

h_{fg1} = Enthalpy of vaporization at pressure, P_1

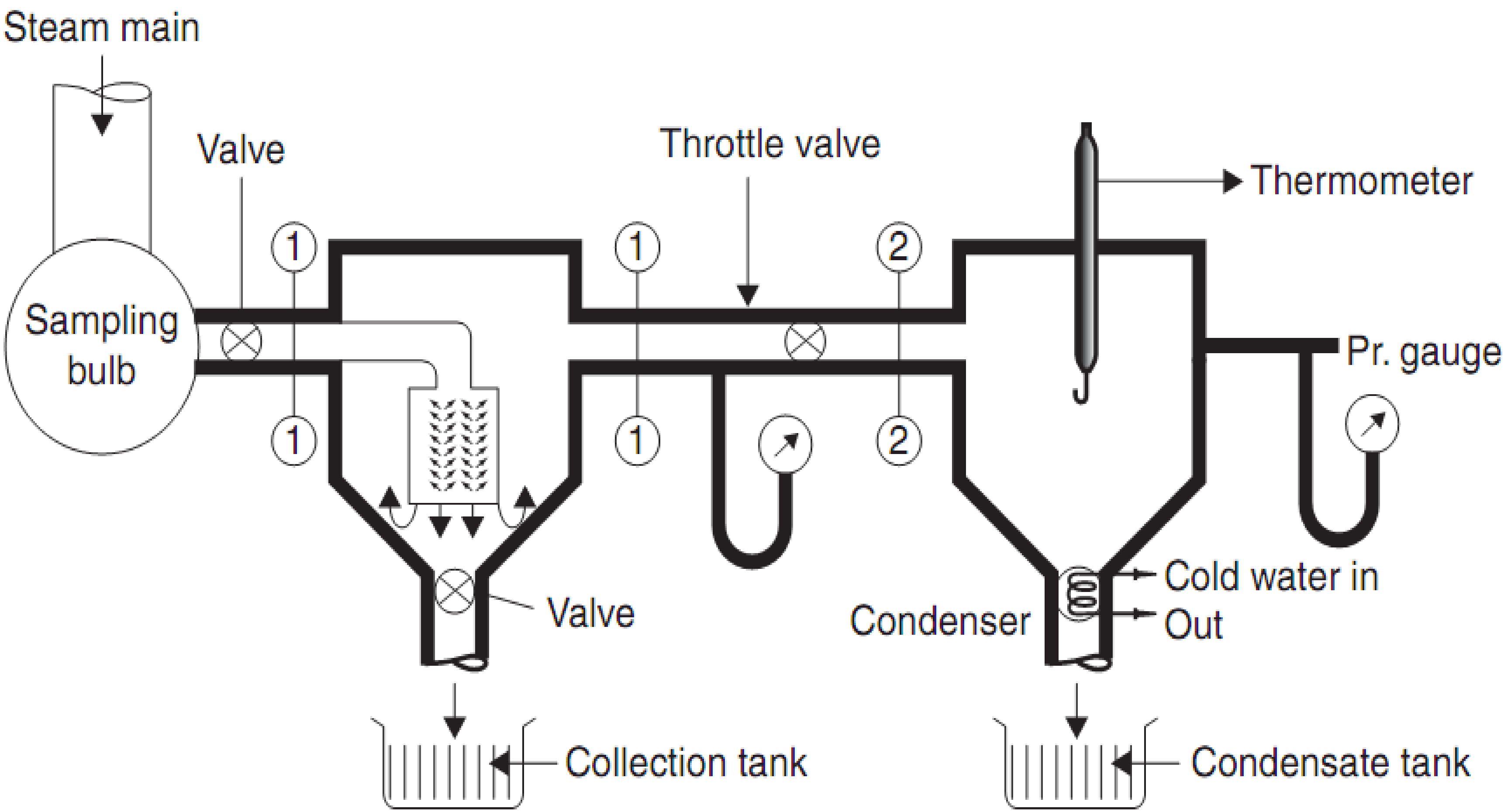
C_{pg} = Specific heat of superheated steam

t_{S2} = Saturation temperature at final pressure, P_2

t_{sup} = Temperature recorded by thermometer

x_1 = Dryness fraction of steam before throttling

2.Separating and Throttling Calorimeter



Let M = Mass of steam passing through throttling calorimeter

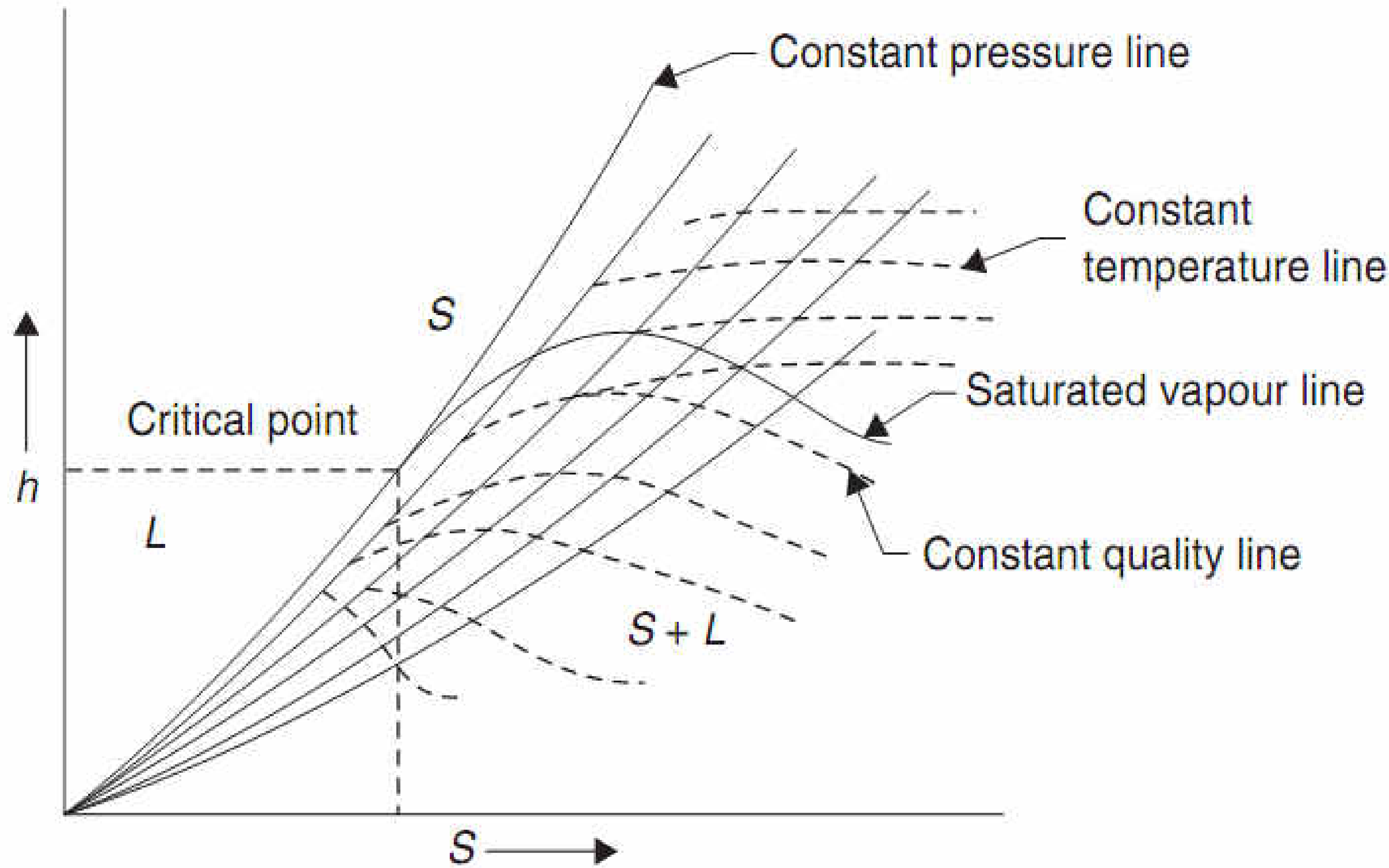
x_2 = The dryness fraction entering into the throttling calorimeter that is determined by throttling calorimeter

M = Mass of water separated out in separating calorimeter

x = Dryness fraction of steam entering the separating calorimeter

Thus, $x = \dfrac{M}{M+m} x_2$ or, $x = x_1 \times x_2$, where $x_1 = \dfrac{M}{M+m}$

Mollier Diagram or h–S Chart



7

Steam Generators

Steam and steam boiler

Generation of steam at constant pressure

Consider 1 kg of ice in a piston -cylinder arrangement as shown.

it is under an Absolute Pressure say P bar and at

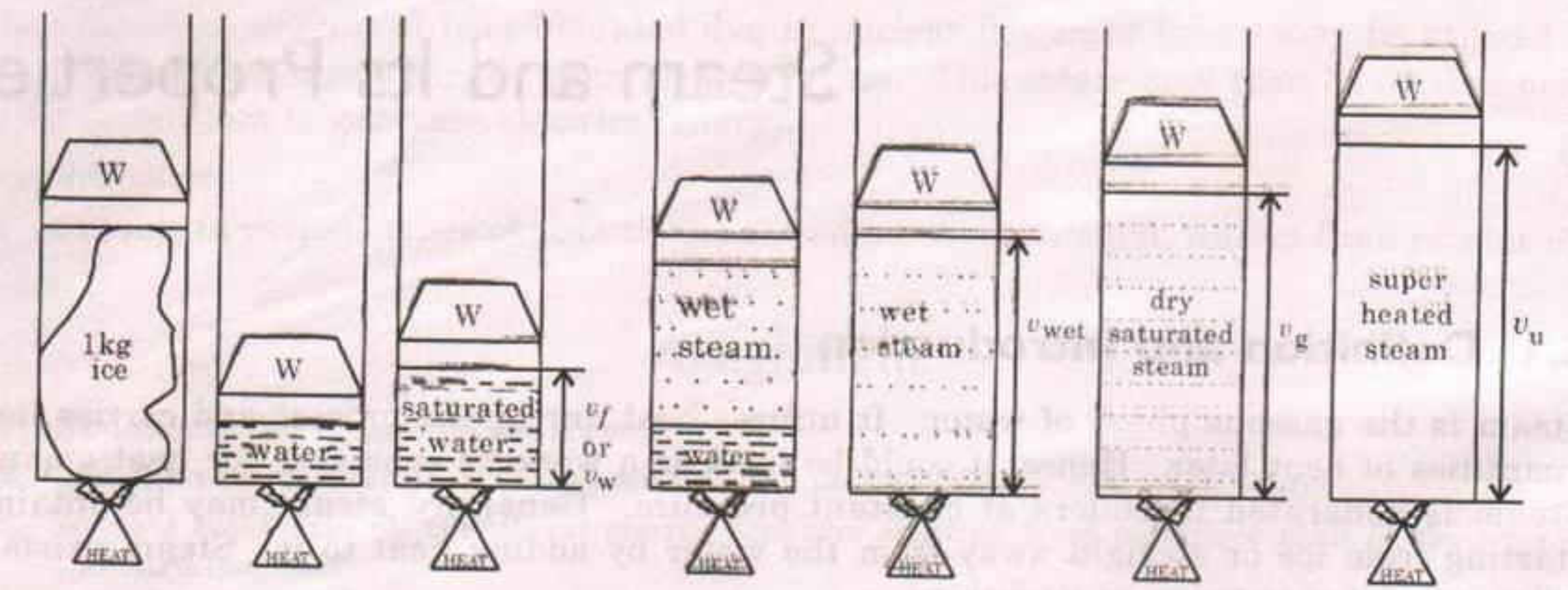


Fig 2.1 *Formation of Steam at Constant Pressure*

Keeping the pressure constant, the gradual heating of the ice

leads to note the following changes in it, These are represented

on a t-h diagram on heating, the temperature of the ice will

gradually rises from p to Q i.e from – t C till reaches the freezing

temperature 0.

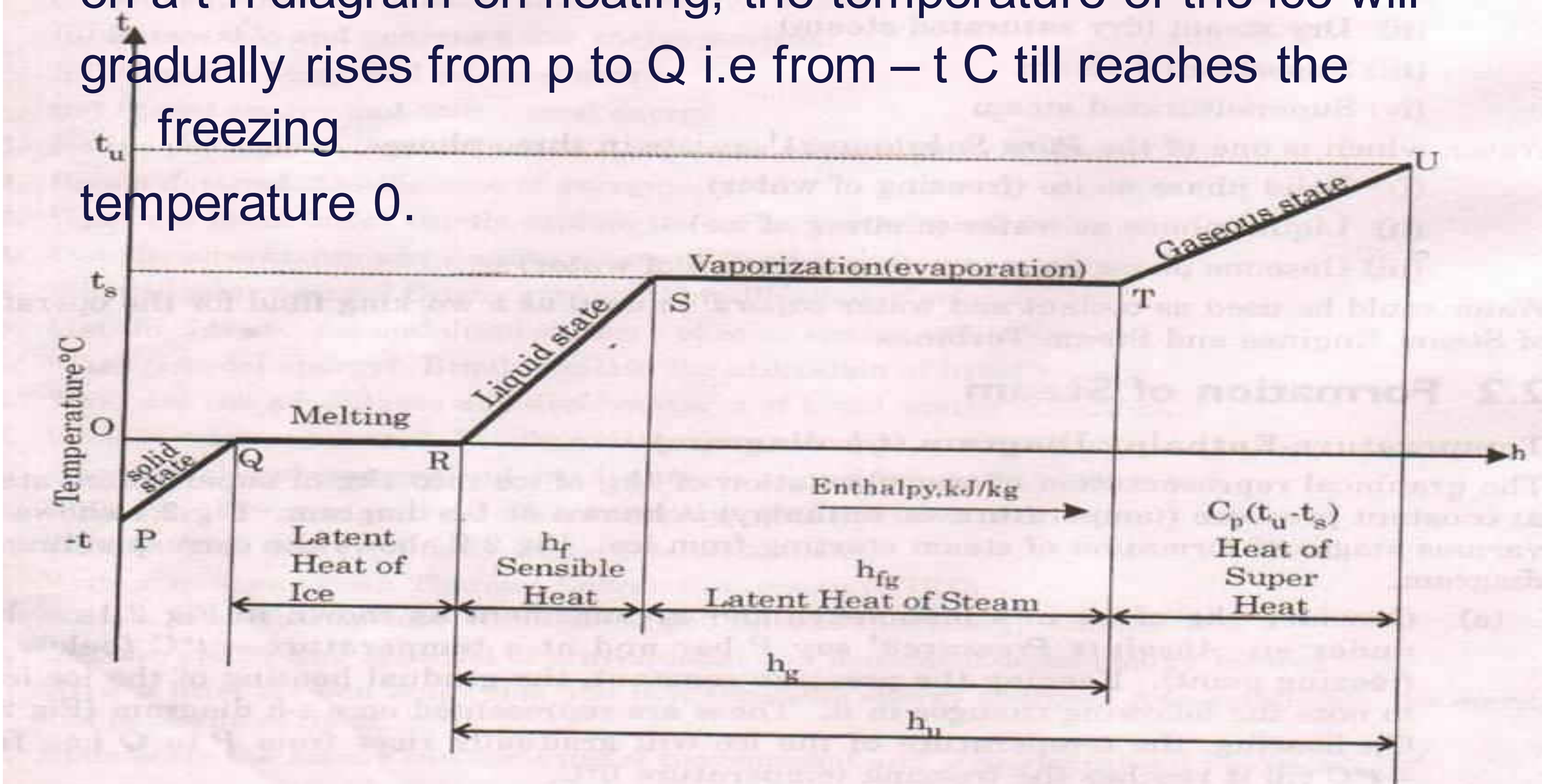


Fig 2.2 *Temperature-Enthalpy Diagram of Formation of Steam at Constant Pressure*

- Adding more heat, the ice starts melting without changing in the temperature till the entire ice is converted into water from Q to R. The amount of heat during this period from Q to R is called Latent heat of fusion of ice or simply Latent heat of ice.

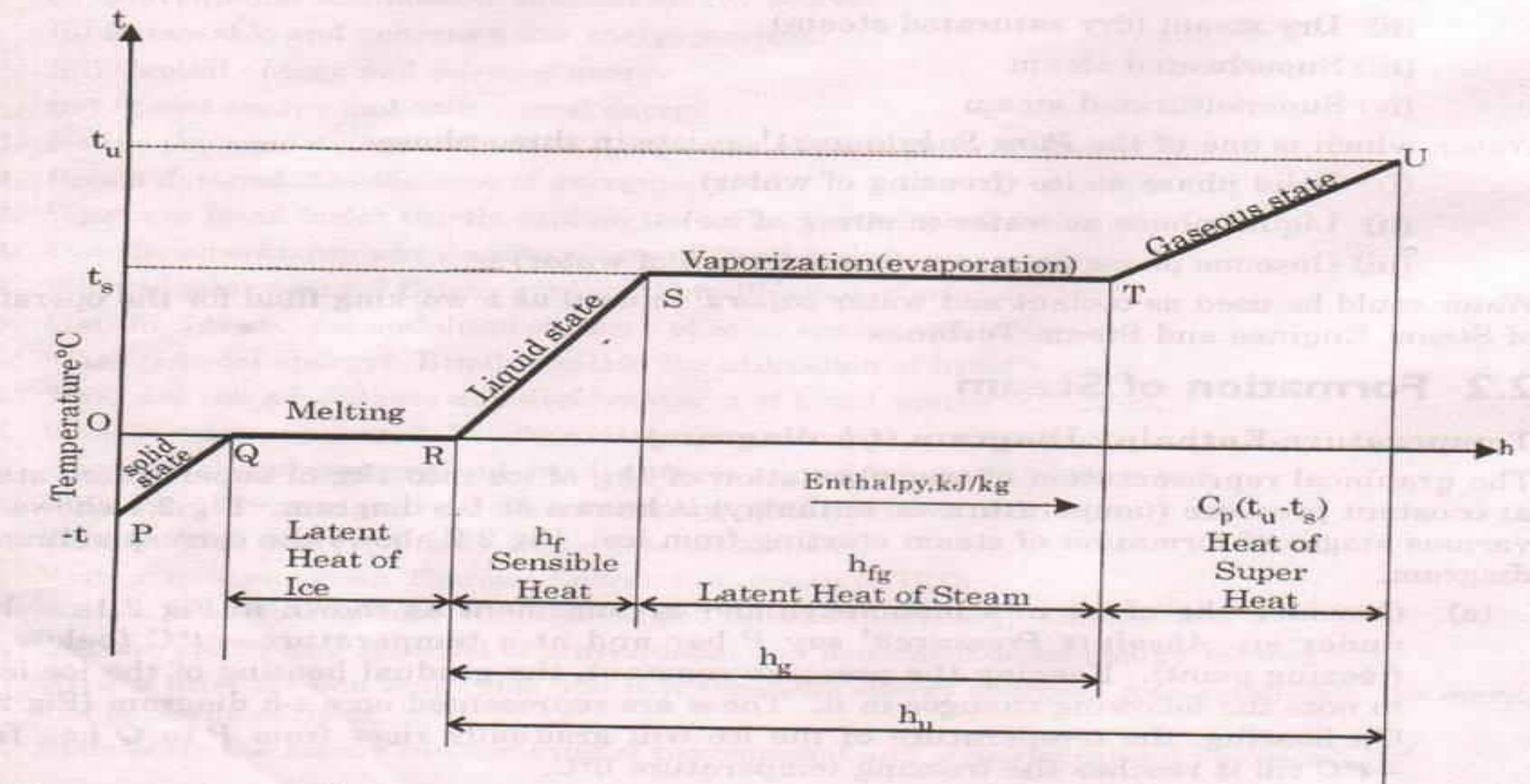


Fig 2.2 *Temperature-Enthalpy Diagram of Formation of Steam at Constant Pressure*

- Continuous heating raises the temperature to its boiling point t C known as Saturation Temperature. The corresponding pressure is called saturation pressure. it is the stage of vaporization at 1.01325 bar atmospheric pressure (760mm of hg at 100'C).

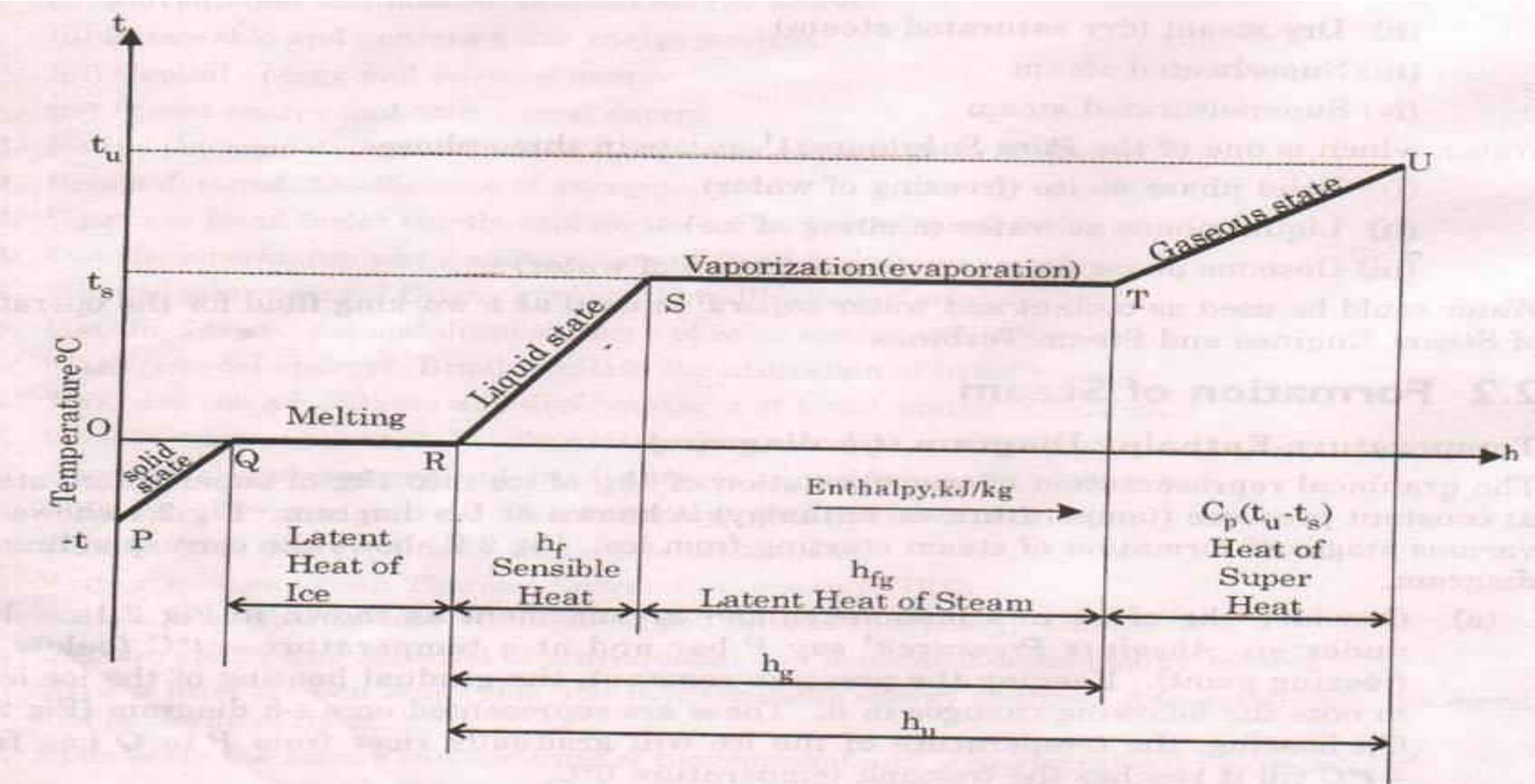


Fig 2.2 *Temperature-Enthalpy Diagram of Formation of Steam at Constant Pressure*

- As pressure increases, the value of saturation temperature also increases. The amount of heat added during R to S is called Sensible Heat or Enthalpy of Saturated Water or Total Heat of Water (h, or h ''').

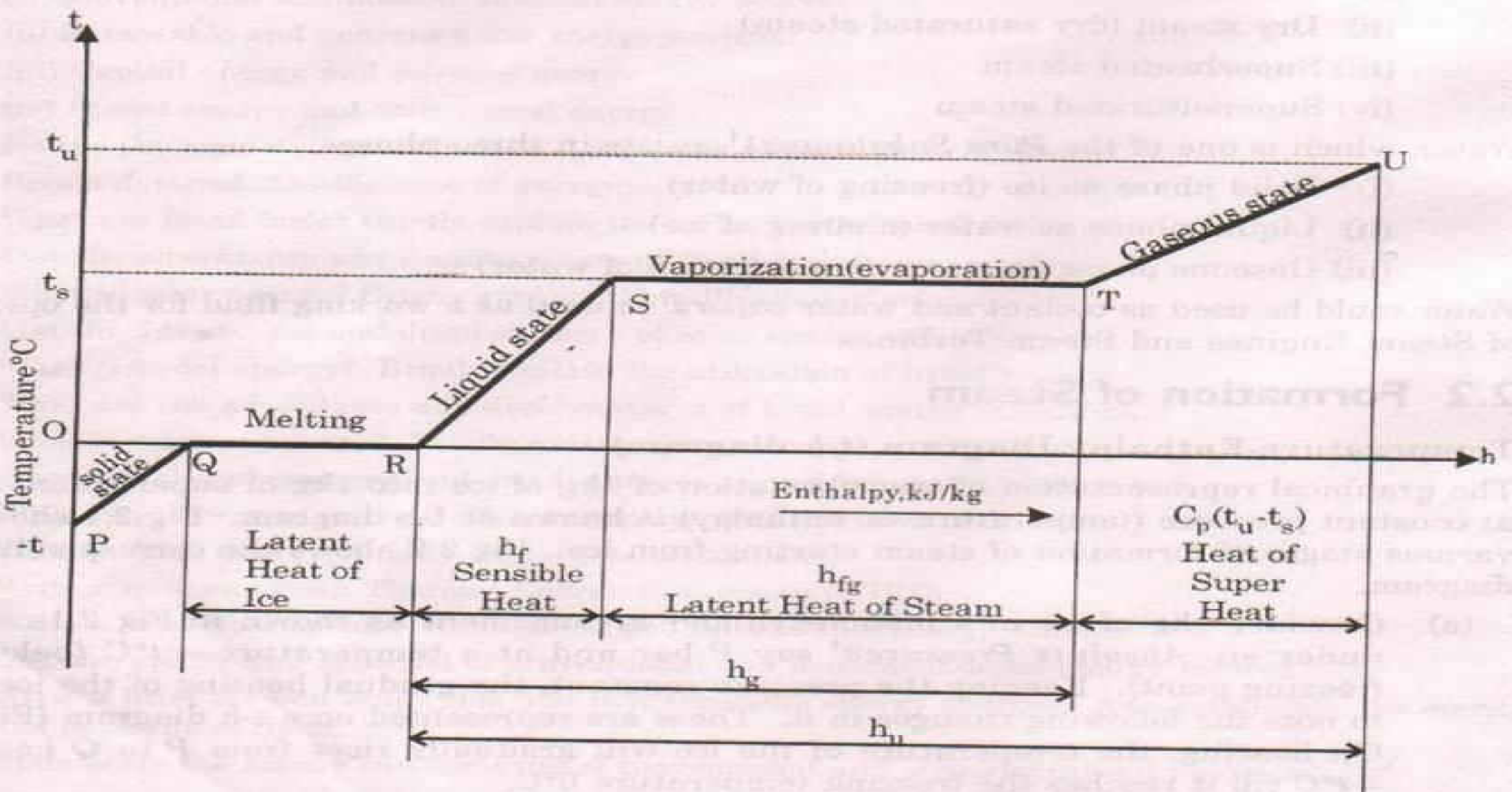


Fig 2.2 *Temperature-Enthalpy Diagram of Formation of Steam at Constant Pressure*

- During the process, a slight increase in volume of water (saturated water) may be noted. The resulting volume is known as Specific volume of Saturated Water (Vf or vW).

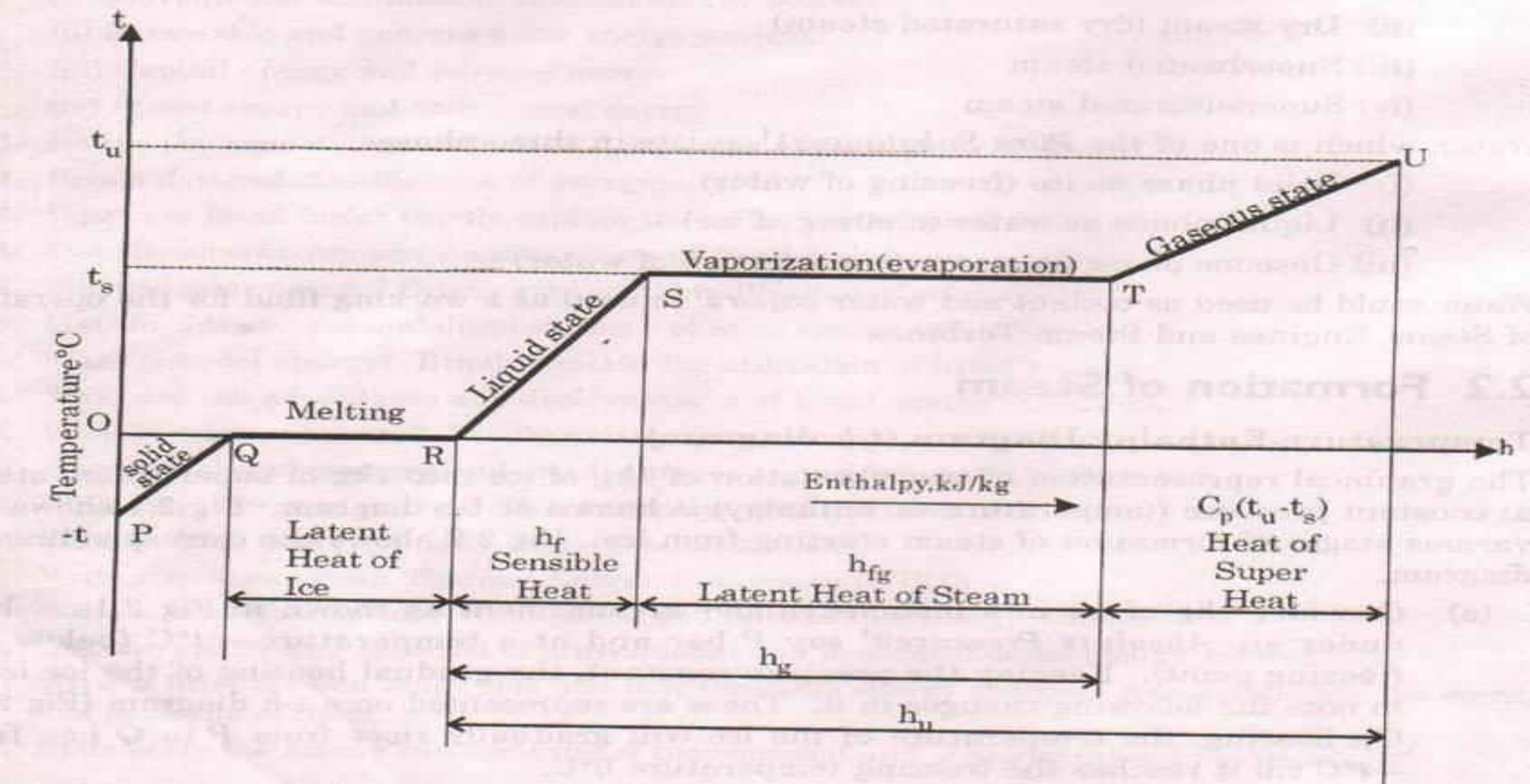


Fig 2.2 *Temperature-Enthalpy Diagram of Formation of Steam at Constant Pressure*

- On further heating beyond S, the water will gradually starts evaporate and starts convert it to steam, but the temperature remains constant. As long as the steam is in contact with water, it is called Wet Steam or Saturated Steam.

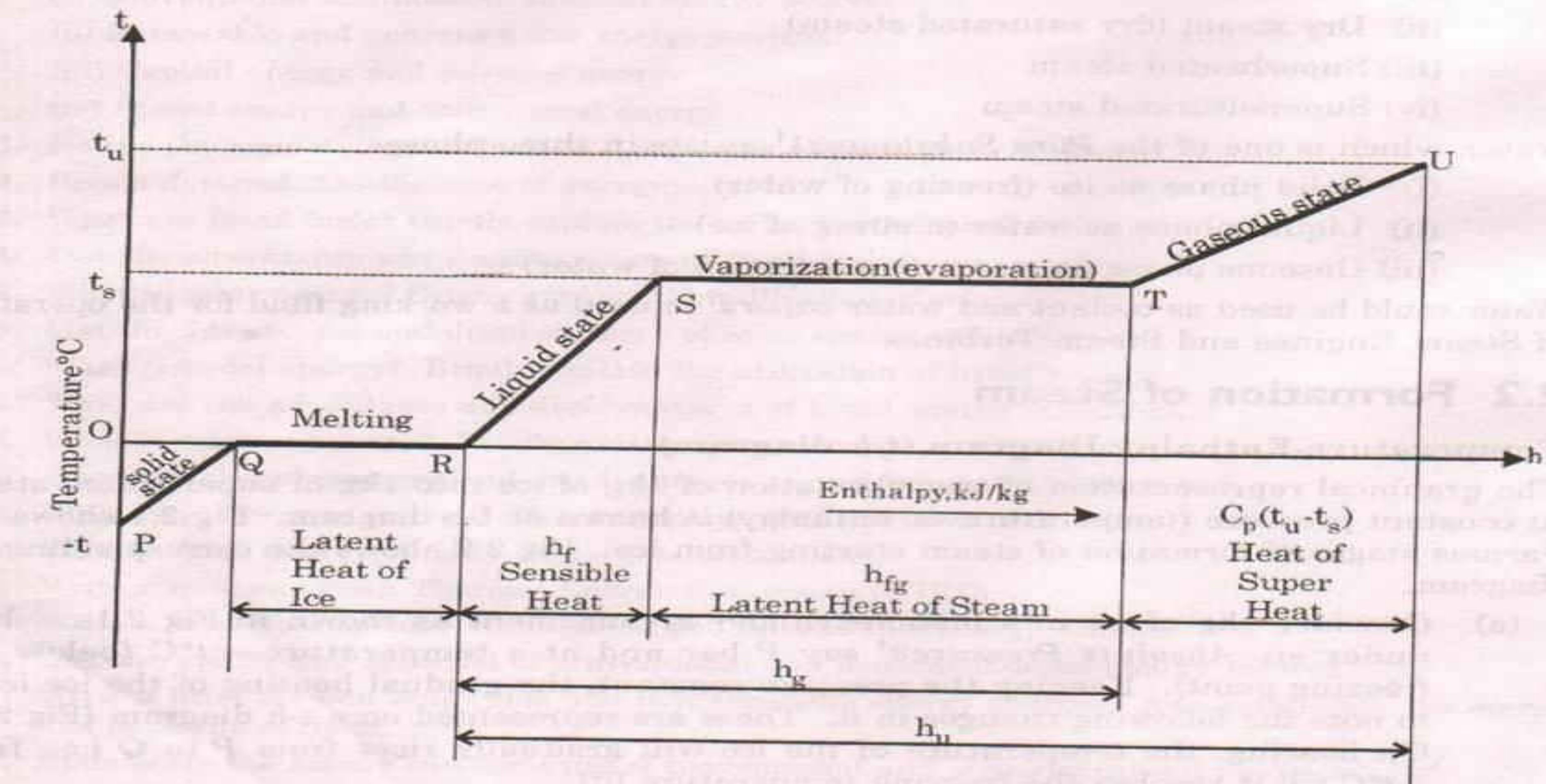


Fig 2.2 *Temperature-Enthalpy Diagram of Formation of Steam at Constant Pressure*

- (e) On further heating the temperature remains constant, but the entire water converts to steam. But still it will be wet steam. The total heat supplied from OOC is called Enthalpy of Wet Steam (h wet). The resulting volume is known as Specific Volume of Wet Steam (v wet)

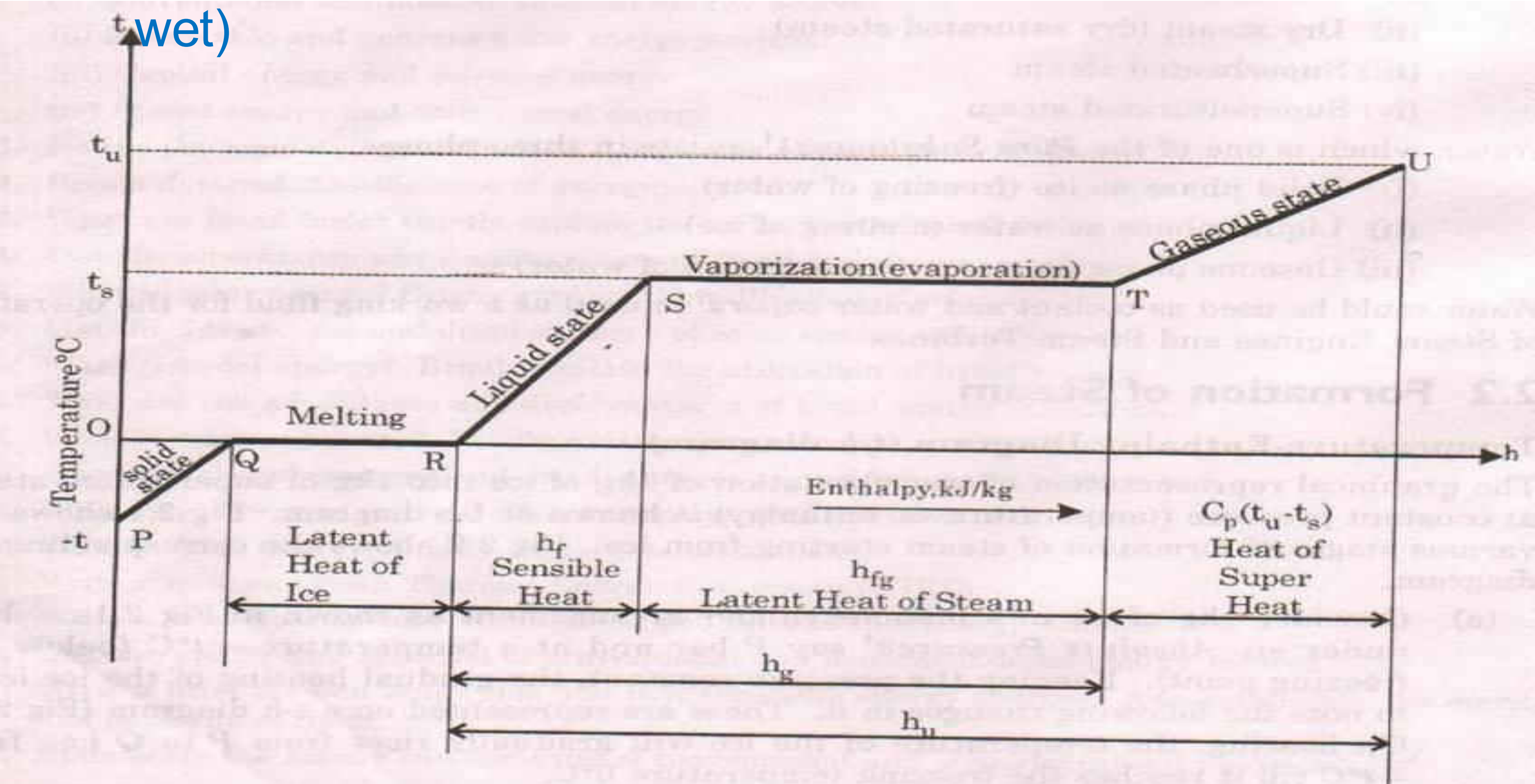


Fig 2.2 *Temperature-Enthalpy Diagram of Formation of Steam at Constant Pressure*

- (f) On further heating the wet steam, the water particles, which are in suspension, will start evaporating gradually and at a particular moment the final particles just evaporates. The steam at that moment corresponding to point T is called Dry Steam or Dry Saturated Steam.

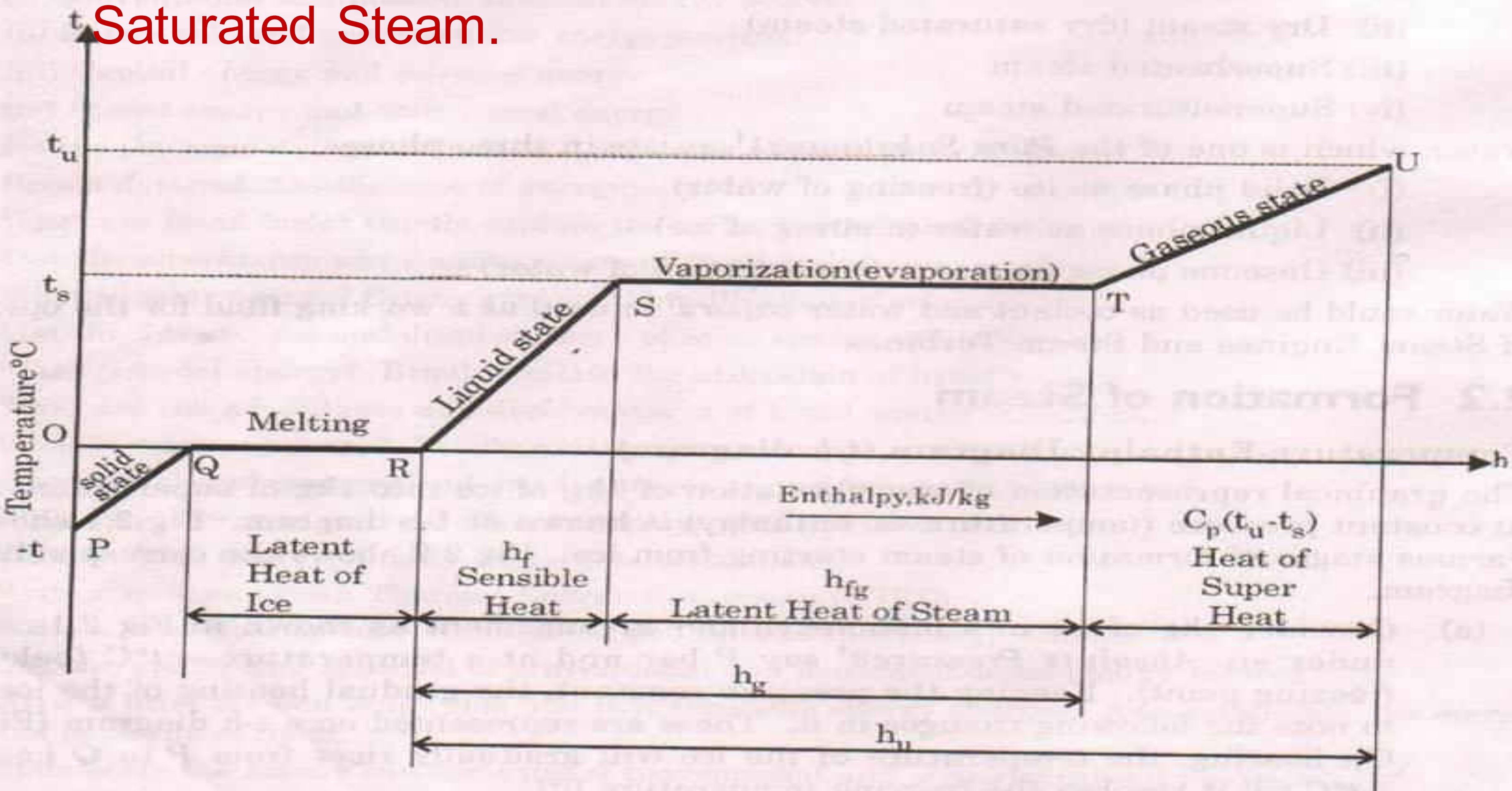


Fig 2.2 *Temperature-Enthalpy Diagram of Formation of Steam at Constant Pressure*

The resulting volume is known as Specific Volume of Dry Steam (vg). This steam not obeys the gas laws. The amount of heat added during S to T is called Latent Heat of Vaporization of Steam or Latent Heat of Steam (hfg). During the process, the saturation temperature remains constant.

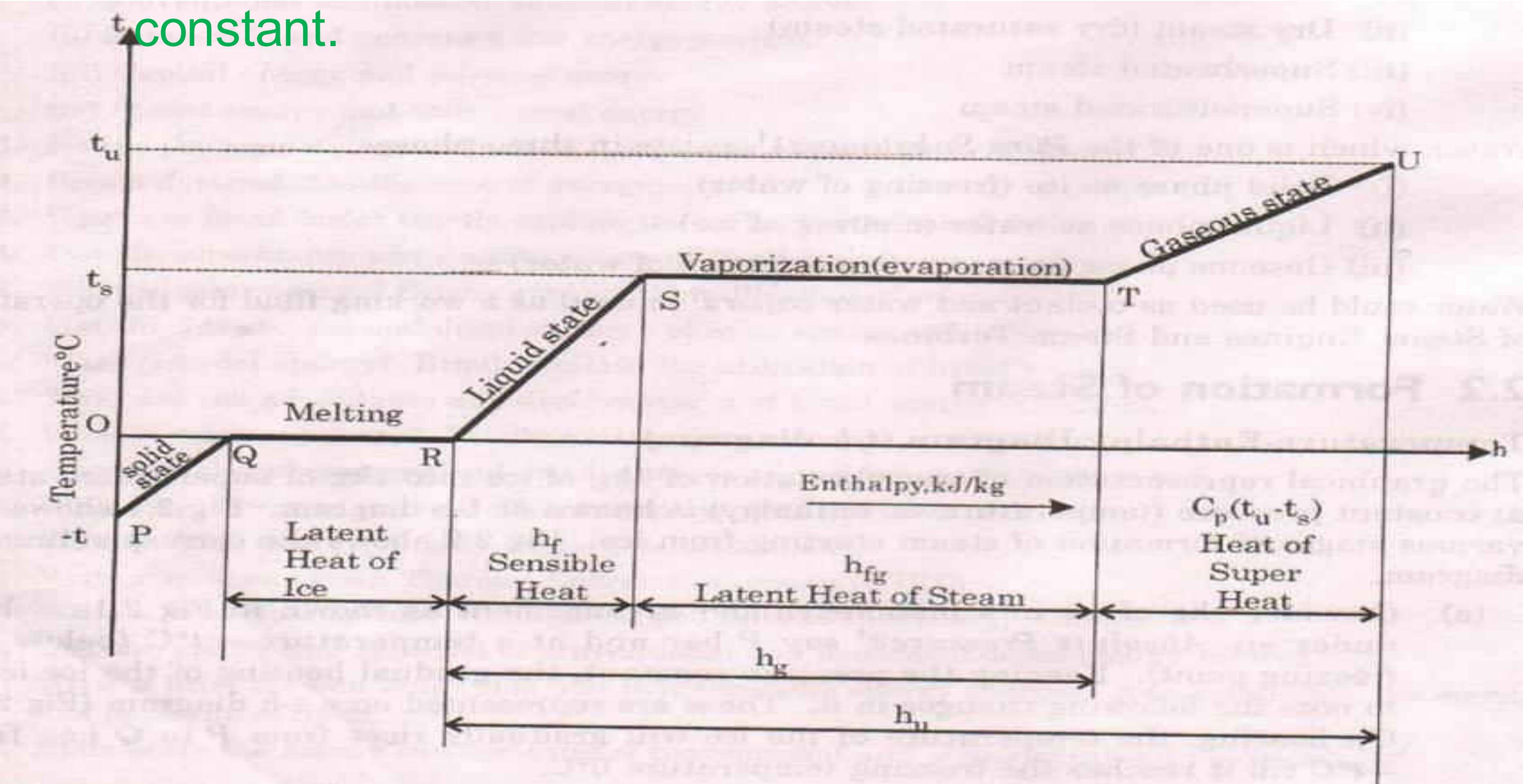


Fig 2.2 *Temperature-Enthalpy Diagram of Formation of Steam at Constant Pressure*

- The total heat supplied from O'C is called Enthalpy of Dry Steam (hg).
- (g) On further heating beyond point T to U the temperature starts from ts to tu, the point of interest. This process is called Super heating. The steam so obtained is called Super Heated Steam

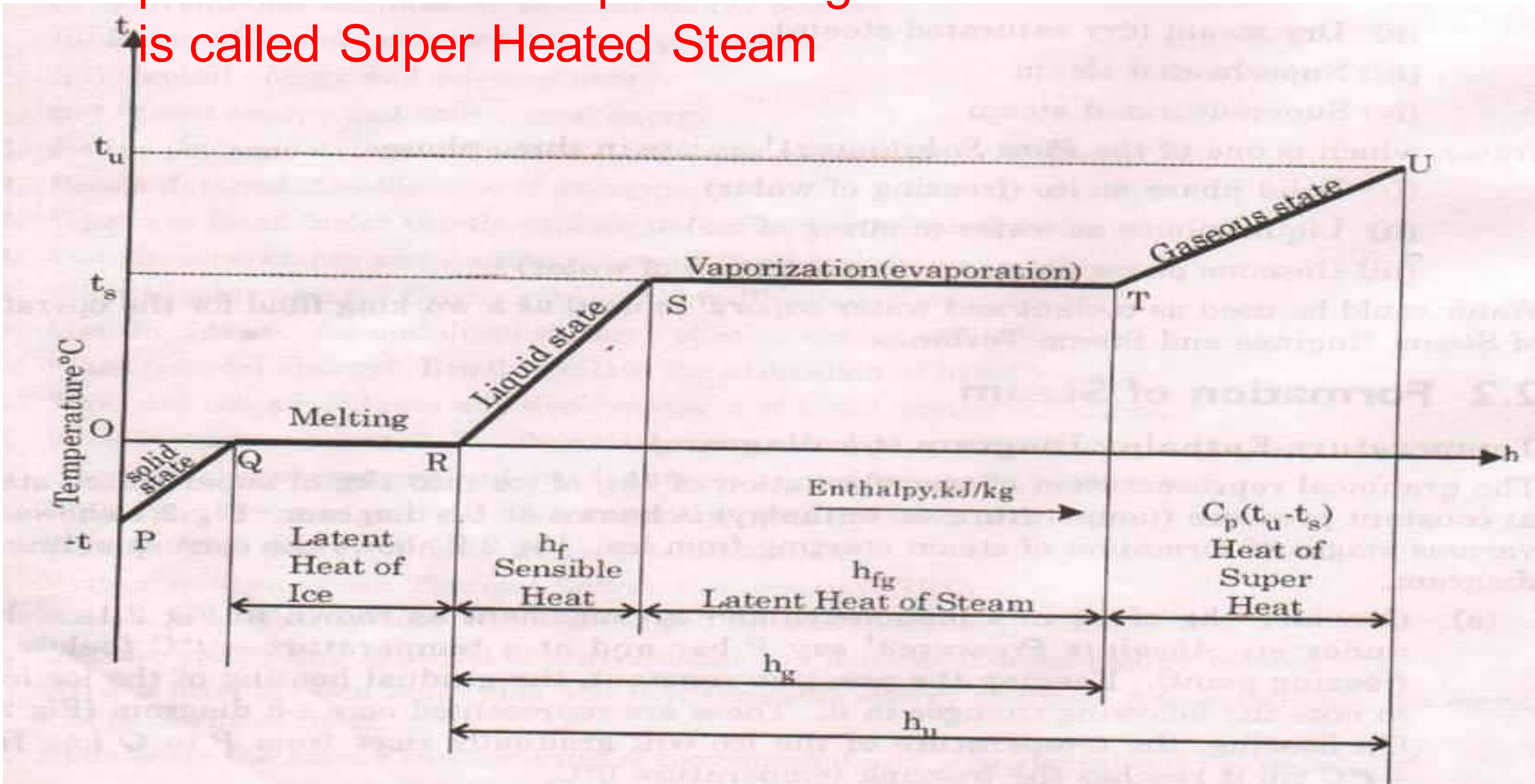


Fig 2.2 *Temperature-Enthalpy Diagram of Formation of Steam at Constant Pressure*

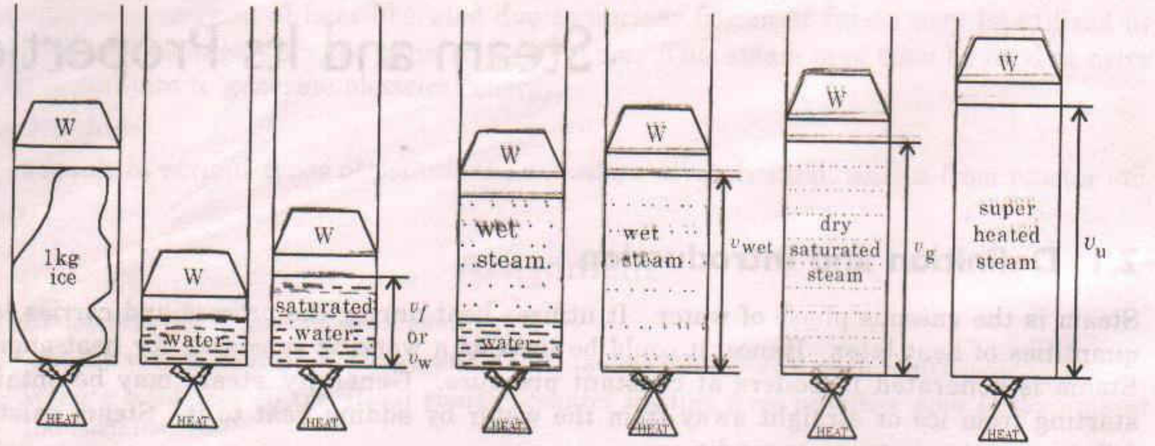


Fig 2.1 *Formation of Steam at Constant Pressure*

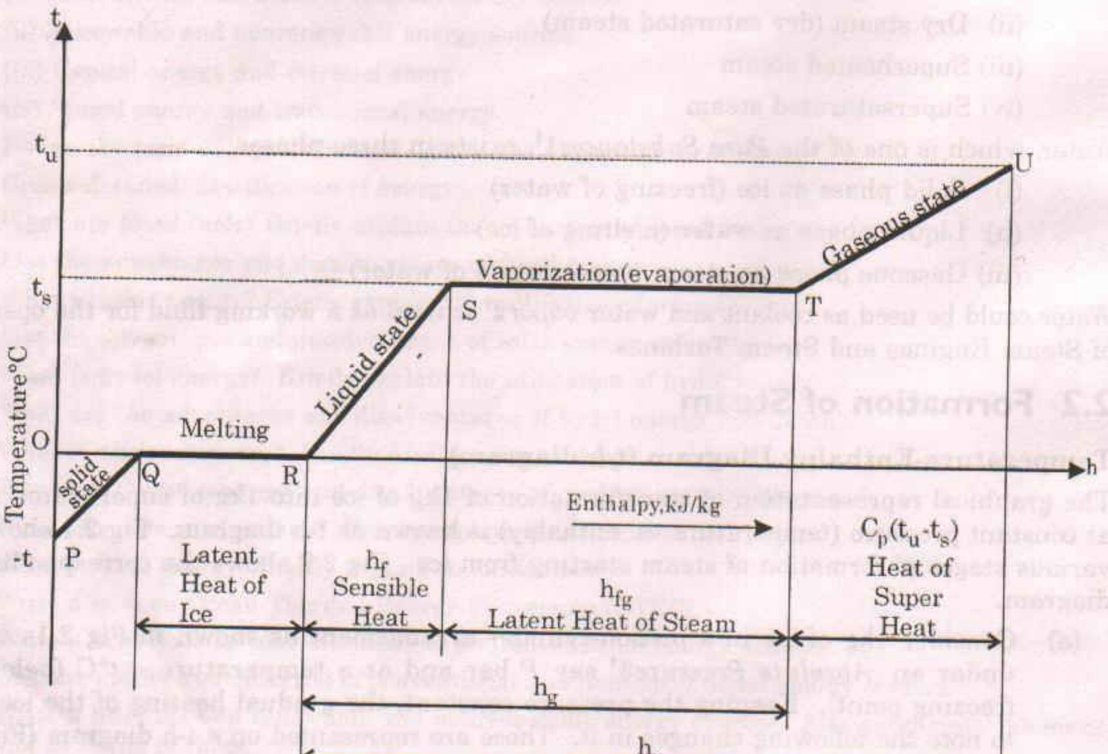


Fig 2.2 *Temperature-Enthalpy Diagram of Formation of Steam at Constant Pressure*

- The point a represents the initial condition of water at 0◦c and pressure P (in bar) as shown in fig.

Line ABCD shows the relation between temp and heat at a specific pressure of p(in bar).

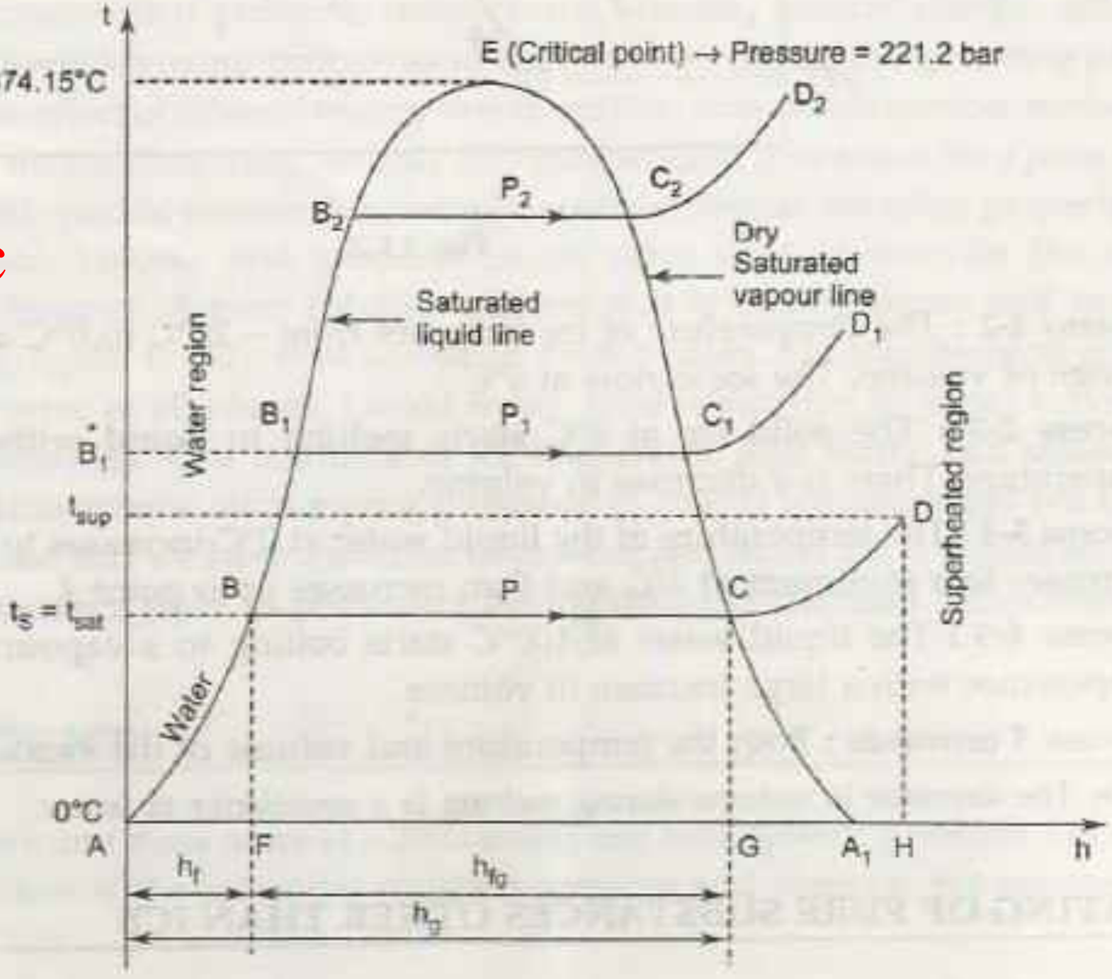


Fig. 11.4

- During the formation of the superheated steam, from water at freezing point , the heat is absorbed in the following three stages
- The heating of water up to boiling temperature or saturation temperature (t) is shown by ab in fig.

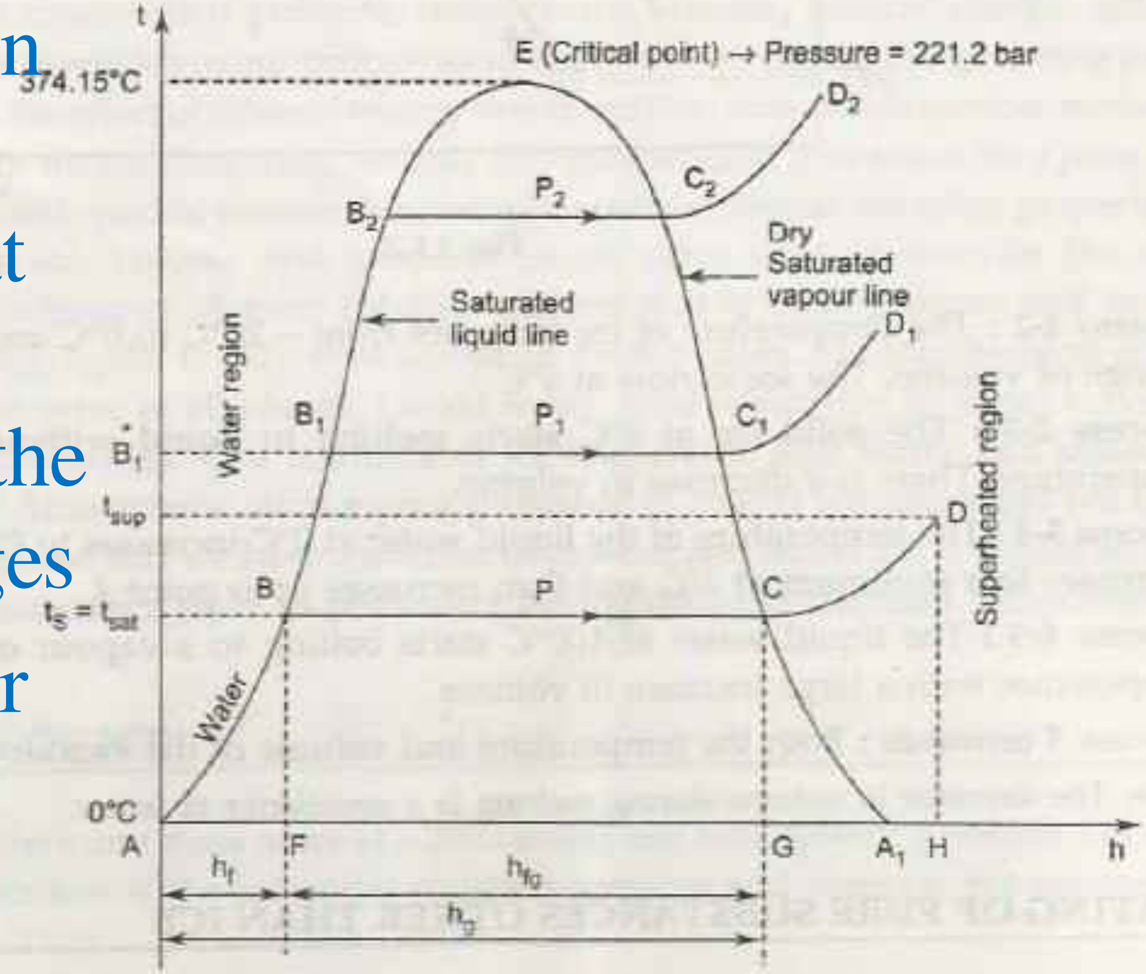


Fig. 11.4

the heat absorbed by the water is AP , known as sensible heat or liquid heat or total heat of water.

2) The change of state from liquid to steam is shown by BC. The heat absorbed during this stage is PQ, known as latent heat of vaporization

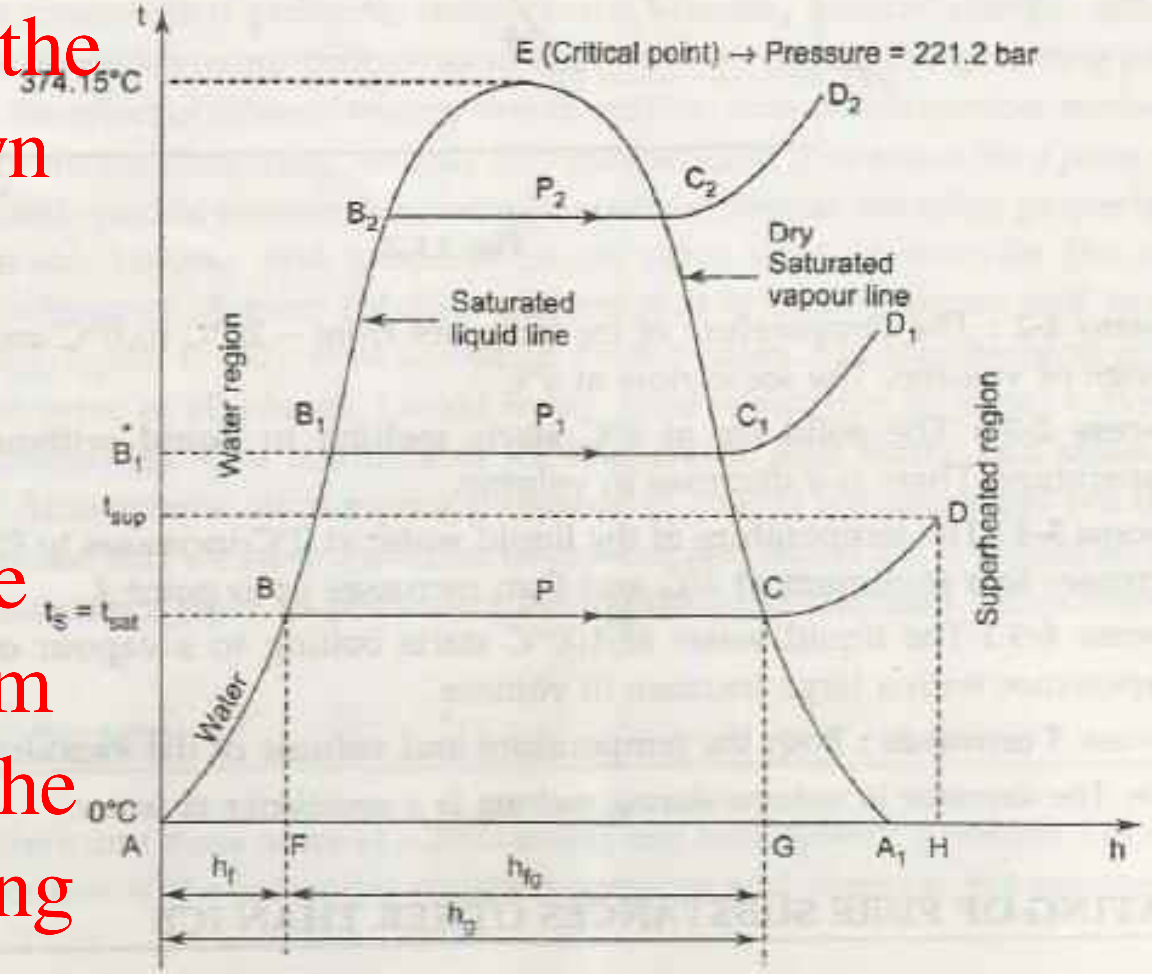


Fig. 11.4

The superheating process is shown by CD.The heat absorbed during this stage is QR, known as heat of superheat.

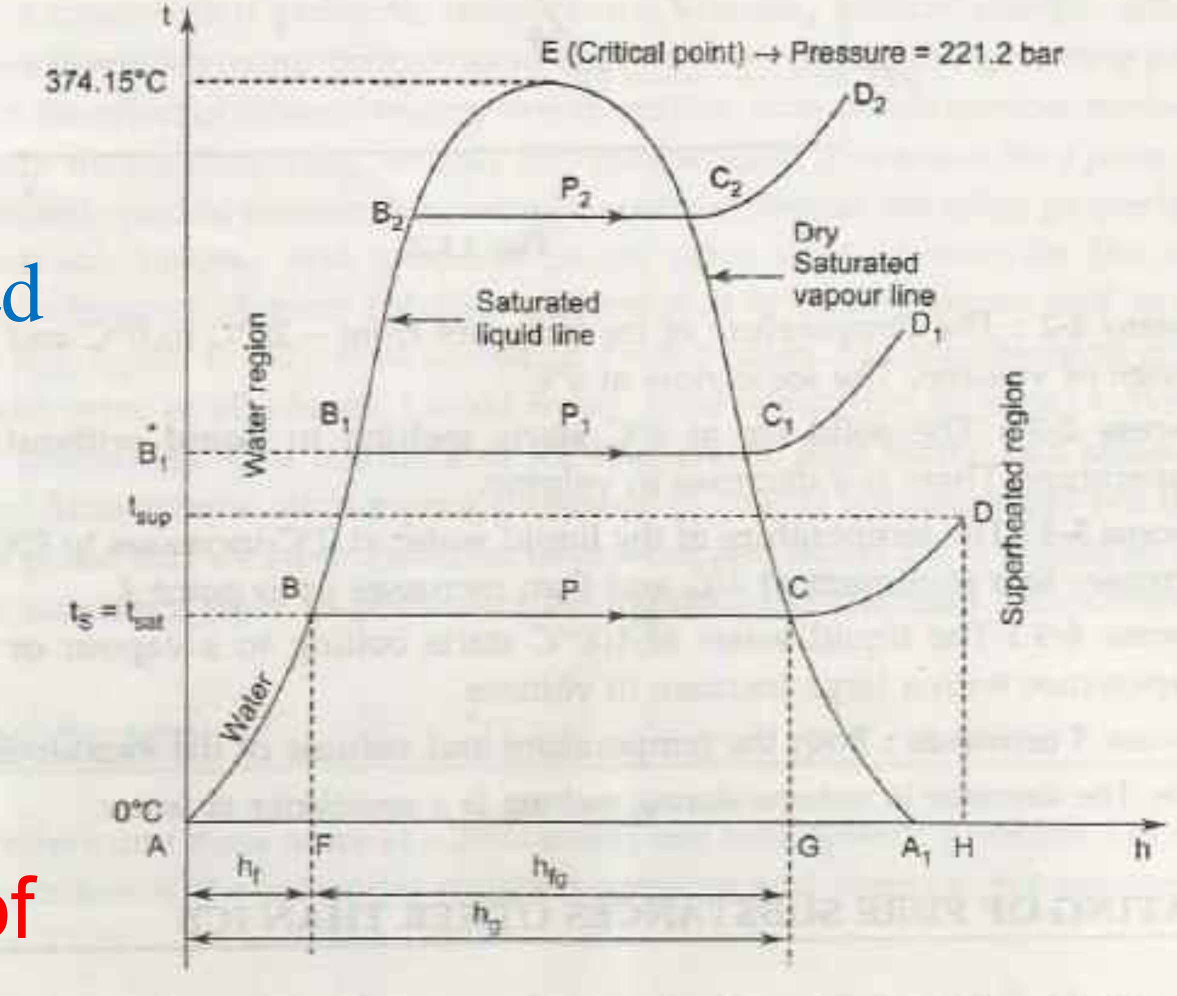


Fig. 11.4

Similarly, a family of curves may be drawn for different pressures as shown in the fig.

The line passing through the points A,B,E,K is known as saturated liquid line which forms boundary line between water and steam.

Similarly , a line passing Through dry steam points L,F,C is known as dry saturated steam line which forms boundary line between wet and superheated steam.

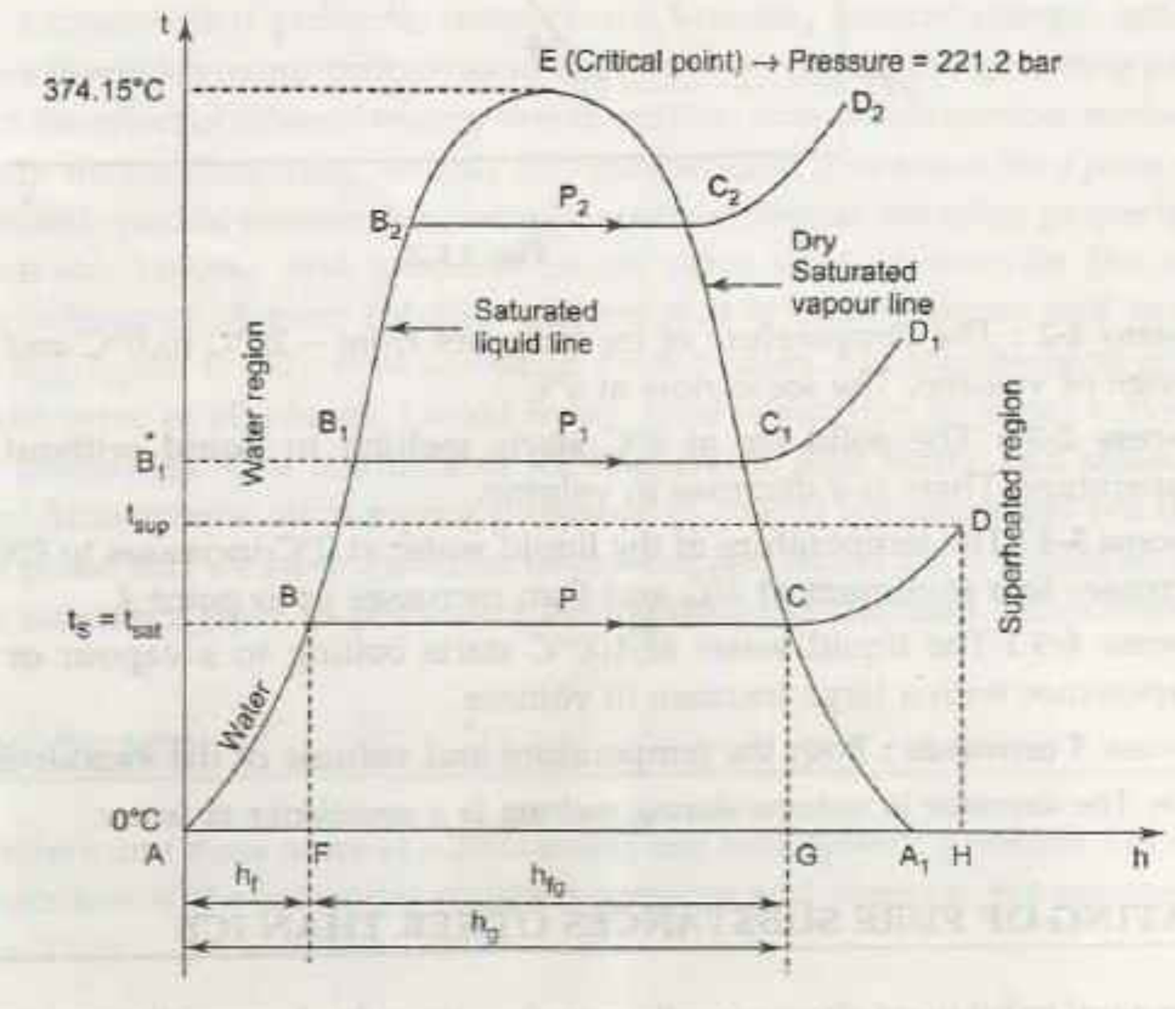


Fig. 11.4

The temperature corresponding to critical point N is known as critical temperature and the pressure is known as critical pressure.

For steam , the critical temperature is 374.15◦c and critical pressure is 221.2 bar.

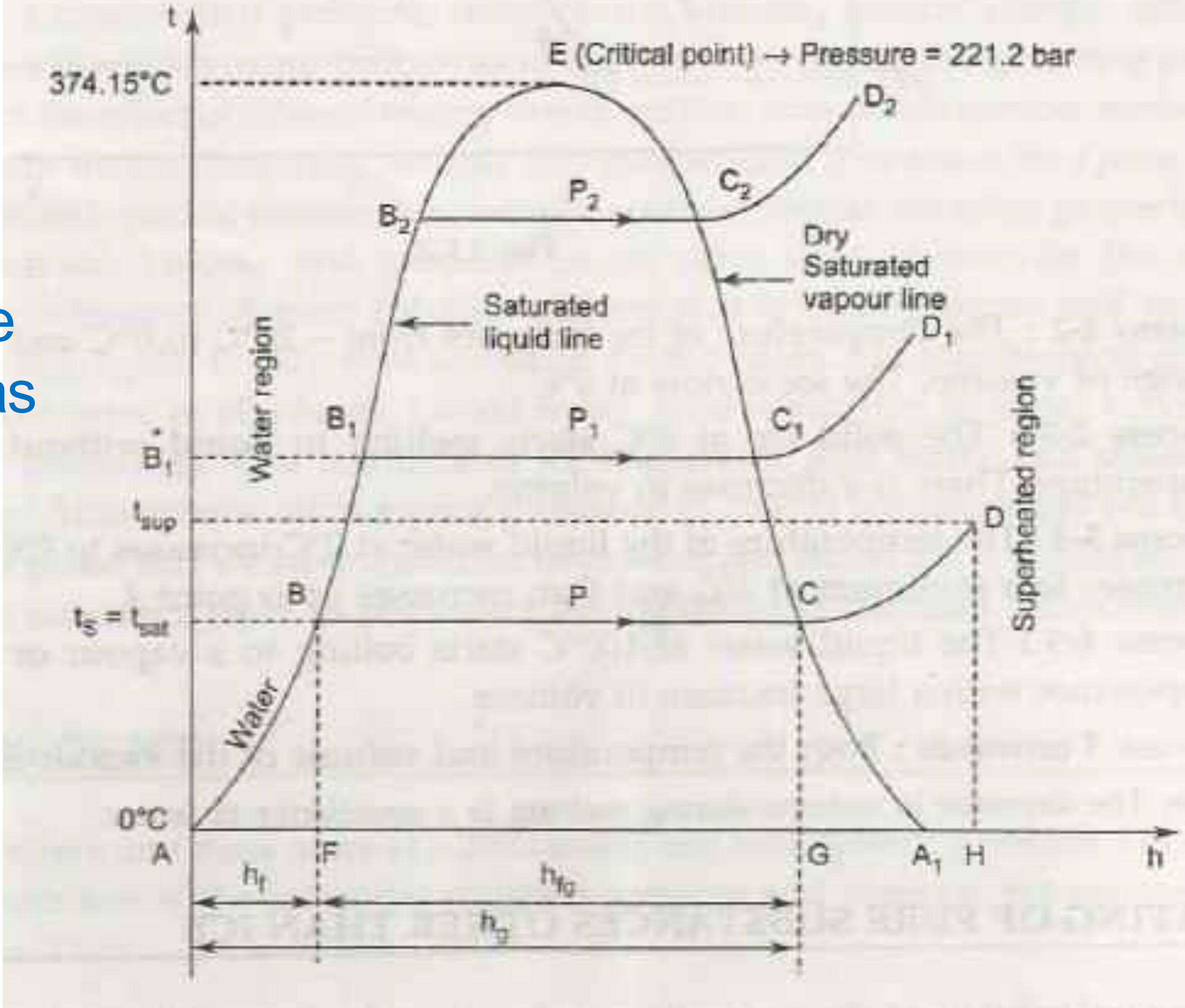


Fig. 11.4

Wet steam: when the steam contains moisture or particles of water in suspension , it is said to be wet steam.

- Dry saturated steam: when wet steam is further heated and it does not contain any suspended particle of water, it is known as dry saturated steam.
- Superheated steam: when the dry steam is further heated at a constant pressure, thus raising its temperature, it is said to be superheated steam.
- Since the pressure is constant therefore volume of superheated steam increases, it may be noted that the volume of 1 kg of superheated steam is considerably greater than the volume of 1 kg of dry saturated steam at same pressure.

- Dryness fraction or quality of wet steam: it is the ratio of the mass of actual dry steam ,to the mass of same quantity of wet steam.

It is generally denoted by ‘x’

Mathematically,

$$X = \frac{mg}{mg + mf} = \frac{mg}{m}$$

Mg= mass of actual dry steam

Mf= mass of water in suspension

M = mass of wet steam

- Sensible heat of water: it is amount of heat absorbed by 1kg of water , when heated at a constant pressure, from the freezing point (0◦c)to the temperature of formation of steam, i.e. saturation temp.
- Latent heat of vaporization: it is amount of heat absorbed to evaporate 1 kg of water , at its boiling point or saturation temperature without change of temperature.

It is denoted by h(fg).

The latent heat of steam is 2257 kJ/kg at atmospheric pressure.

- Enthalpy or total heat of steam: it is amount of heat absorbed by water from freezing point to saturation temperature plus heat absorbed during evaporation.

enthalpy= sensible heat+ latent heat

It is denoted by h(g)

For wet steam h= h(f)+ x h(fg)

Dry steam h=h(g)= h(f)+h(fg)

Superheated steam h(sup)= total heat for dry steam + heat for superheated steam .

=h(f)+h(fg)+Cp(t sup –t)

=h(g)+Cp(t sup- t)

Cp= mean specific heat at constant pressure for superheated steam

t sup= temp. of superheated steam

t= saturation temp at the given constant pressure.

The difference (t sup - t) is known as degree of superheat.

- Specific volume of steam

It is the volume occupied by the steam per unit mass at a given temperature and pressure.

It is expressed in m3/kg.

Classification of boilers

- **1. Horizontal, Vertical or Inclined Boiler.**

  -If the axis of the boiler is horizontal, the boiler is called horizontal,
- -If the axis is vertical, it is called vertical boiler and
- -If the axis is inclined it is called as inclined boiler.

Classification of boilers

- **2. Fire Tube and Water Tube**

  -In the fire boilers, the hot gases are inside the tubes and the water surrounds the tubes.
  Examples: Cochran, Lancashire and Locomotive boilers.

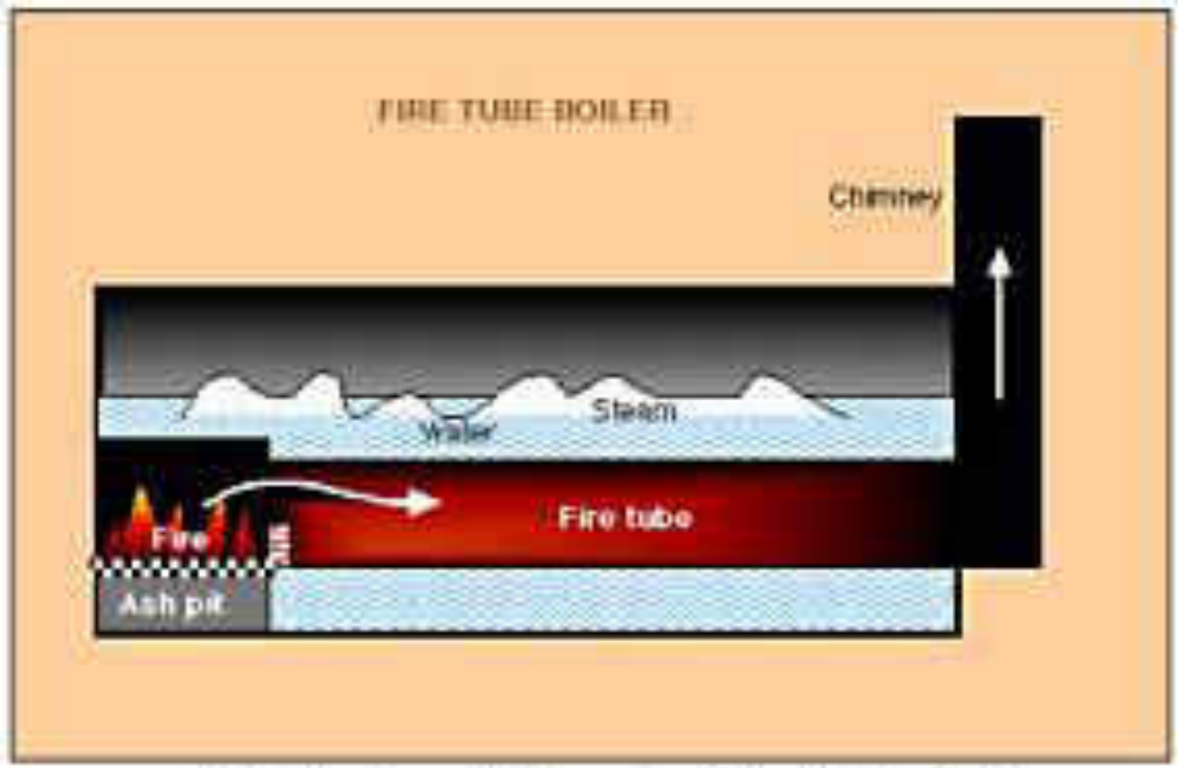
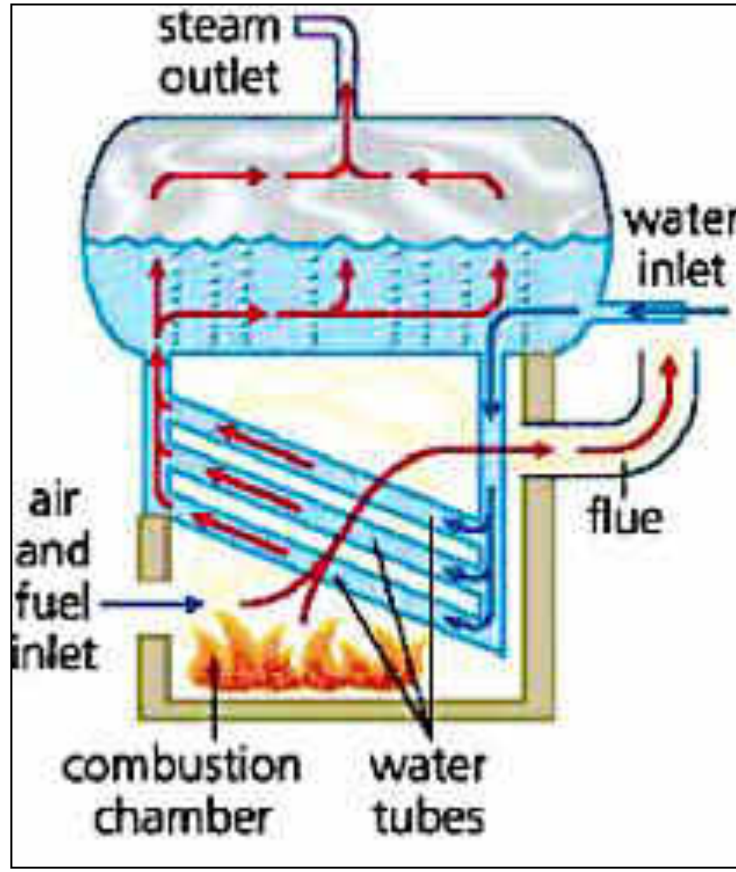


Fig: Sectional View of a Fire Tube Boiler

Classification of boilers

-In the water tube boilers, the water is inside the tubes and hot gases surround them. Examples: Babcock and Wilcox,
Yarrow boiler etc.



Classification of boilers

- **3. Externally fired and internally fired**

  The boiler is known as externally fired if the fire is outside the shell.
  Examples: Babcock and Wilcox boiler
  -
- In case of internally fired boilers, the furnace is located inside the shell.
  Examples: Cochran, Lancashire boiler etc.

Classification of boilers

- **4. Forced circulation and Natural Circulation**

  -In forced circulation type of boilers, the circulation of water is done by a forced pump.
  Examples: Lamont, Benson Boiler etc.

- -In natural circulation type of boilers, circulation of water in the boiler takes place due to natural convention currents produced by the application of heat.
  Examples: Lancashire, Babcock and Wilcox boiler etc.

Classification of boilers

- **5. Higher Pressure and Low Pressure Boilers**
  -The boiler which produce steam at pressures of 80 bar and above are
  called high pressure boilers.
  Examples: Babcock and Wilcox, Benson Boiler etc.

- -The boilers which produce steam at pressure below 80 bar are called low
  pressure boilers.
  Examples: Cochran, Cornish, Lancashire and Locomotive boiler etc.

Classification of boilers

- **6. Stationary and Portable**

  Stationary boilers are used for power plant steam, for central station utility power plants, for plant process steam etc.

- Mobile boilers or portable boilers include locomotive type, and other small units for temporary use at sites.

Classification of boilers

- **7. Single Tube and Multi Tube Boiler**

  The fire tube boilers are classified as
- -single tube and
- -multi-tube boilers,
  depending upon whether the fire tube is one or more than one.
  Examples: Cornish, simple vertical boiler are the single tube boiler and rest
  of the boilers are multi-tube boiler.

  8. According to passes
  -single pass
  -Multi pass

Cochran boiler

- It is a multi-tubular vertical fire tube boiler having a number of horizontal fire tubes. it is the modification of a simple vertical boiler where the heating surface has been increased by means of a number of fire tubes.

  It consists of

  Shell
- grate
- Fire box
- Flue pipe
- Fire tubes
- Combustion chamber
- Chimney
- Man-hole

Cochran boiler

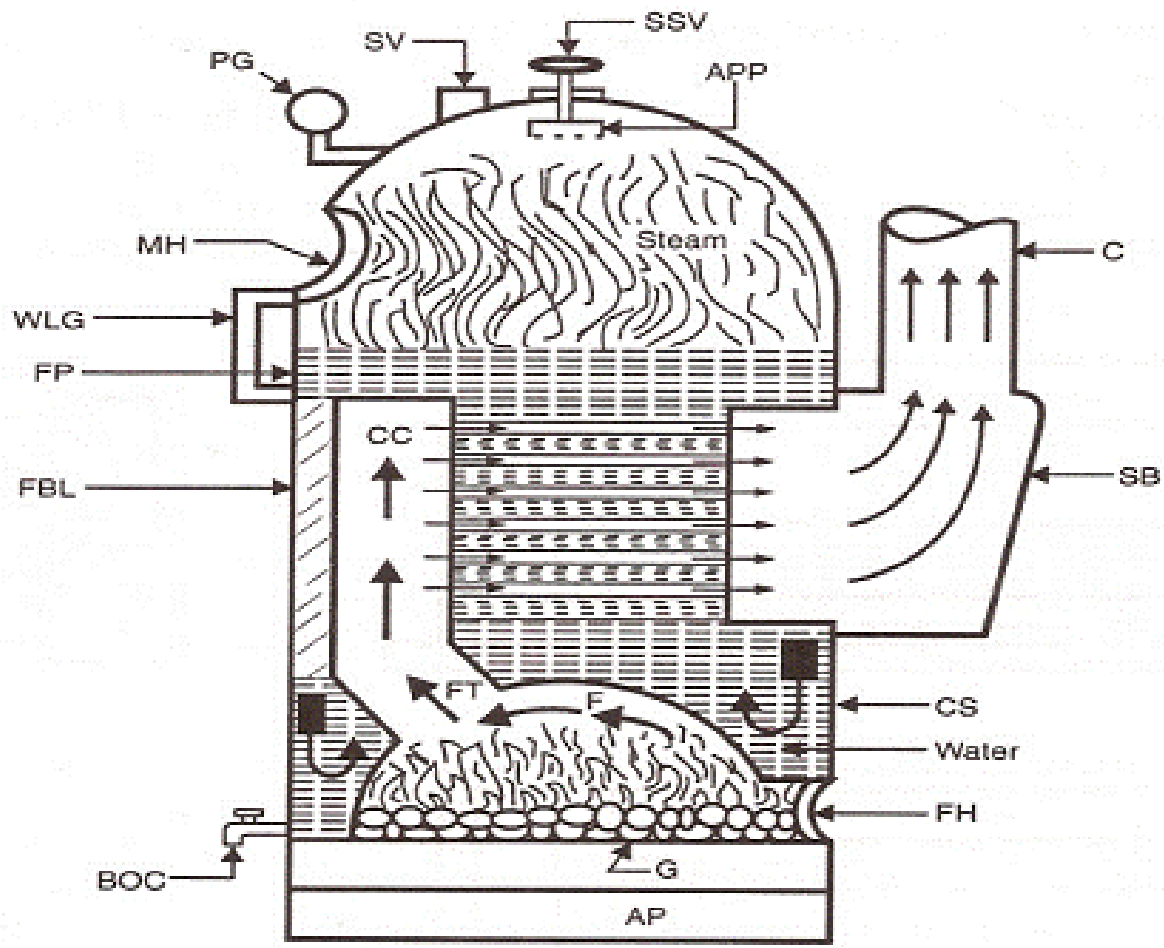
- **Shell**
  It is hemispherical on the top, where space is provided for steam.

  Grate
  It is placed at the bottom of the furnace where coal is burnt.

  Fire box (furnace)
  It is also dome-shaped like the shell so that the gases can be deflected back till they are passed out through the flue pipe to the combustion chamber.

  Flue pipe:
  It is a short passage connecting the fire box with the combustion chamber.

  Fire tubes:
  A number of horizontal fire tubes are provided, thereby the heating surface is increased.



CS = Cylindrical shell
CC = Combustion chamber
FBL = Fir brick lining
F = Furnace (dome shaped)
BOC = Blow off cock
SSV = Steam stop valve
APP = Antipriming pipe
PG = Pressure gauge

FT = Flue tube
SB = Smoke box
C = Chimney
FH = Fire hole
G = Grate
AP = Ash pit
SV = Safety valve
MH = Man hole
WLG = Water level gauge

Cochran boiler

- **Combustion chamber:**
  It is lined with fire bricks on the side of the shell to prevent overheating of the boiler. Hot gases enter the fire tubes from the flue pipe through the combustion chamber.

  Chimney:
  It is provided for the exit of the flue gases to the atmosphere from the smoke box.

  Manhole:
  It is provided for inspection and repair of the interior of the boiler shell.

  Normal size of a Cochran boiler:
  Shell diameter – 2.75 meters:
  Height of the shell – 6 meters.

Cochran boiler

- **Working of the Cochran boiler:**
- Coal is fed into the grate through the fire hole and burnt. Ash formed during burning is collected in the ashpit provided just below the grate and then it is removed manually.

  The hot gases from the grate pass through the flue pipe to the combustion chamber. The hot gases from the combustion chamber flow through the horizontal fire tubes and transfer the heat to the water by convection.

  The flue gases coming out of fire tubes pass through the smoke box and are exhausted to the atmosphere through the chimney.
  Smoke box is provided with a door for cleaning the fire tubes and smoke box.

Cochran boiler

- **The following mountings are fitted to the boiler:**

  Pressure gauge: this indicates the pressure of the steam inside the boiler.

  Water gauge: this indicates the water level in the boiler. The water level in the boiler should not fall below a particular level, otherwise the boiler will be over heated and the tubes may burn out.

  Safety valve: the function of the safety valve is to prevent an increase of steam pressure in the boiler above its normal working pressure.

  Steam stop valve: it regulates the flow of steam supply to requirements.

  Blow-off cock: it is located at the bottom of the boiler. When the blow-off cock is opened during the running of the boiler, the high pressure steam pushes (drains) out the impurities like mud, sand, etc., in the water collected at the bottom.

  Fusible plug: it protects the fire tubes from burning when the water level in the boiler falls abnormally low.

Cochran boiler

- **Salient features of Cochran boiler:**
- The dome shape of the furnace causes the hot gases to deflect back and pass through the flue. The un-burnt fuel if any will also be deflected back.
- Spherical shape of the top of the shell and the fire box gives higher area by volume ratio.
- It occupies comparatively less floor area and is very compact.
- It is well suited for small capacity requirements.

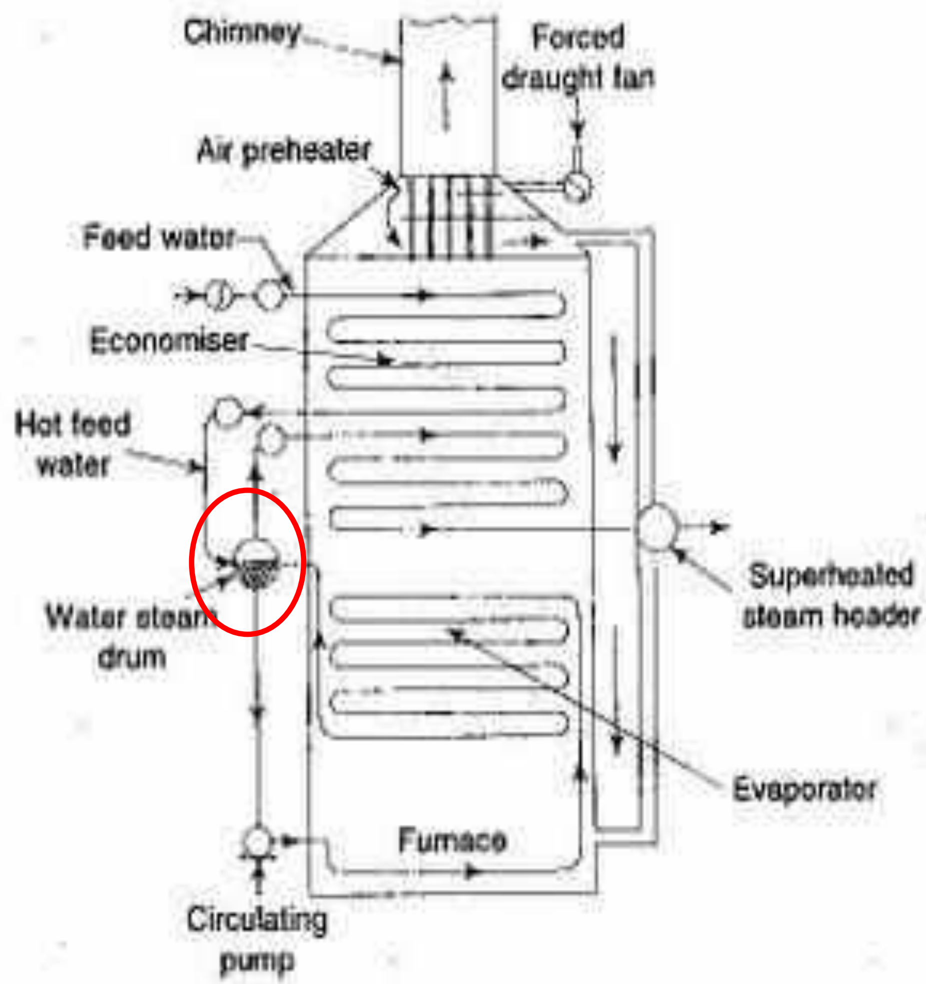
la Mont boiler

- A forced circulation boiler was first introduced by La-Mont in the year 1925 which is used in Europe and America. This is a modern high pressure boiler (water tube type steam boilers) working on forced circulation system.

  Working principle of La Mont Boiler
  The image shows the flow circuit of La Mont Boiler.

la Mont boiler

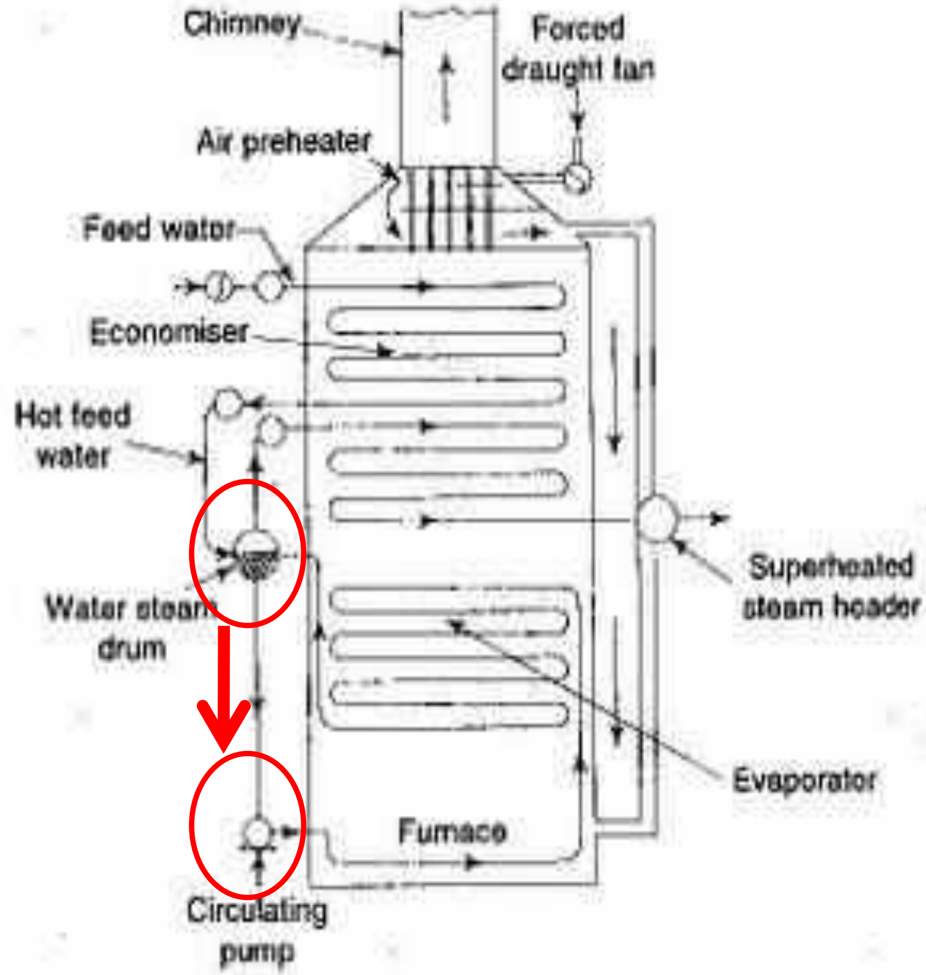
- **Steam separator drum** The la Mont boiler consists of a steam separator drum which is placed wholly outside the boiler setting .
- The drum receives a mixture of steam and water from the evaporator tubes and feed water from the economizer.
- The steam is separated from water in the drum.



High pressure Lamont boiler

la Mont boiler

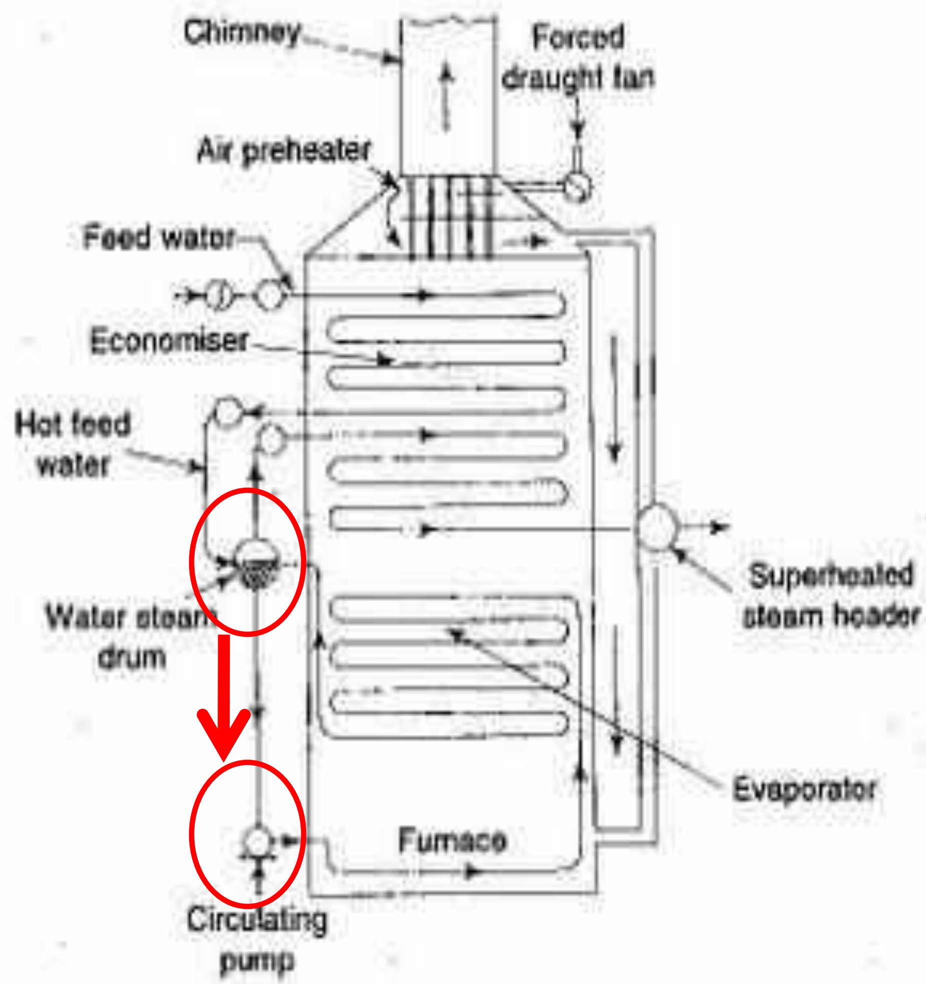
- **Circulating pump**
  The water from the drum is then drawn to the circulating (centrifugal) pump through the down-comer.
- The pump circulates water ("forced circulation") equal to 8 to 10 times the weight of steam evaporated.
- This prevents the tubes from being overheated.



High pressure Lamont boiler

la Mont boiler

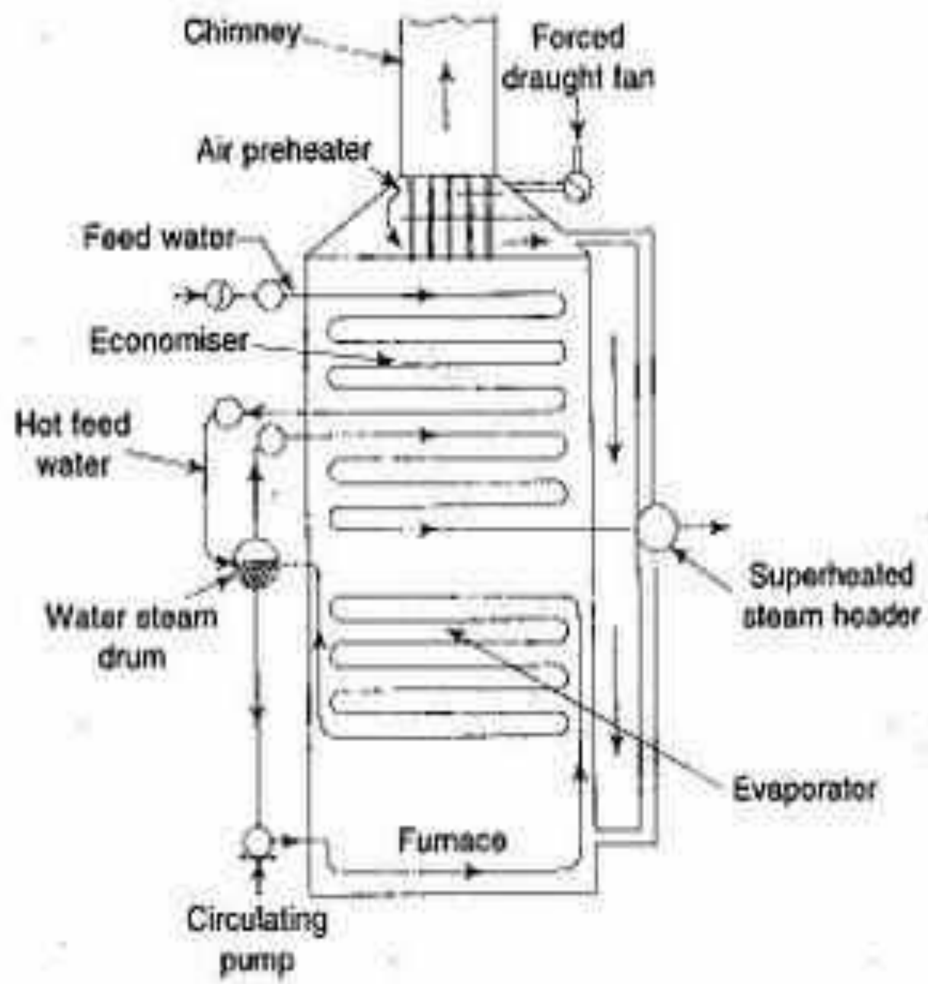
- **Distributing header**
  The circulating pump delivers the feed water to the distributing header with orifices at a pressure above the drum pressure.



High pressure Lamont boiler

la Mont boiler

- **Evaporator**
  The header distributes water through orifices into the evaporator tubes acting in parallel.
- Orifice in the header controls the flow of water to the evaporator tubes.
- Here part of the water is evaporated and a mixture of steam and water from these tubes enters the drum.

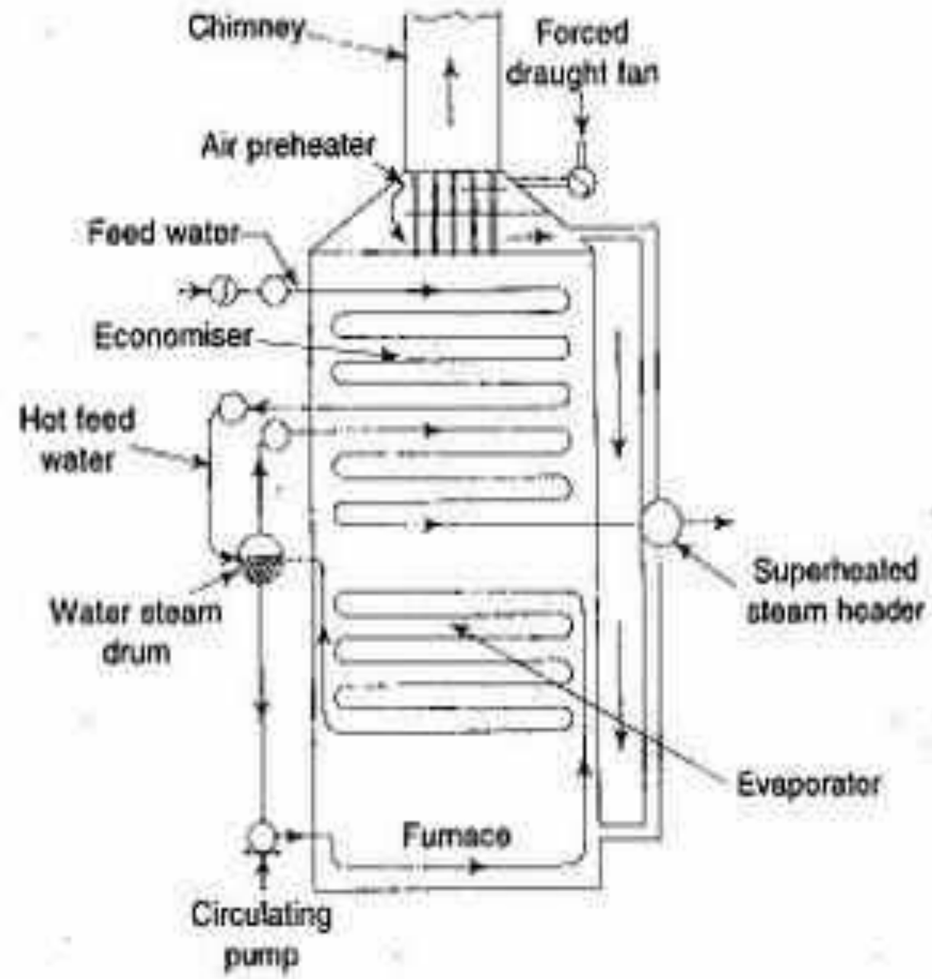


High pressure Lamont boiler

la Mont boiler

- **Convection super heater**

- The steam produced in the boiler is nearly saturated.
- This steam as such should not be used in the steam turbine.
- The presence of moisture in it will cause corrosion of turbine blades, etc. to raise the temperature of steam and thereby to increase the turbine efficiency, super heater is used.

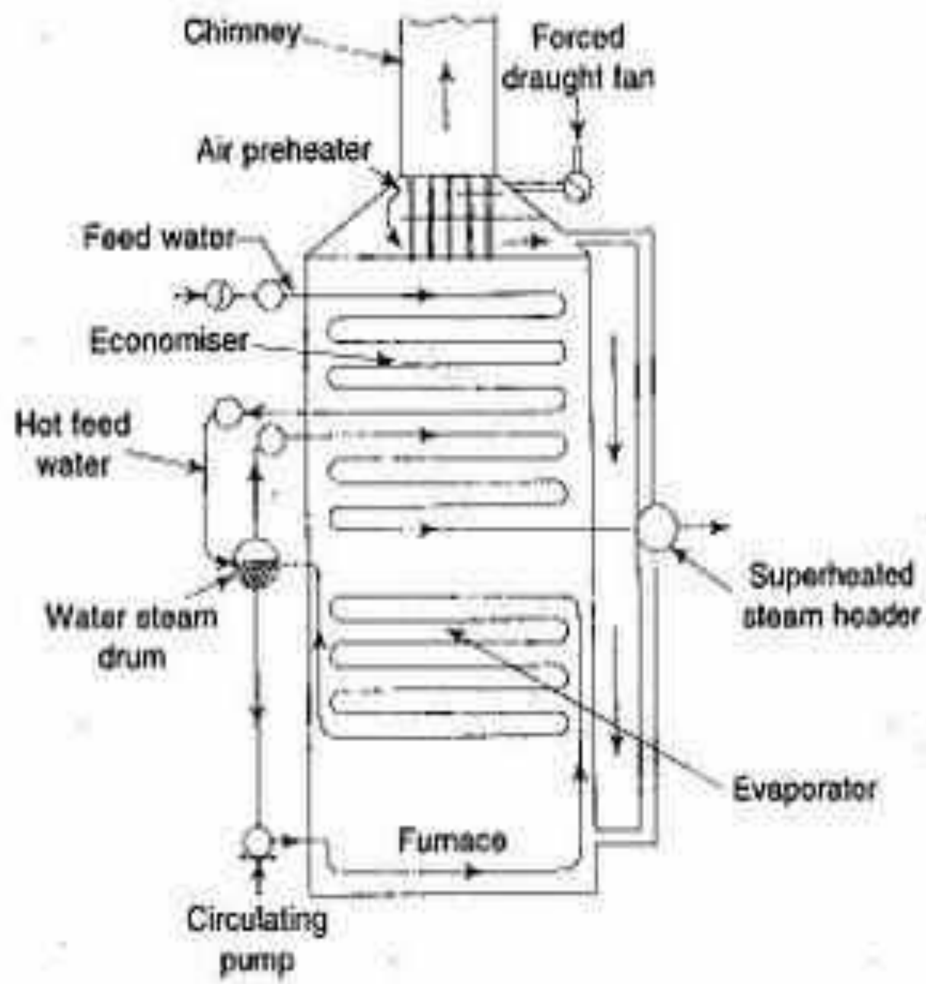


High pressure Lamont boiler

la Mont boiler

- **Steam outlet**
  Superheated steam from the super heater passes out to the steam turbine through the steam outlet.

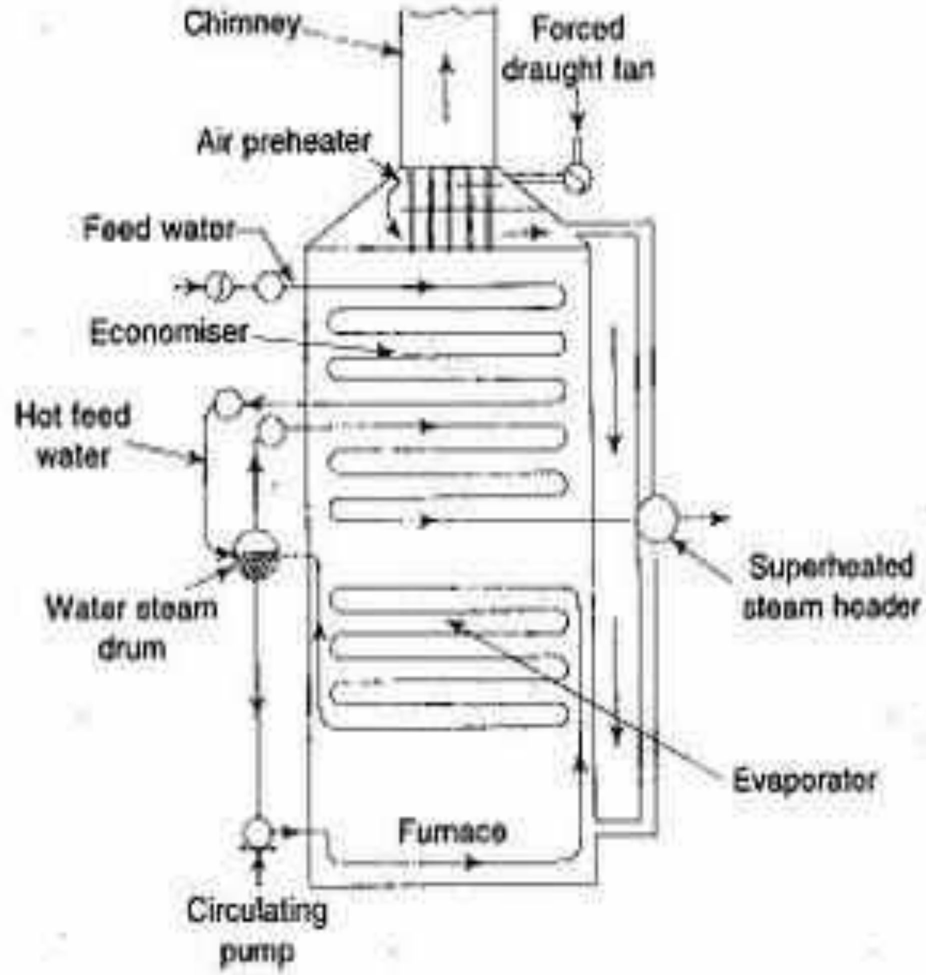
  Economizer
  The quantity of superheated steam thus delivered to turbine is continuously made up in the form of feed water.
- Feed water supplied by the feed pump is heated in the economizer on its way to the steam separator drum.



High pressure Lamont boiler

la Mont boiler

- The economizer is a device used to preheat the feed water using the hot gases leaving the boiler.
- Before the gases are let off to the atmosphere, they are made to flow in a definite passage in the economizer so that some of the heat in the hot gases, which otherwise gets wasted, can be used to preheat the feed water.
- The preheated water requires only a small amount of heat to be supplied in the boiler, resulting in some saving of the fuel burnt.



High pressure Lamont boiler

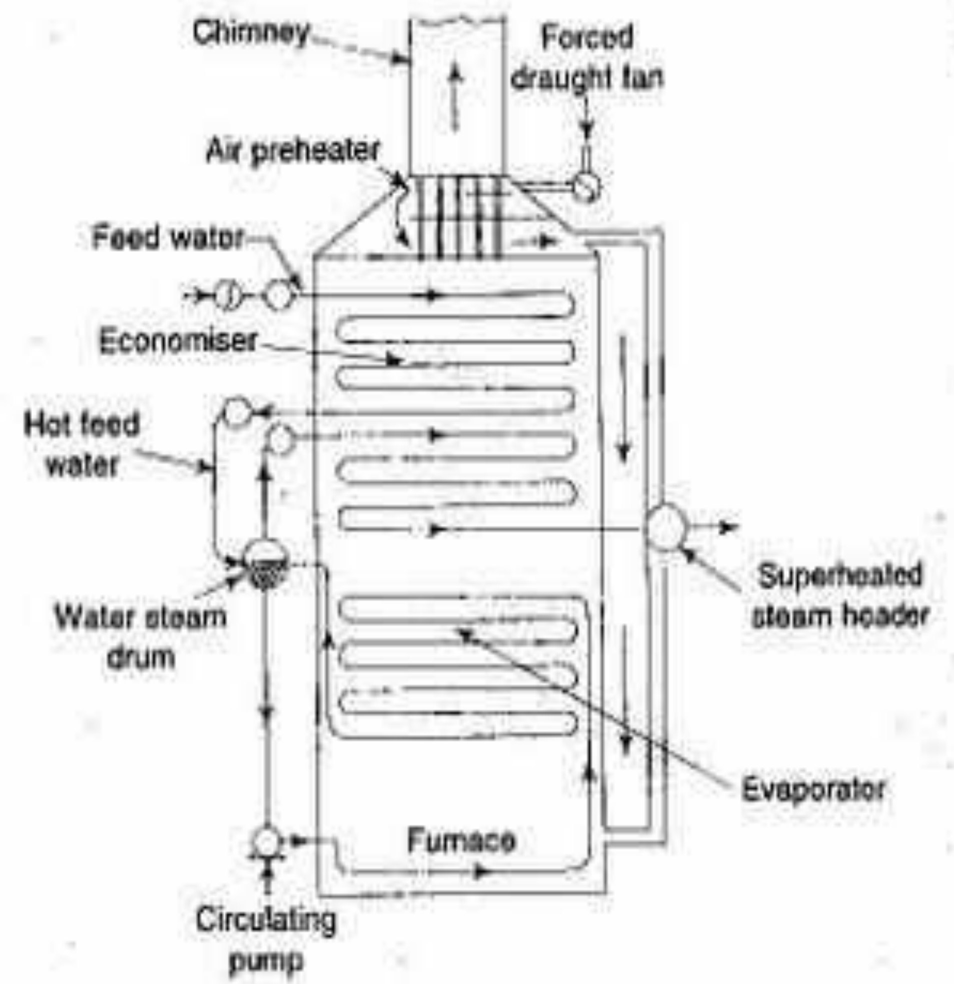
la Mont boiler

Air pre heater

Since the heat of the exit gases cannot be fully extracted through the economizer, the air pre heater is employed to recover some of the heat escaping in these gases.

These exit gases preheat the air from the blower in the air pre heater.

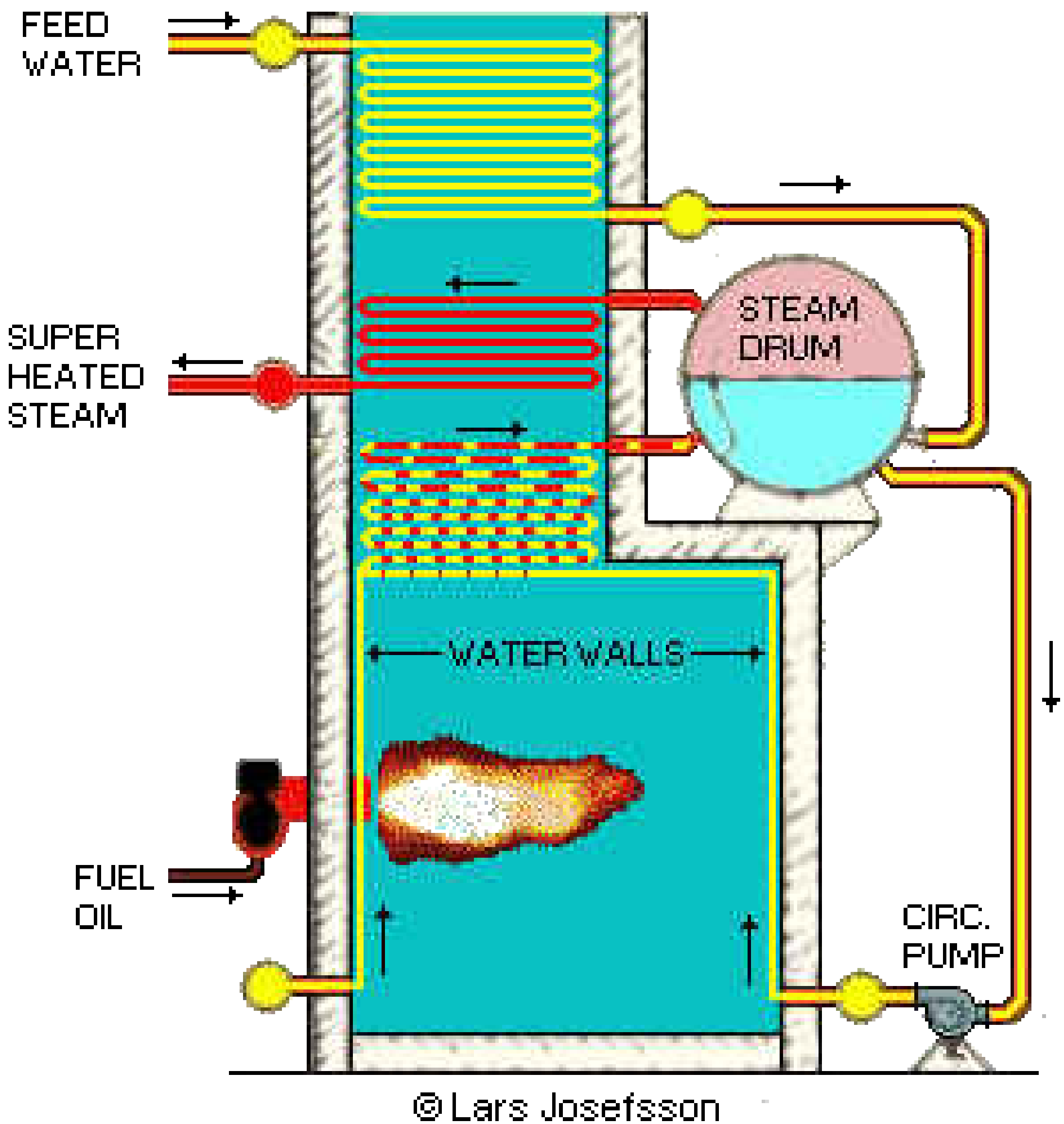
The preheated air is supplied to the furnace for combustion.



High pressure Lamont boiler

Capacity

The capacity of la-mont boiler is about 50 Tonnes/hr of superheated steam at a pressure of 170 kgf/sq.cm. and at a temperature of 500’C..


FEED
WATER
SUPER
HEATED
STEAM
STEAM
DRUM
WATER WALLS
FUEL
OIL
CIRC.
PUMP
© Lars Josefsson

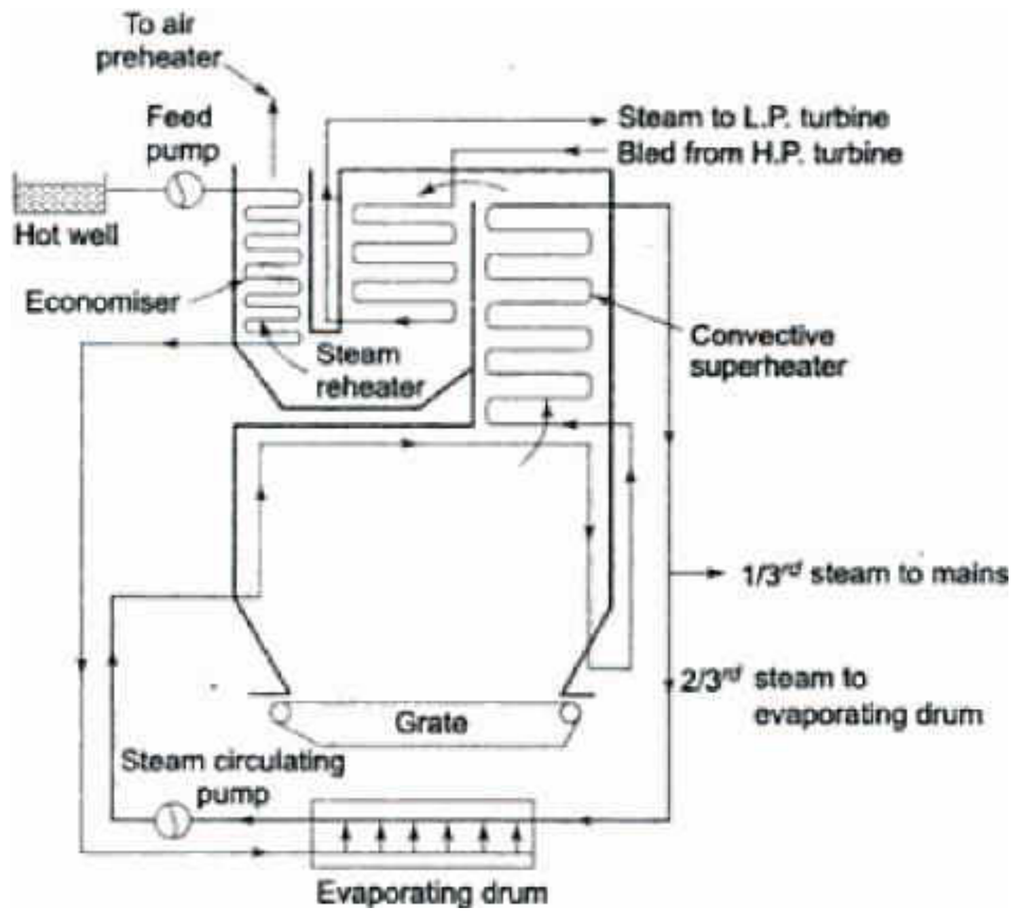
Loeffler Boiler

- **Salient features of Loeffler Boiler**
  The novel feature of the Loeffler Boiler is to evaporate water solely by means of superheated steam. The furnace heat is supplied only to economiser and superheater. In other words, steam is used as a heat absorbing medium.

  The major difficulty experienced in La-Mont boiler is deposition of salt and sediment on the inner surfaces of water tubes.
- The deposition reduces the heat transfer, ultimately, the generating capacity. This difficulty was solved in Loeffler boiler by preventing the flow of water into the boiler tubes.
- Feed water is evaporated in the drum using part of the superheated steam coming out from the water-heater.
- Thus only the dry saturated steam passes through the tubes. Poor feed water can, therefore, be used without any difficulty in the boiler, which is great advantage of this boiler.

Loeffler Boiler

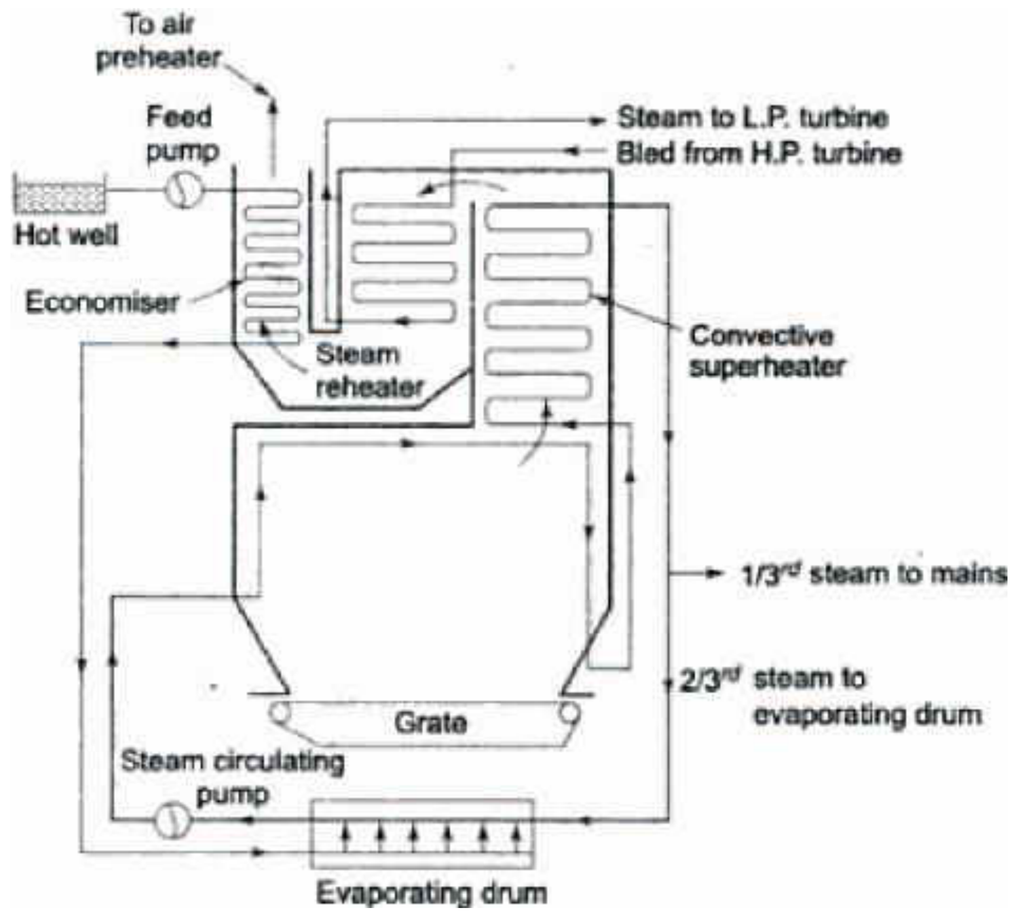
- **Economiser**
  The feed water from the feed tank is supplied to the economiser by feed pump.
- In the economiser the feed water is made to flow through a number of tubes surrounding which the hot gases leaving the furnace pass over.
- There is a heat exchange from the hot gases to the feed water, which is preheated in the economiser.



Loeffler Boiler

- **Evaporated Drum**

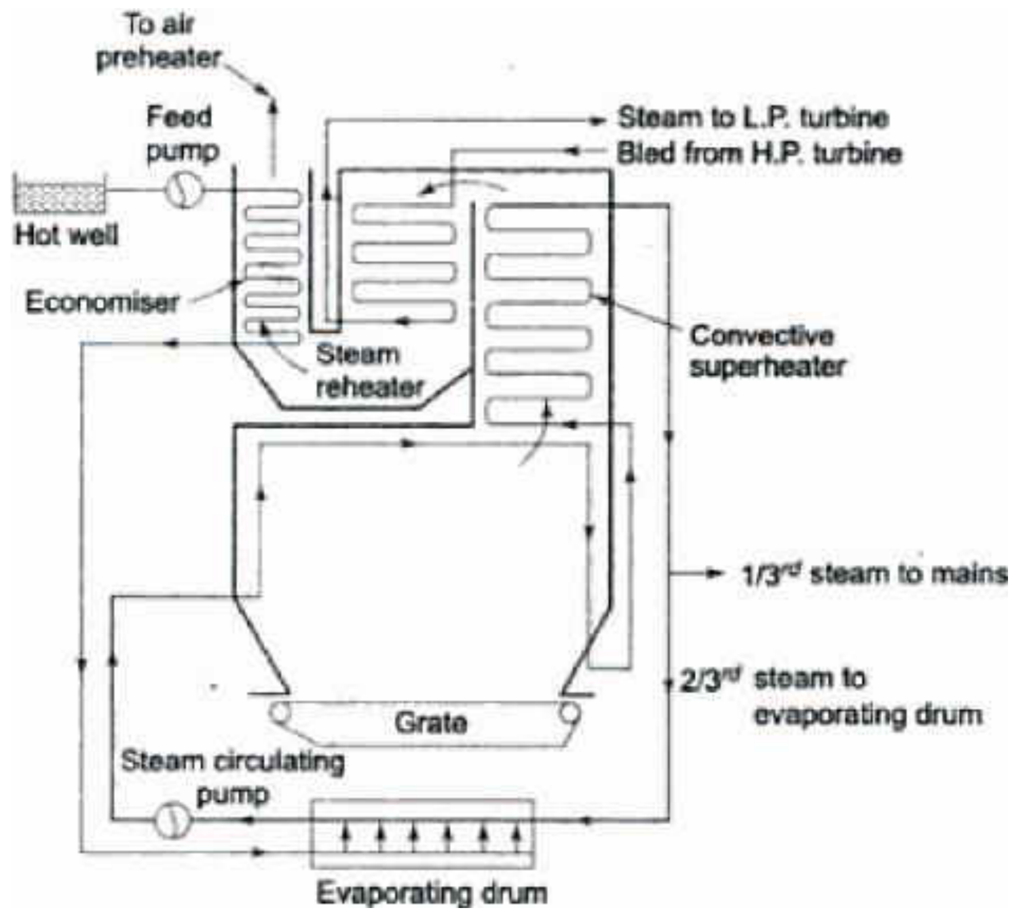
- It is housed away from the furnace.
- It contains a mixture of steam and water.
- The feed water from the economiser tubes enters the evaporator drum into which is also passed two-thirds of the superheated steam generated by the boiler.
- The superheated steam gives its superheat to the water in the drum and evaporates it to saturated steam.



Loeffler Boiler

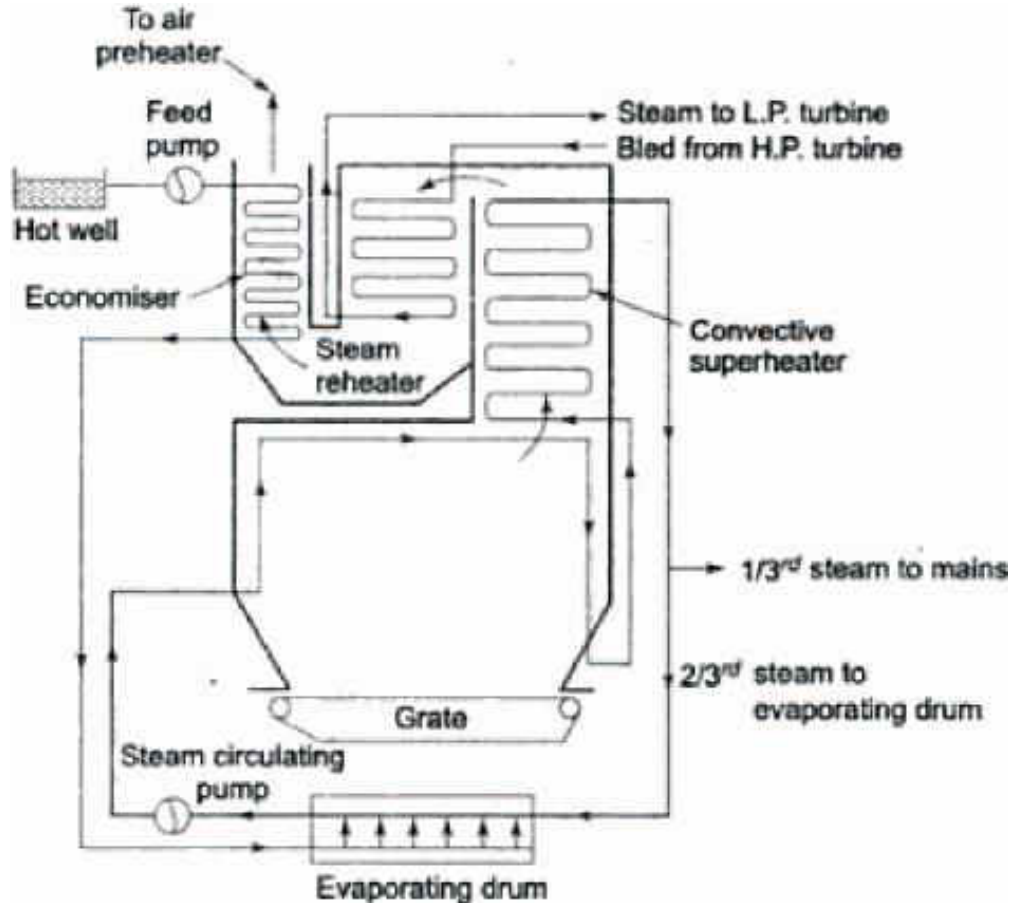
- **Mixing Nozzles**
  The nozzles distribute and mix the superheated steam throughout the water in the evaporator drum.

  Steam circulating pump
  A steam circulating pump forces this saturated steam from the evaporator drum to the radiant super heater through the tube of the furnace wall.



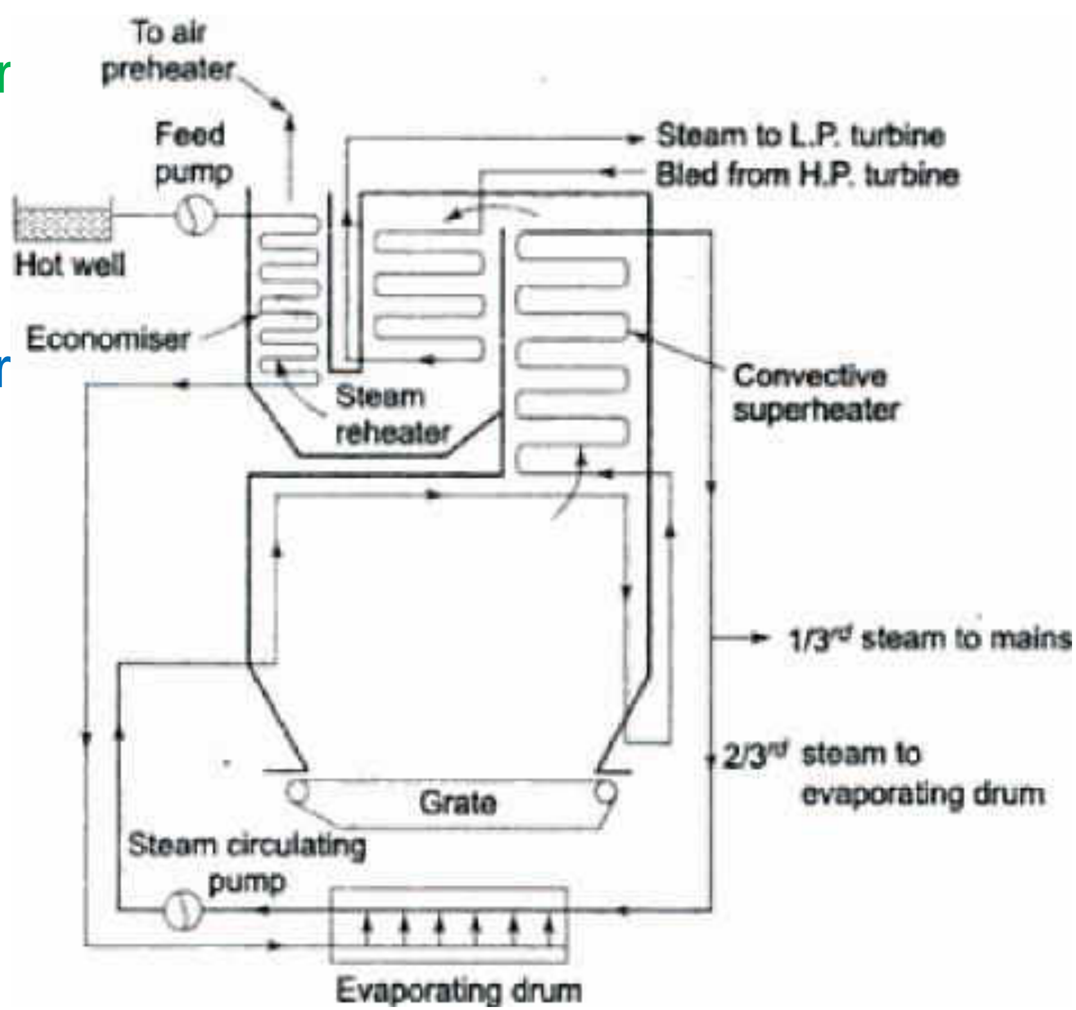
Loeffler Boiler

- **Radiant super heater** The radiant super heater is placed in the furnace.
- The hot gases in the furnace are used for superheating the saturated steam from the drum.
- The radiant super heater receives heat from the burning fuel through radiation process.



Loeffler Boiler

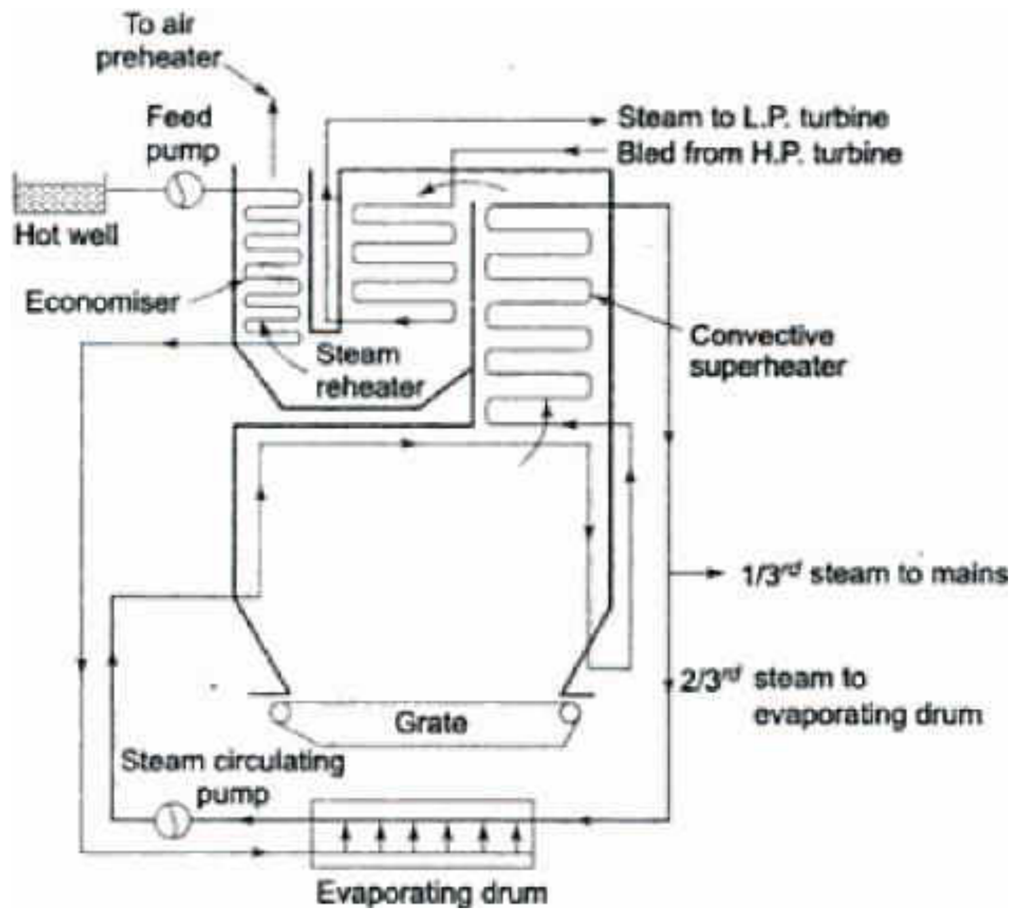
- **Convection super heater**
  Steam from the radiant super heater enters the convection super heater where it is finally heated to the desired temperature of 500'C.
- The convection super heater receives heat from the flue gases entirely by convective heat transfer.
- Both radiant and convection super heater are arranged in series in the path of the flue gases.



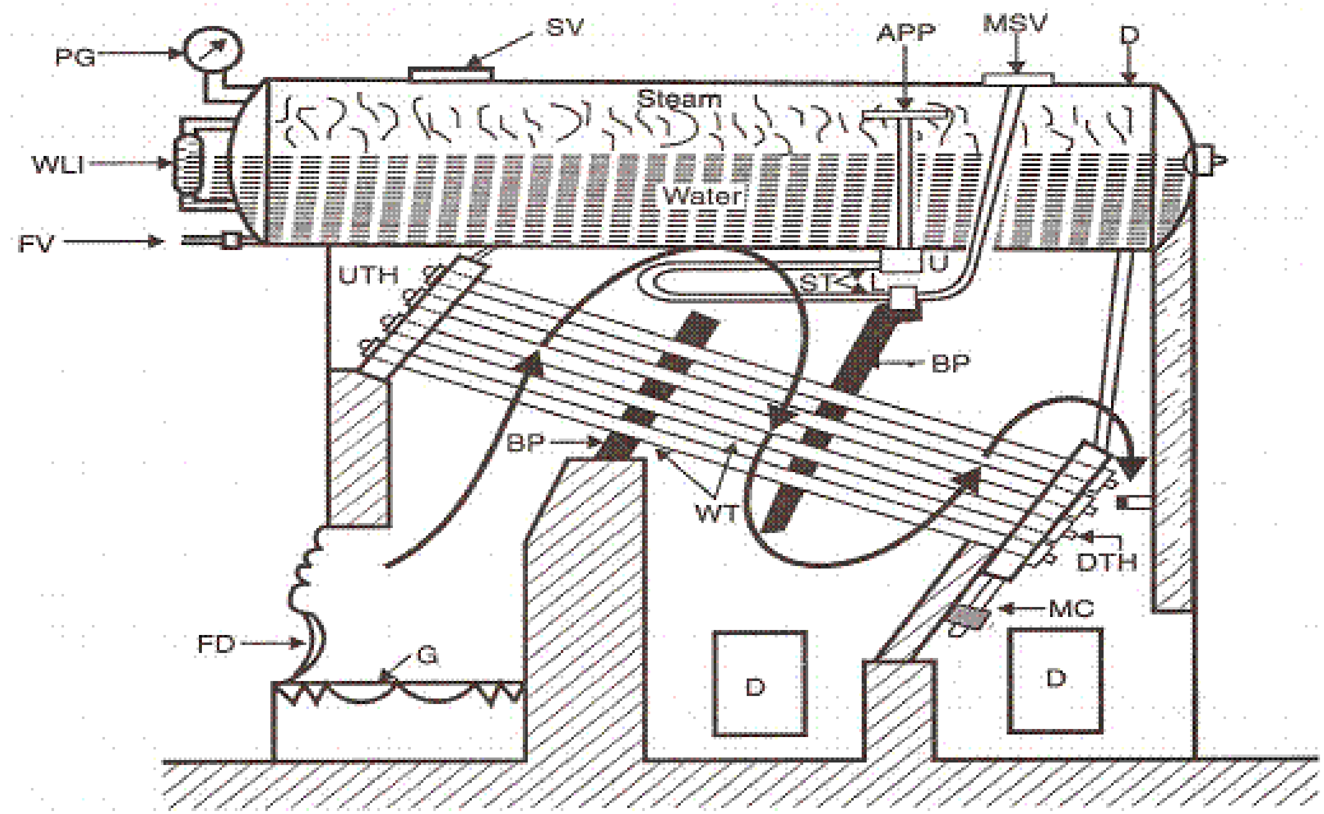
Loeffler Boiler

- **Steam outlet**
  About one-third of the superheated steam from the convection super heater passes to the steam turbine while the remaining two-thirds is passed on to evaporator drum to evaporated the feed water to saturated steam.

  Capacity
  Capacity of the Loeffler boiler is about 100 Tonnes/Hr of superheated steam generated at a pressure of 140 kgf/sq.cm and at a temperature of 500'C.



Babcock and Wilcox boiler



D = Drum
DTH = Down take header
WT = Water tubes
BP = Baffle plates
D = Doors
G = Grate
FD = Fire door
MC = Mud collector
WLI = Water level indicator
PG = Pressure gauge
ST = Superheater tubes
SV = Safety valve
MSV = Main stop valve
APP = Antipriming pipe
L = Lower junction box
U = Upper junction box
FV = Feed valve

Babcock and Wilcox boiler

- **Babcock and Wilcox boiler:** Babcock and Wilcox is a water-tube boiler is an example of horizontal inclined tube boiler it also a High Pressure Boiler.
- Construction: Babcock and Wilcox boiler with longitudinal drum. It consists of a drum connected to a series of front end and rear end header by short riser tubes.
- To these headers are connected a series of inclined water tubes of solid drawn mild steel. The angle of inclination of the water tubes to the horizontal is about 15° or more.

Babcock and Wilcox boiler

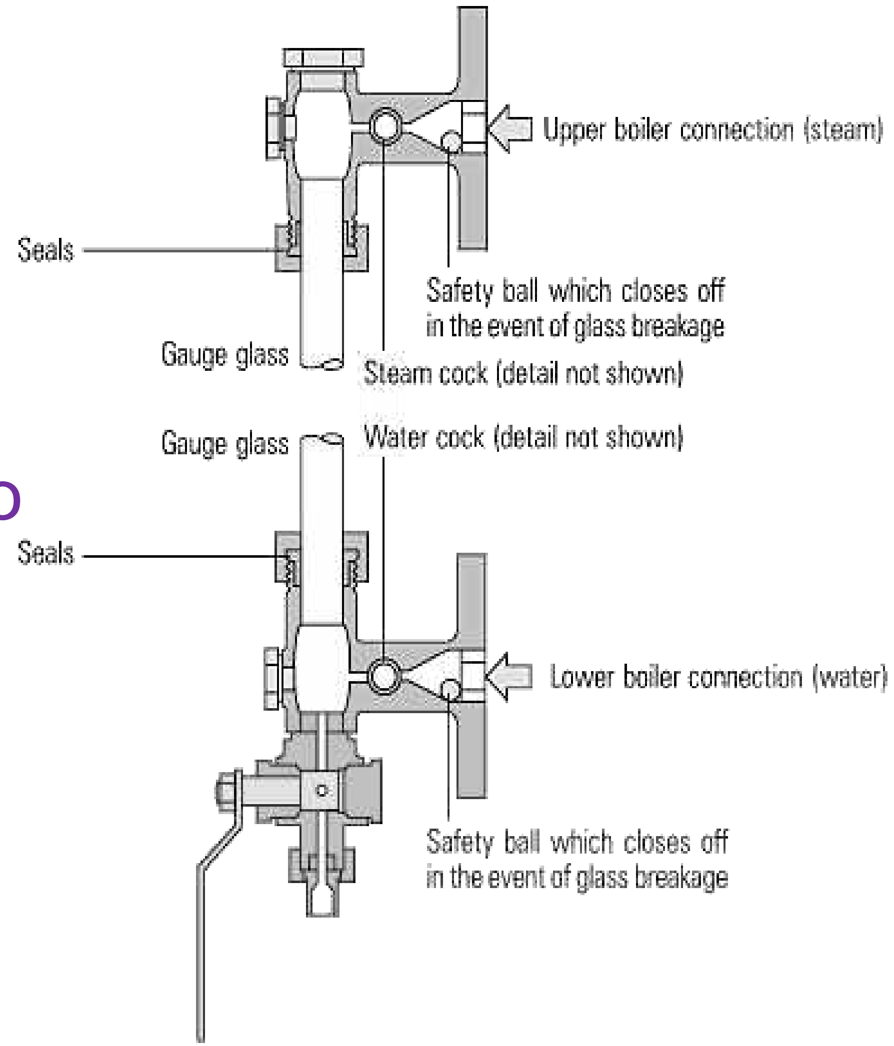
- Working: The fire door the fuel is supplied to grate where it is burnt.
- The hot gases are forced to move upwards between the tubes by baffle plates provided.
- The water from the drum flows through the inclined tubes via down take header and goes back into the shell in the form of water and steam via uptake header.
- The steam gets collected in the steam space of the drum. The steam then enters through the anti priming pipe and flows in the super heater tubes where it is further heated and is finally taken out through the main stop valve and supplied to the Steam turbine or Steam engine when needed.

Mounting of high pressure boiler

- **Mounting of high pressure boiler:** There are different fittings and device which are necessary for the operation and safety of a boiler. The various mountings used on the boiler
- 1. **Water level indicators:** The function of a water level indicator is to indicate the level of water in the level constantly. It is also called water gauge.

Water level indicator

- **Working of Water gauge:**
- The water gauge shows the level of water in the boiler drum. It warns the operator if the water level goes below a fixed mark, so that corrective action may be taken in time to avoid any accident.

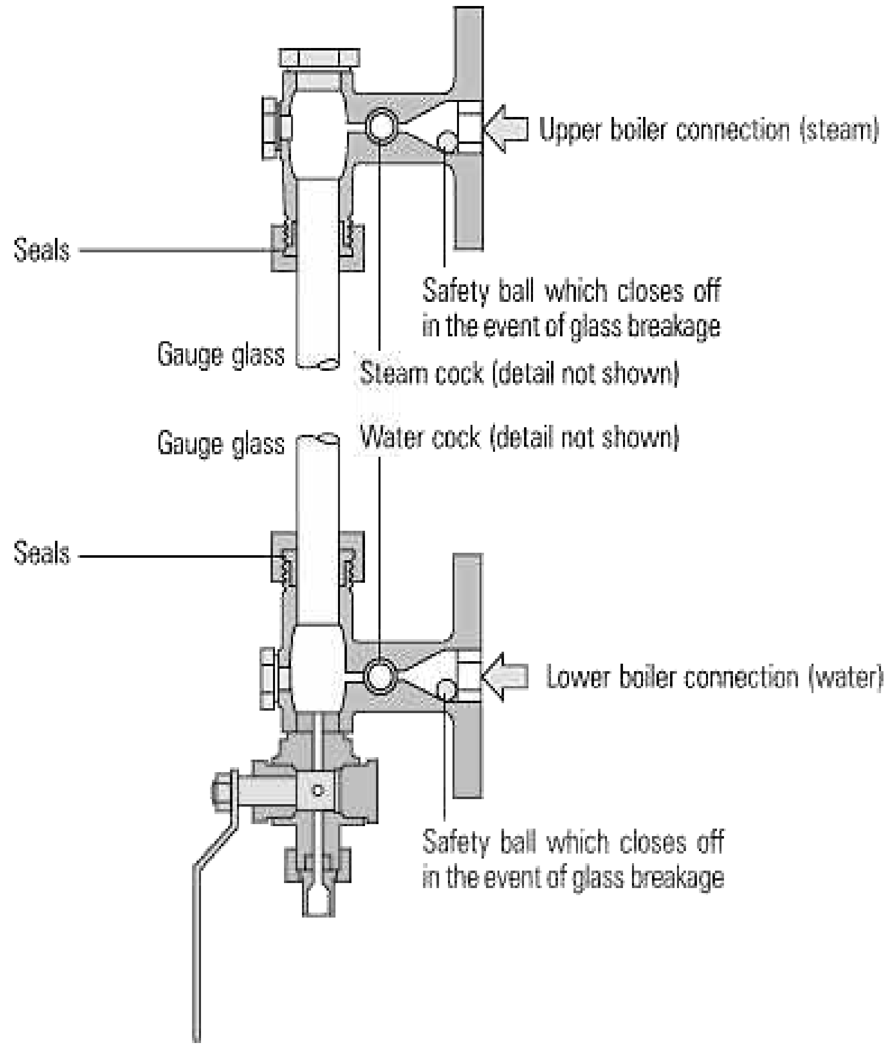


- For the observation of the water level in the boiler, the water and steam cocks are opened and drain cock is closed.

Water level indicator

- The steam enters from the upper metal tube M1 into the glass tube and water enters from the lower metal tube M2 into the glass tube.
- Hence, water stands in the glass tube at the same level as in the boiler.

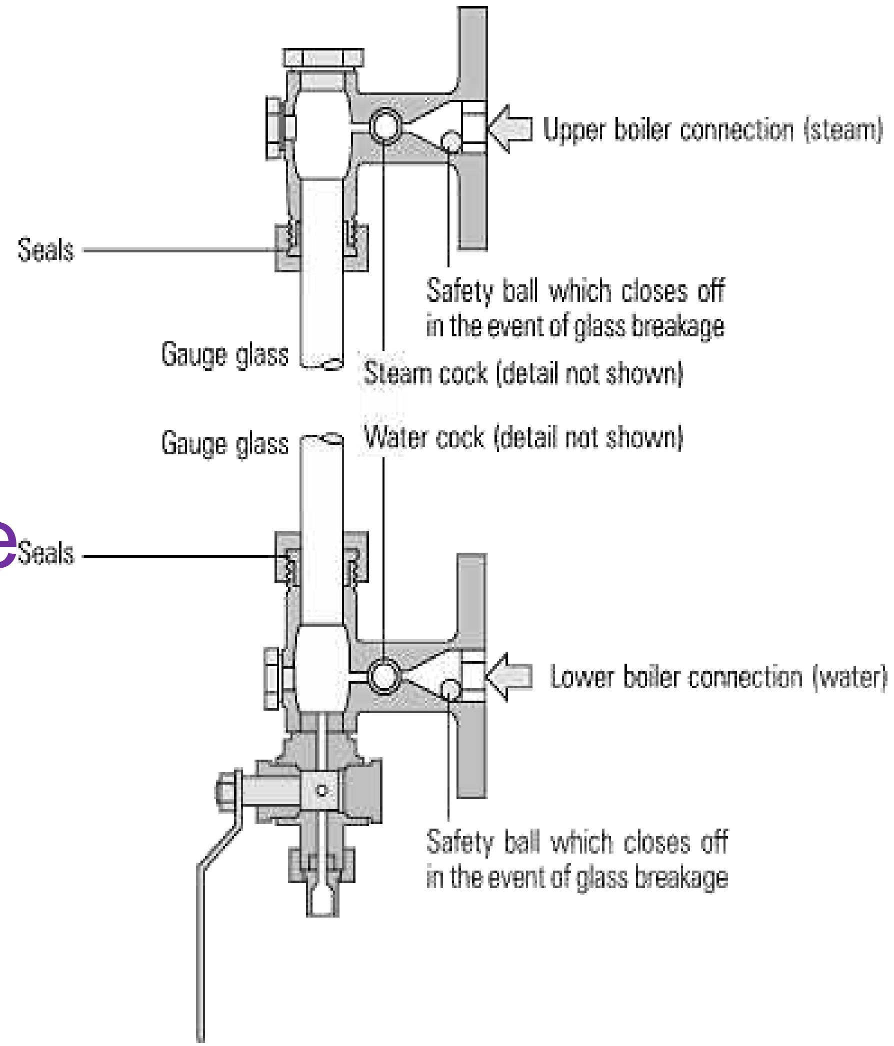
- The junctions of the metal tubes and the glass tube are provided with two balls. In case the glass tube is broken, the balls are pushed to the top and bottom ends of the glass tube.



Water level indicator

Thus the flow of both water and steam out of the boiler is prevented.

When the boiler is not working, the water gauge can be taken out from the boiler for cleaning purposes by removing the bolts.



Fusible plug

- 2. **Fusible plug:** The function of a fusible plug is to prevent the boiler against damage due to overheating for low water level.

- **Working of fusible plug:**
- During the normal operation, the fusible plug is submerged in water which keeps the temperature of the fusible metal below its melting point.
  But when the water level falls below the top of the fusible plug, it is uncovered by the water.
- The fusible plug therefore melts by the heat of the furnace. Thus the copper plug drops down and is held within the gun metal body by the ribs.
- The opening so made allows the steam rush into the furnace and extinguish the fire. The damage to the fire box which could burn up, is avoided.

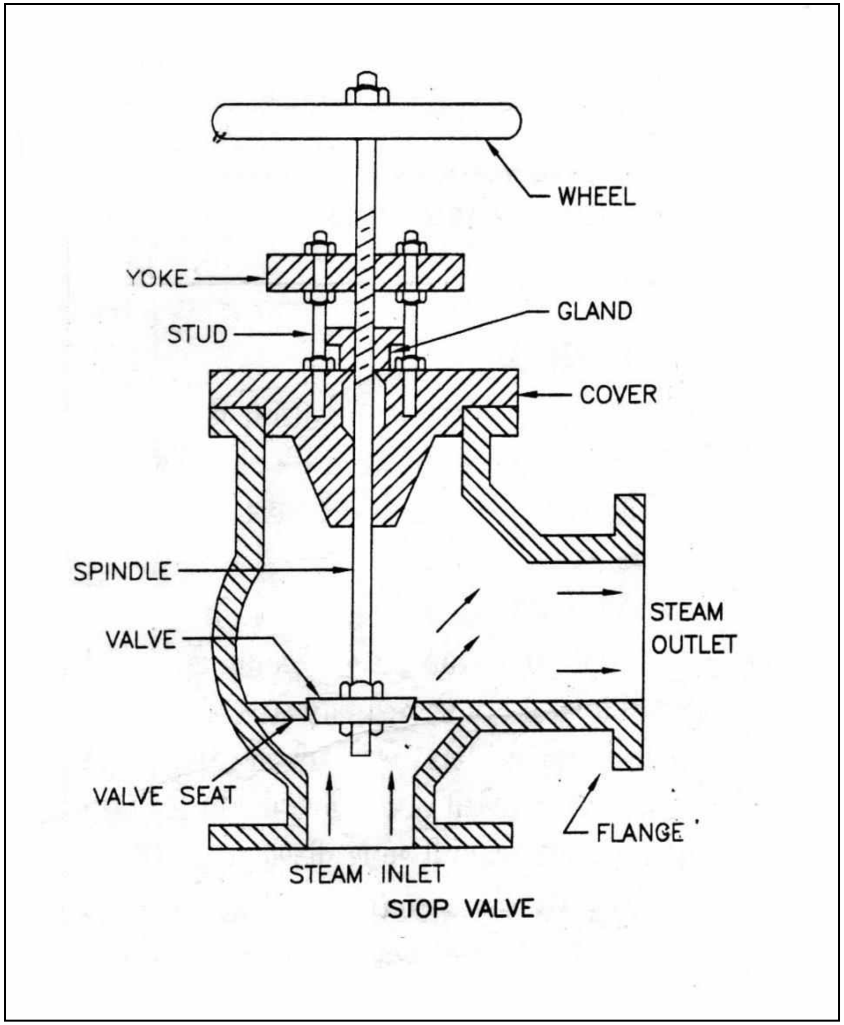
Steam stop valve

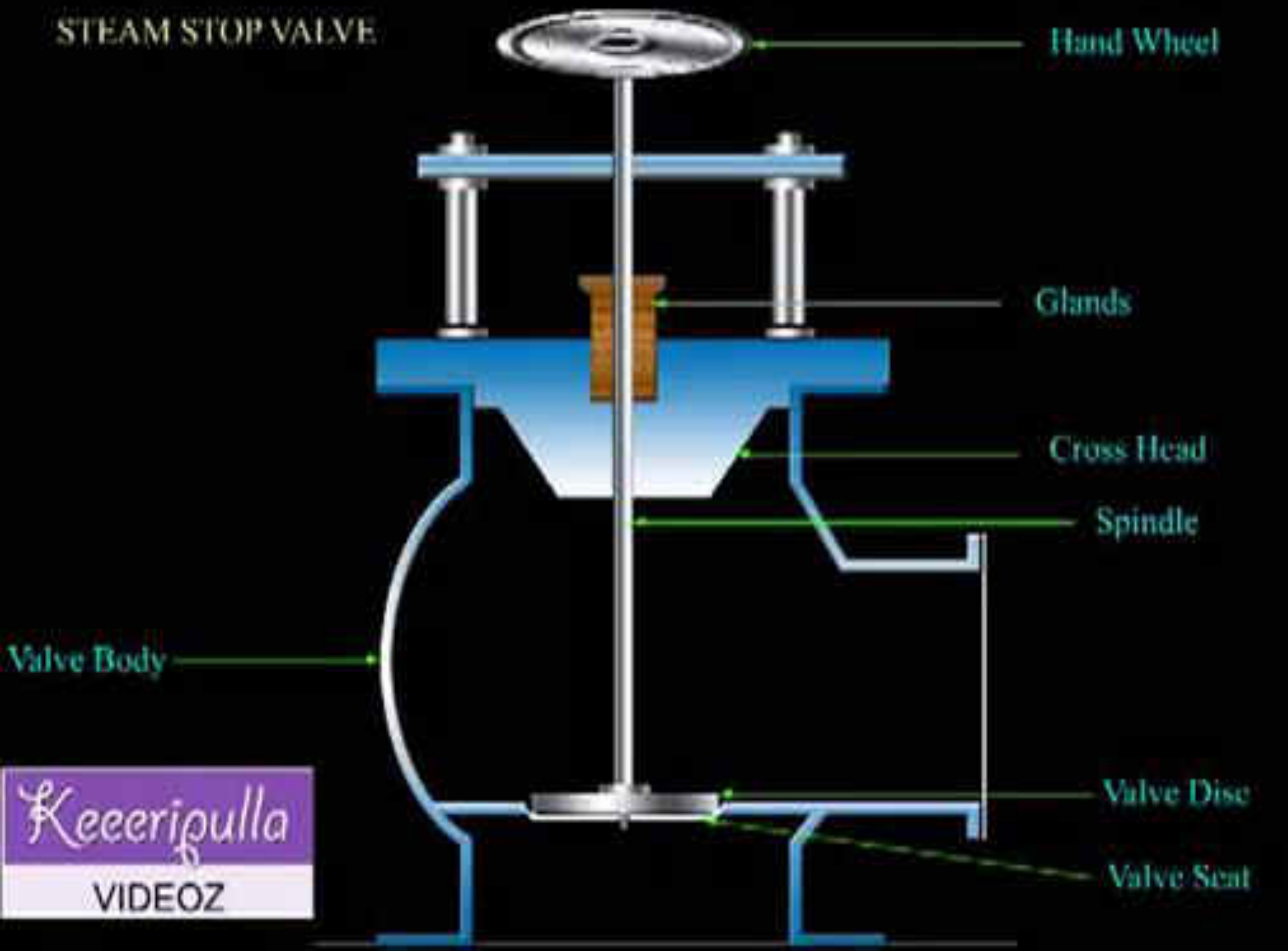
- **Steam stop valve:** A junction valve is a valve which is placed directly over a boiler and connected to a steam pipe which carries steam to the engine.
- If a valve is placed in the steam pipe leading steam to the engine and placed near the engine. It usually termed as stop valve. The larger sizes are called Junction valve and smaller sizes Stop valve
- • Function: to shut off or regulate the flow of steam from the boiler to the steam pipe or steam from the steam pipe to the engine.

Steam stop valve

- When the hand wheel is turned, the spindle which is screwed through the nut is raised or lowered depending upon the sense of rotation of wheel.

  The passage for flow of steam is set on opening of the valve.




STEAM STOP VALVE
Hand Wheel
Glands
Cross Head
Spindle
Valve Body
Valve Disc
Valve Seat
Keeeripulla
VIDEOZ

Feed check valve

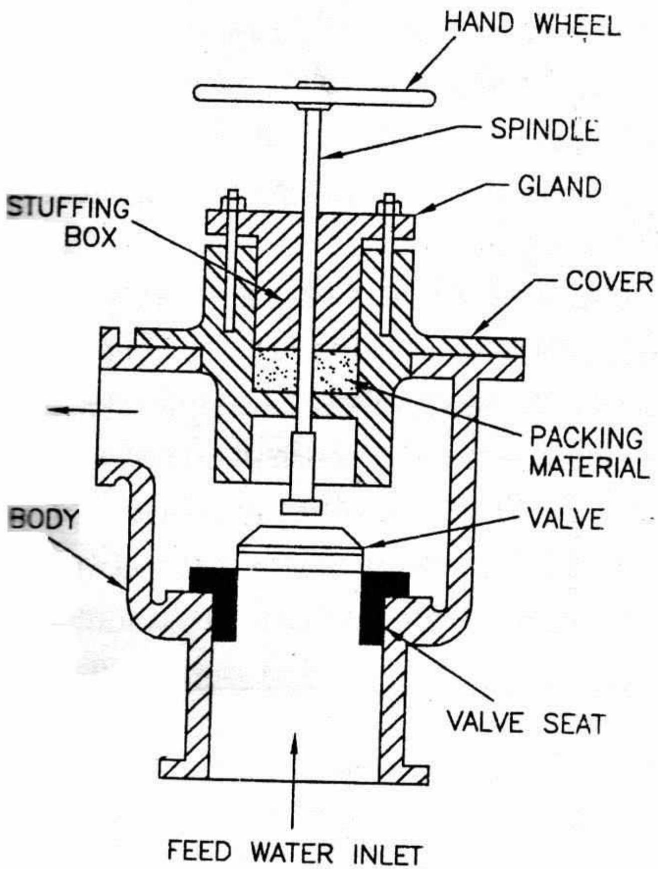
- 4. **Feed check valve:** The function of a feed check valve is to control the supply of water to the boiler and to prevent the exception of water from the boiler when the pump pressure is less as pump is stopped.

- i) To allow the feed water to pass into the boiler.

- ii) To prevent the back flow of water from the boiler in the event of the failure of the feed pump.

- The feed check valve is fitted in the water space of the boiler slightly below the normal level of the water.

Feed check valve



FEED CHECK VALVE

Working same as that non return valve

Blow off cock

Function: To drain out the water from the boiler for internal cleaning, inspection, repair or other purposes.

- It may discharge a portion of water when the boiler is in operation to blow out mud, scale or sediments, periodically.
- It is fitted on the boiler shell directly or to a short branch pipe at the lowest part of the water space.

Blow off cock

- The commonly used blow off cock is shown in figure.
- It is consists of conical plug fitted accurately in to similar casing.
- The plug has rectangular opening, which may be brought in line of passage by rotating the plug.
- This causes the water to discharge from boiler.
- The discharge of water may be stopped by rotating the plug again.

Safety Valve

Function : The function of safety valve is to release the excess steam when the pressure of steam inside the boiler exceeds the rated pressure.

- There are 4 types of safety valves:
  i) **Lever Safety Valve**

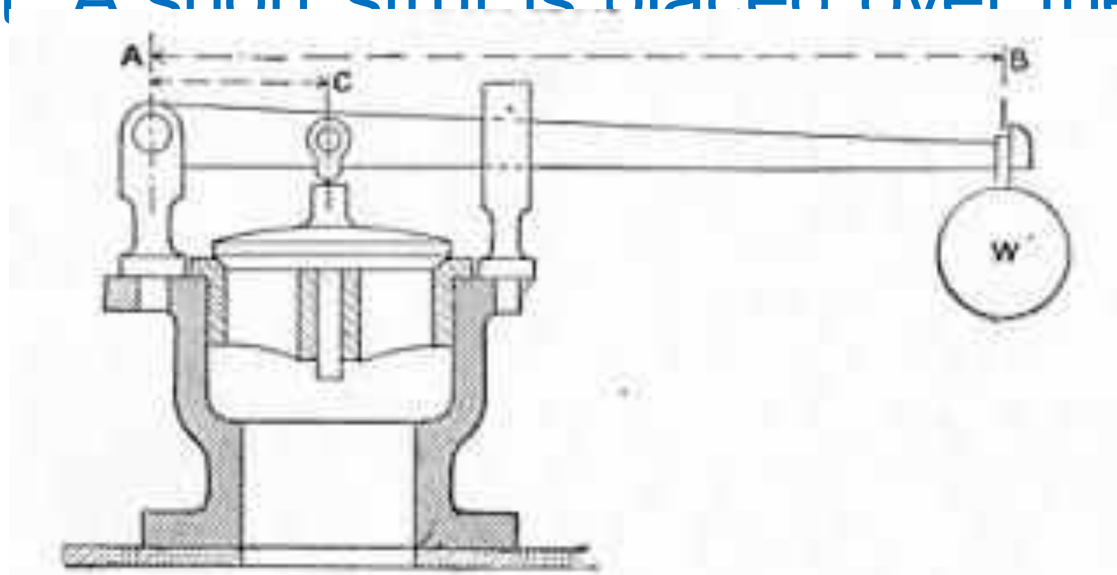
  ii)**Dead Weight Safety Valve**

  iii) **Spring Loaded safety Valve**

  iv)**High steam and low water safety valve**

Lever Safety Valve

- It consists of a valve resting over a gun metal seat. The valve seat is fixed on a mounting block, fitted over the boiler shell.
- One end of the lever is hinged to a rod of the mounting block, while the other end carries a weight. A short strut is placed over the valve.



Dead Weight Safety Valve

ii)**Dead Weight Safety Valve**

- It is mainly used for low pressures, low capacity, stationary boilers of the Cornish and Lancashire types.

- Merits:
  1)Simplicity of design
  2)Gives quite a satisfactory performance during operation.
  3)It cannot be easily tempered from the pressure adjustment view.

- Demerits:
  1)Unsuitable for use on any boiler where extensive vibration and movement are experienced(e.g. locomotive and marine work).

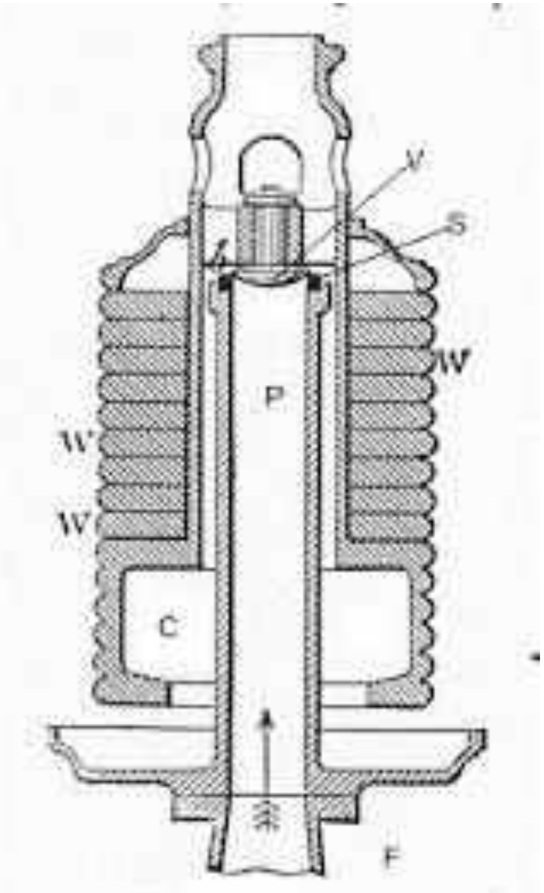
2)It is not suitable for high pressure boilers because a large amount of weight is required to balance the steam pressure.

Dead Weight Safety Valve

- The Dead Weight safety valve consists of a valve **V** which is made of gun metal to prevent rusting.
- It rests on the gun metal seat **S** and is fixed to the top of a vertical steam pipe **P**.
- The pipe has a flange F at the bottom for fixing at the top of the boiler shell.

A weight carrier **C** is suspended from the top of the boiler.

- It carries cast iron rings (i.e., weight **W**). the total weight must be sufficient to the keep the valve on its seat against the normal working pressure.



V - VALVE , S - VALVE SEAT
P - STEAM PIPE , W - WEIGHT
C - WEIGHT CARRIER

Dead Weight Safety Valve

- **Working of Dead Weight safety valve:**
- When the steam pressure in the boiler exceeds the normal working pressure, it lifts the valve with its weight.
- The excess steam therefore escapes through the pipe to the atmosphere, until the pressure reaches its normal value.
  It is the simplest type of safety valve; it is suitable for stationary boilers only, because it cannot withstand the jerks and vibration of mobile (marine) boilers.
- Another disadvantage of this valve is the heavy weight required to balance the steam pressure. Hence, it is not suitable for high pressure boilers

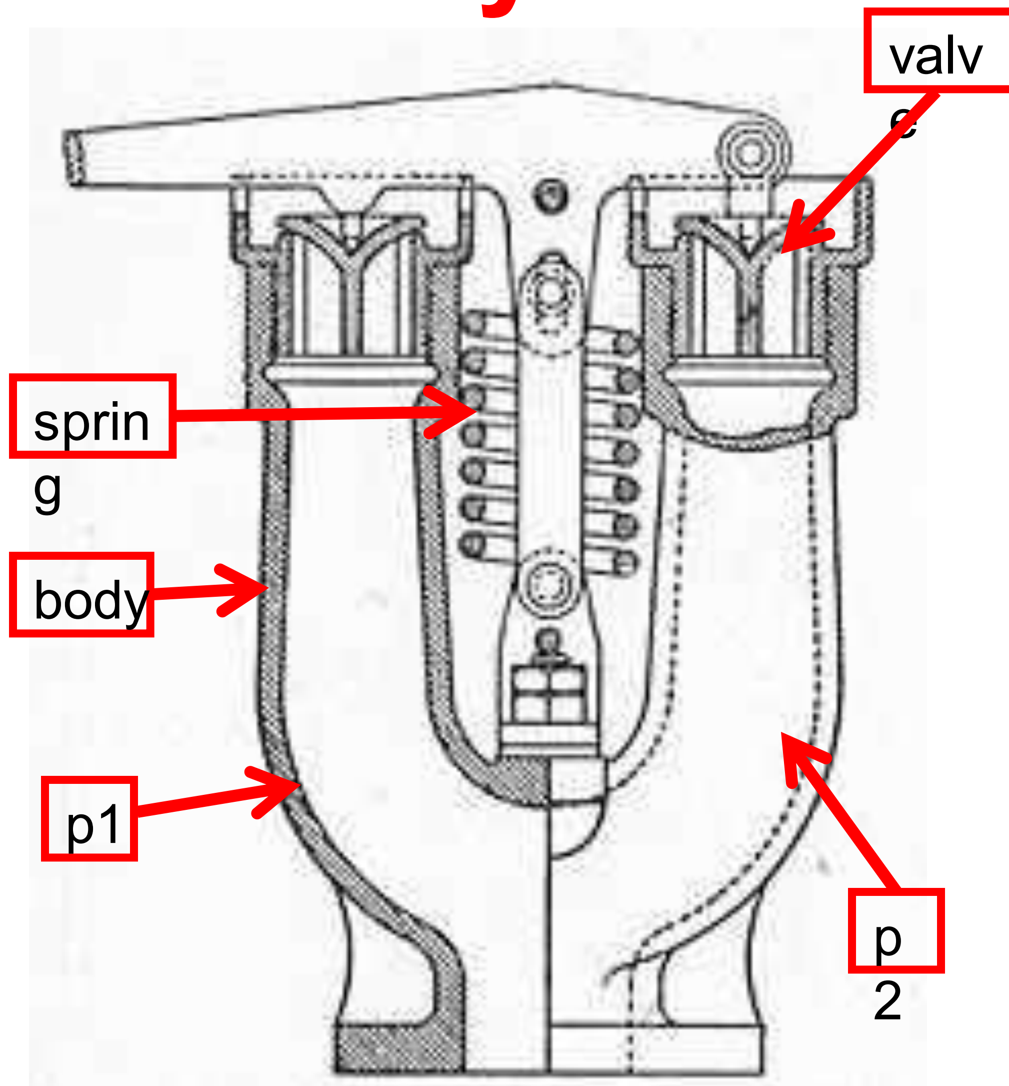
Spring Loaded safety Valve

- iii) **Spring Loaded safety Valve**
- • For locomotives and marine engines both the lever and dead weight types are unsuitable for obvious reasons,
  and the valve must be spring loaded, as such valve is unaffected by vibration or deviation from the vertical.

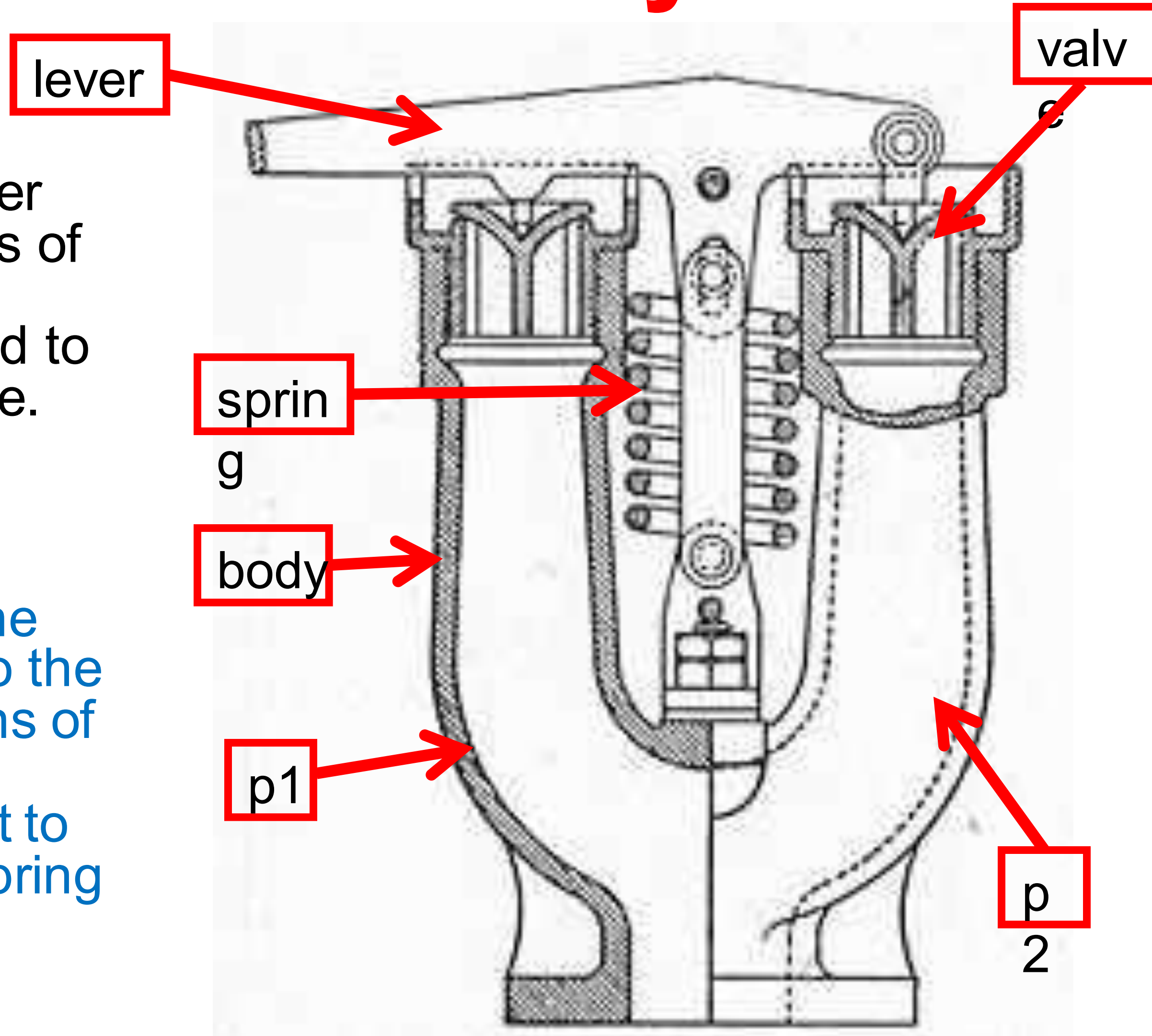
- • Disadvantage :
  One disadvantage of this valve is that the load on the valve increases as the valve lifts, so that pressure required
  just to lift the valve is less than that required to open it fully.

Spring Loaded safety Valve

- **Description of spring loaded safety valve:**
- It is loaded with a spring instead of weights. Hence it is called spring loaded safety valve.
- It consists of a cast iron body having two branch pipes P1 and P2. Two separate valves are placed over the valve seating's, which are fixed to the top of the branch pipes.



Spring Loaded safety Valve



- A lever is placed over the valves by means of two conical pivots. The lever is attached to a spring at its middle. The spring pulls the lever in downward direction.
- The lower end of the spring is attached to the valve body by means of a shackle. Thus the valves are held tight to their seats by the spring force.

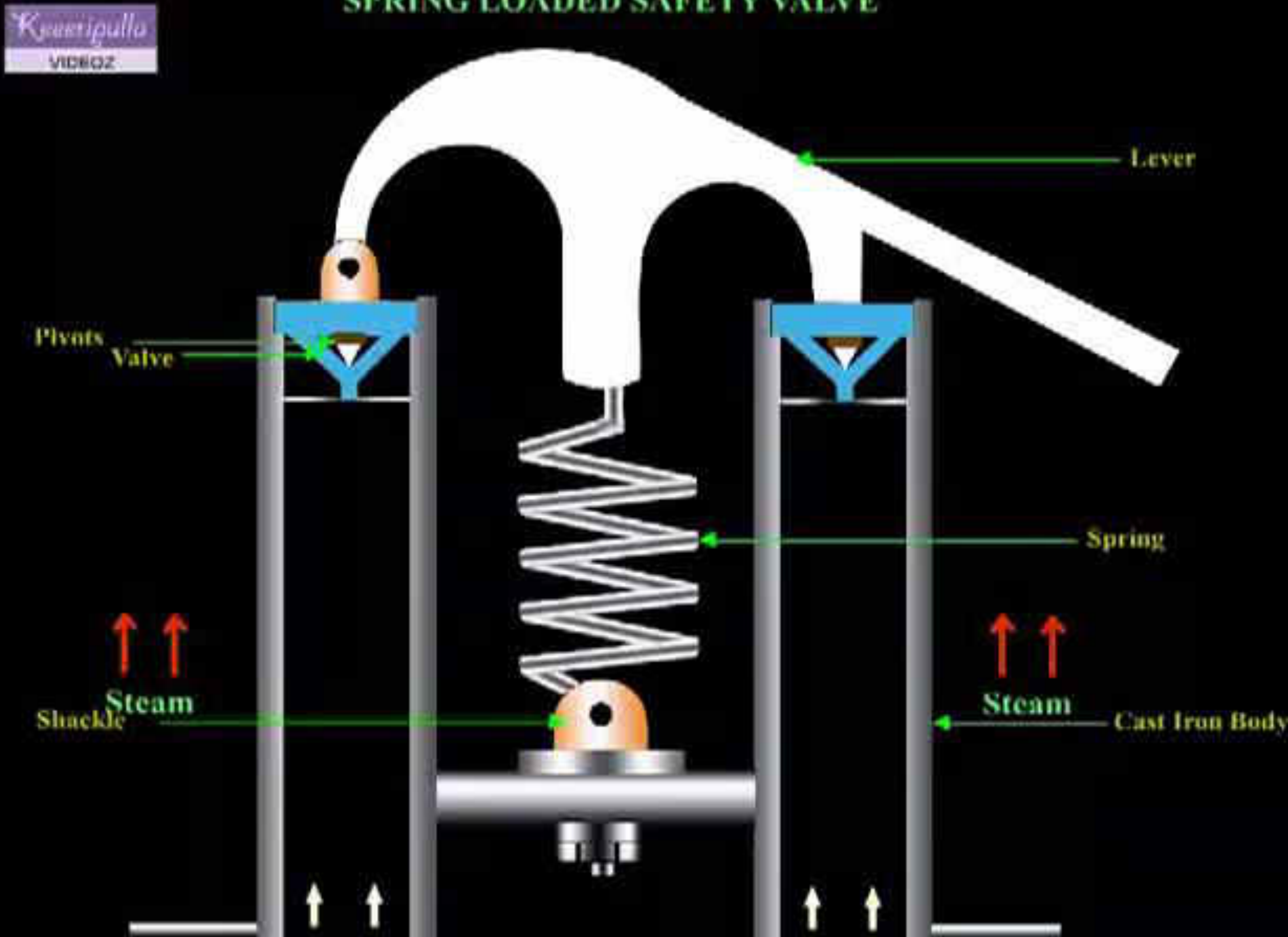
Spring Loaded safety Valve

working

When the steam pressure exceeds the normal working pressure , the valves rise up against the action of the spring and allow the steam to escape from the boiler till the pressure in the boiler reaches its working pressure

- The spring loaded safety valve is much lighter and compact compared with other safety valves.
- For locomotive or marine service, the safety valve should be such that it is unaffected by jerks and vibration likely to occur in such device.
- Hence spring loaded safety valve is preferred for locomotive and marine services, in addition to stationary boilers.

SPRING LOADED SAFETY VALVE



High steam and low water safety valve

It serves the following purposes.

(i) The steam automatically escapes out when the level of water falls below a certain level.

(ii) It automatically discharges the excess steam when the pressure of the steam rises above a certain pressure.

Use : It is generally used on Lancashire or Cornish boiler.

It cannot used in mobile boilers.


A
B
C
D
A. MAIN WEIGHT
B. FULCRUM
C. COUNTER WEIGHT
D. VALVE
E. SPINDLE
F. FULCRUM
G. KNIFE EDGE
H. FLOAT
I. BALANCE WEIGHT
F
H
COMBINE HIGH STEAM &
LOW WATER SAFETY VALVE

High steam and low water safety valve

- It consist of a main valve (known as lever safety valve) and rest on its seat.
- In the centre of the main valve, a seat for a hemispherical valve is formed for low water operation.
- This valve is loaded directly by the dead weights attached to the valve by a long rod.
- There is a lever , which has its fulcrum. The lever has weight suspended at the end.

High steam and low water safety valve

- When it is fully immersed in water. It is balanced by a weight at other end of lever F.
- When the water level falls, the weight comes out of water(E) and weight at other end will not be sufficient to balance weight(F).
- Therefore weight comes down E.

There are two projections on the lever to the left the fulcrum which comes in contact with a collar attached to the rod.

When weight E comes down, the hemispherical valve is lifted up and the steam escapes with a loud noise, which warns operator.

Pressure gauge

The function of a pressure gauge is to measure the pressure exerted inside the vessels.

It is usually constructed to indicate up to double the maximum working pressure.

Its dial is graduated to read pressure in kgf/cm2 gauge.

There are two type of pressure gauges, they are Bourdon tube type pressure gauge and Diaphragm tube type pressure gauge

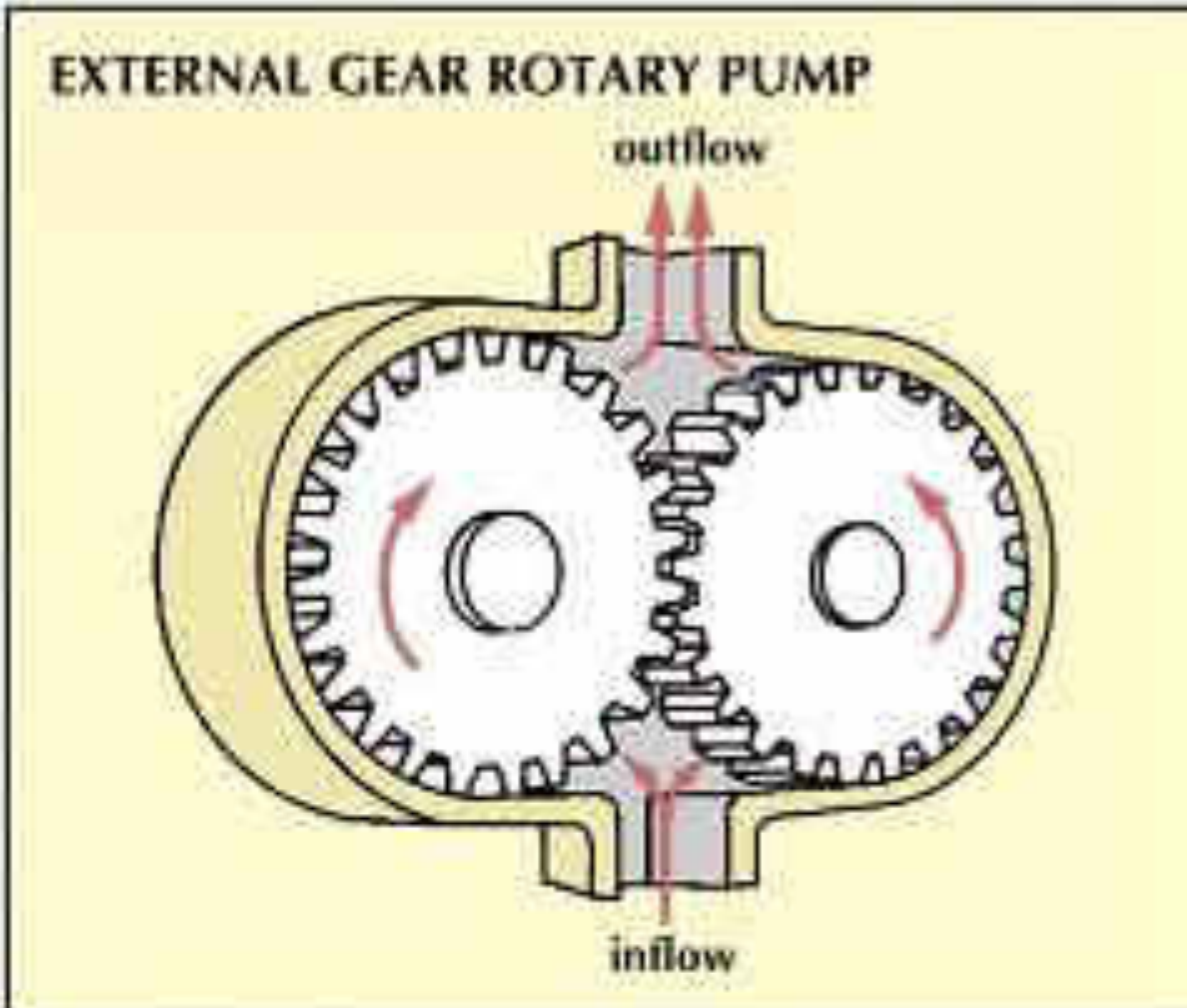
Manhole and mud box

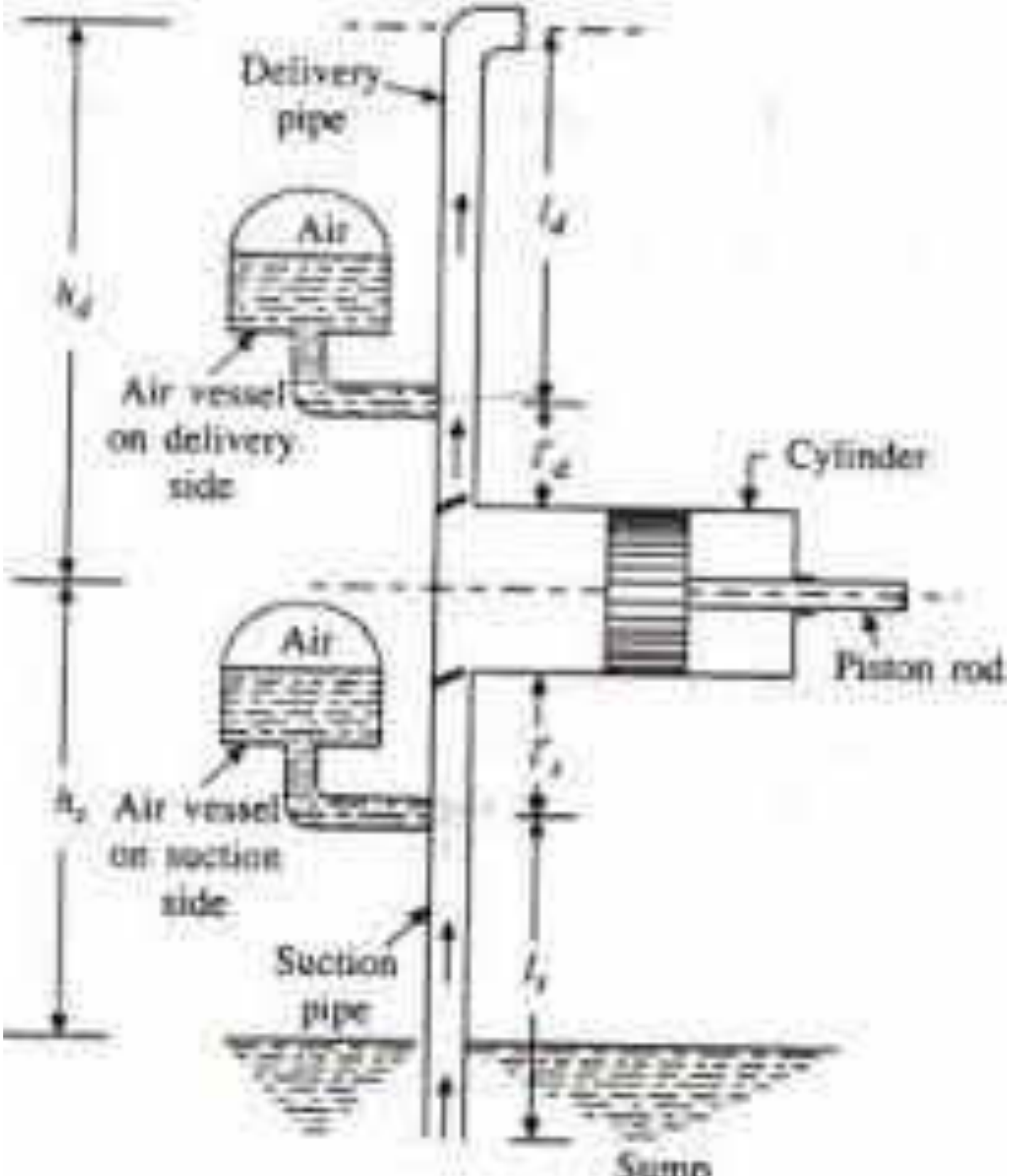
Manhole provides opening for cleaning, inspection and maintenance purpose.

- Mud box is a collection chamber (as shown in Babcock and Wilcox boiler) for collecting the Mud.

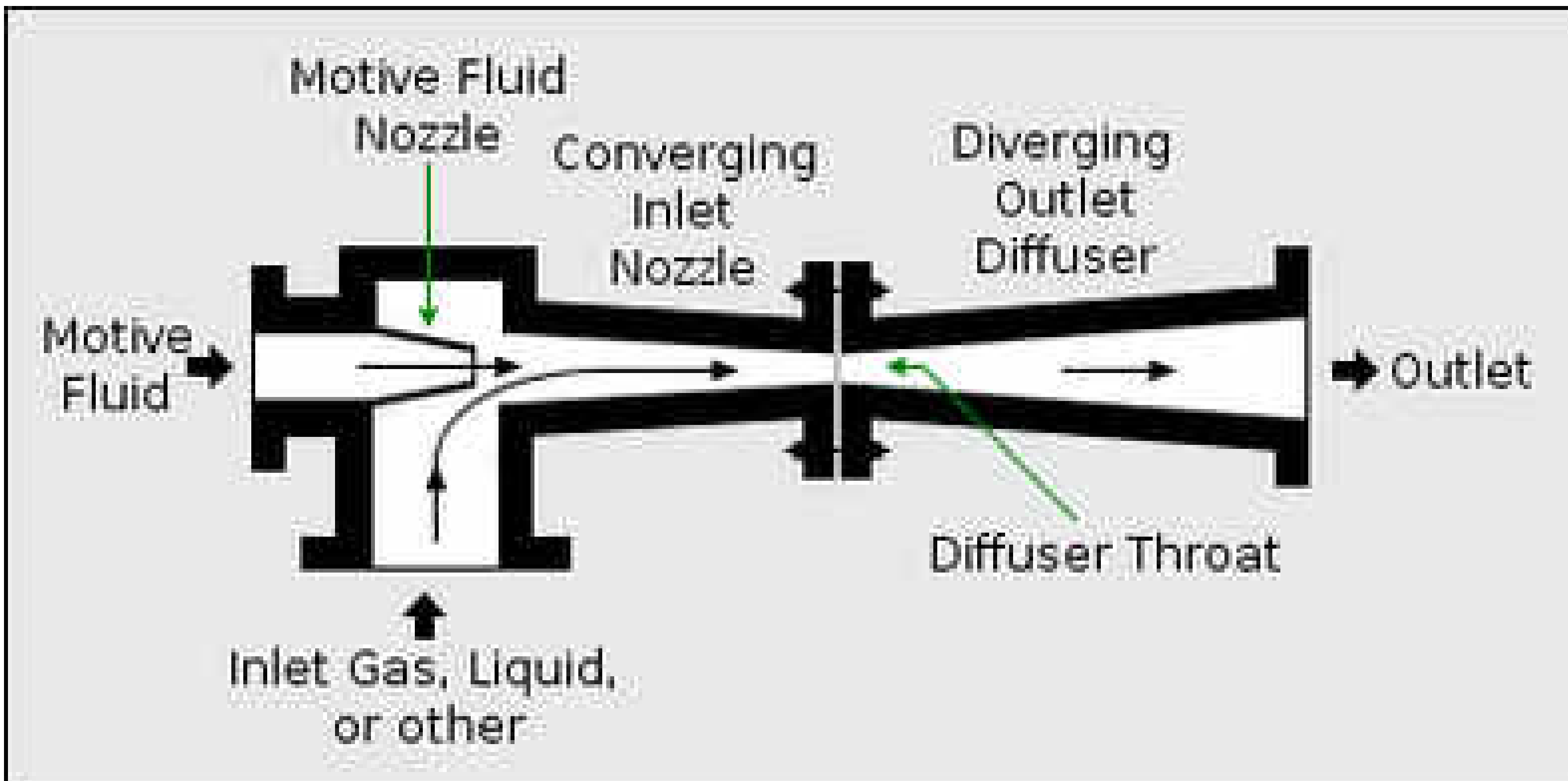
Accessories of high pressure boiler

- There are auxiliary plants required for steam boiler for their proper operation & for increase of their efficiency. The various accessories are
- 1. **Feed pump:** The feed pump is a pump which is used to deliver feed water to the boiler .
- It is desirable that the quantity of water supplied should be at least equal to that evaporated and supplied to the engine.
- Two type of pumps which are commonly used as feed pump are Reciprocating pump and Rotary pump.

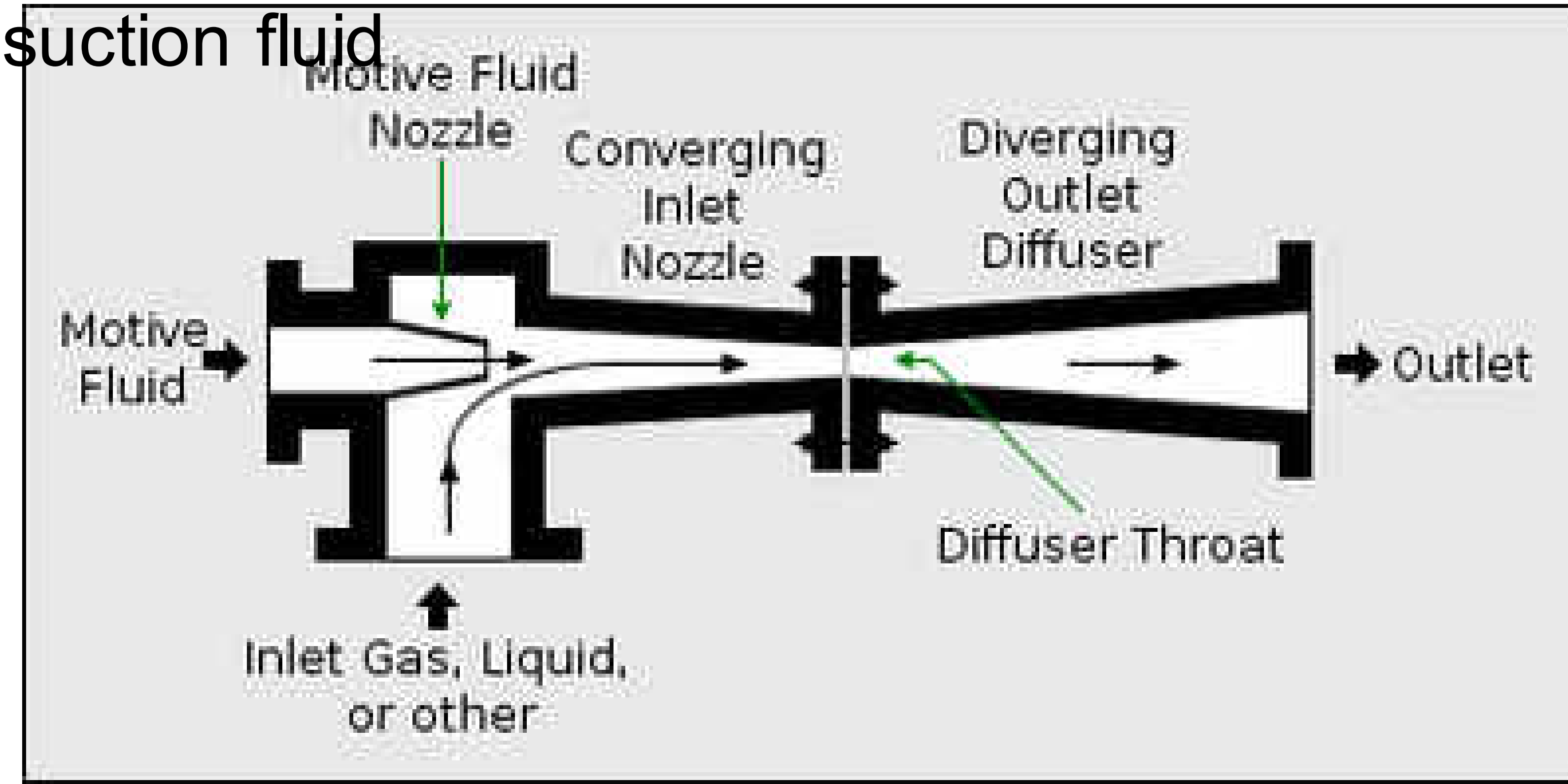

EXTERNAL GEAR ROTARY PUMP
outflow
inflow


Delivery pipe
Air
Air vessel on delivery side
Cylinder
Piston rod
Air
Air vessel on suction side
Suction pipe
Sump

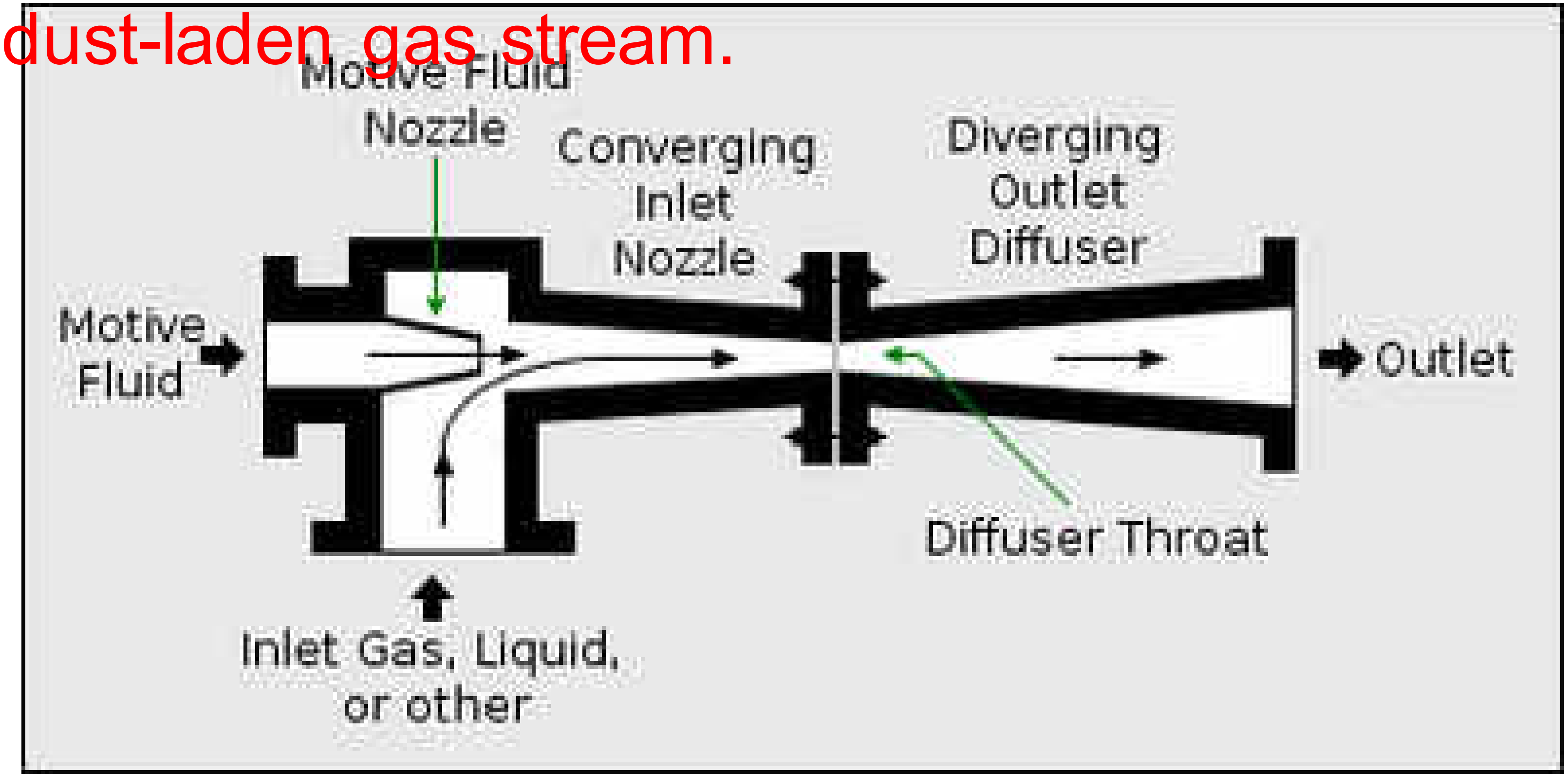
- 2. **Injector:** The function of an injector is to feed water in to the boiler.
- It is commonly employed for vertical and locomotive boiler and does not find its applications in large capacity high pressure boiler.



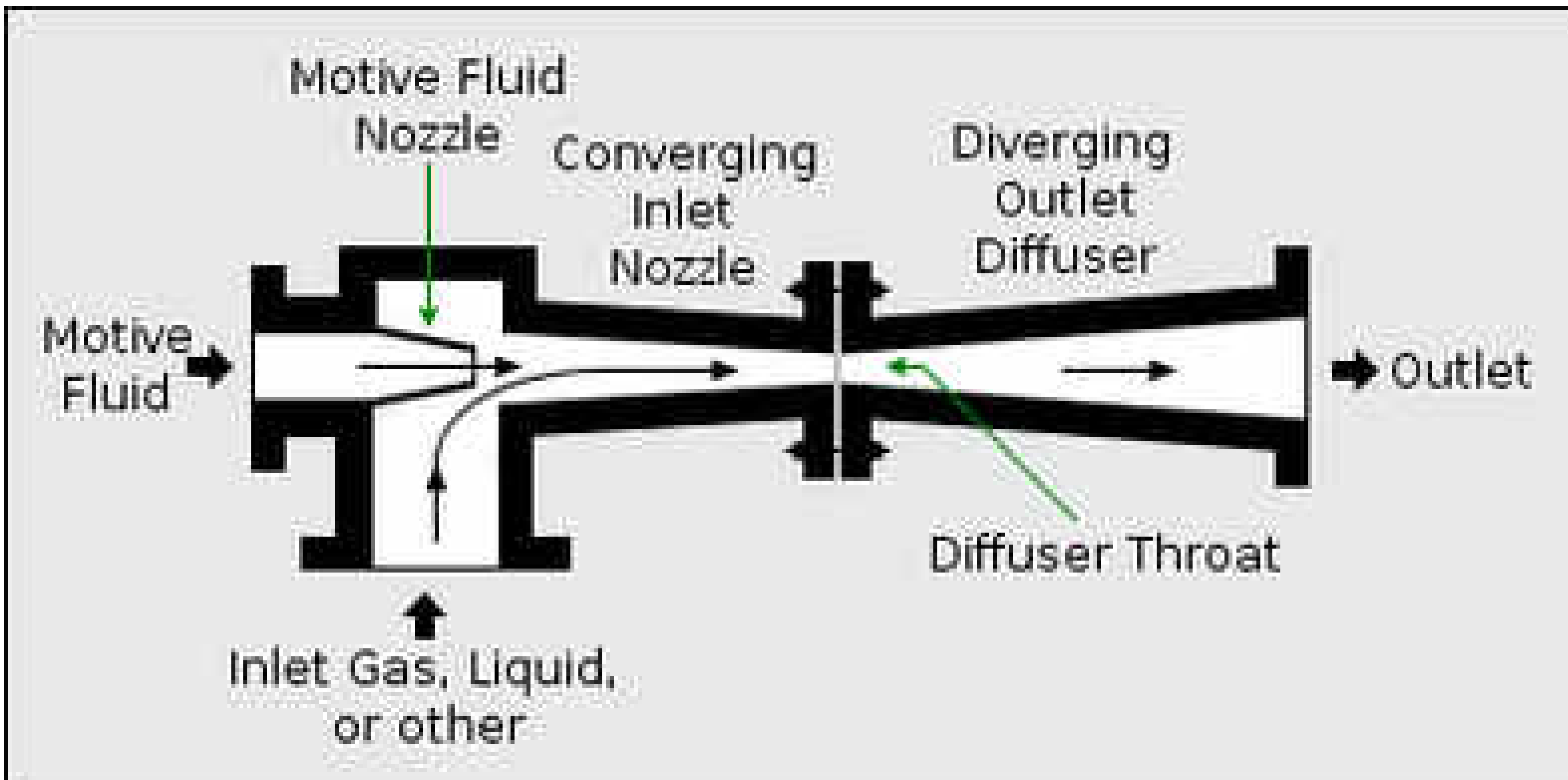
- An **injector**, **ejector**, **steam ejector**, **steam injector**, **eductor-jet pump** or **thermocompressor** is a type of pump that uses the Venturi effect of a converging-diverging nozzle to convert the pressure energy of a motive fluid to velocity energy which creates a low pressure zone that draws in and entrains a suction fluid



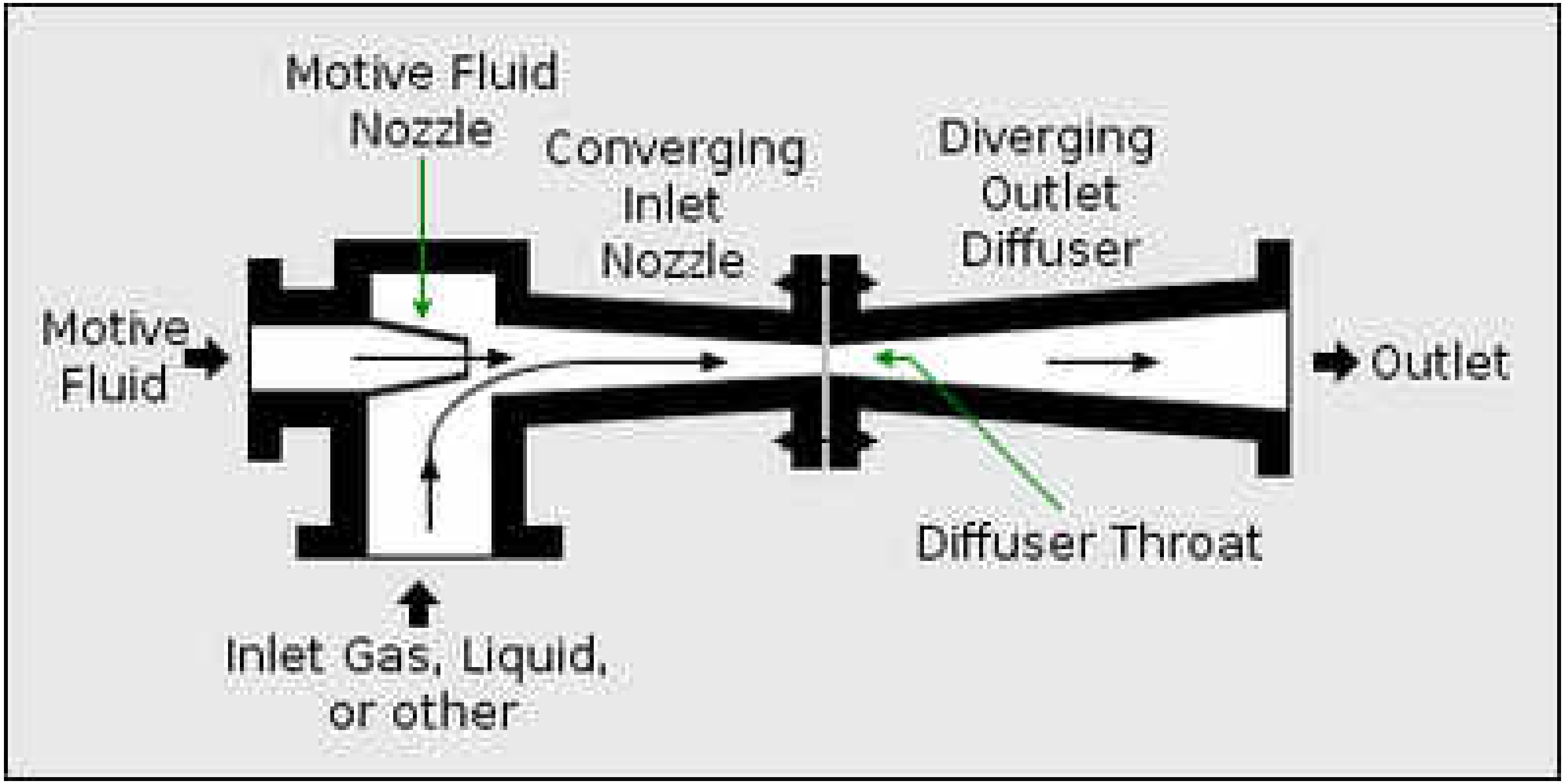
- After passing through the throat of the injector, the mixed fluid expands and the velocity is reduced which results in recompressing the mixed fluids by converting velocity energy back into pressure energy. The motive fluid may be a liquid, steam or any other gas. The entrained suction fluid may be a gas, a liquid, a slurry, or a dust-laden gas stream.



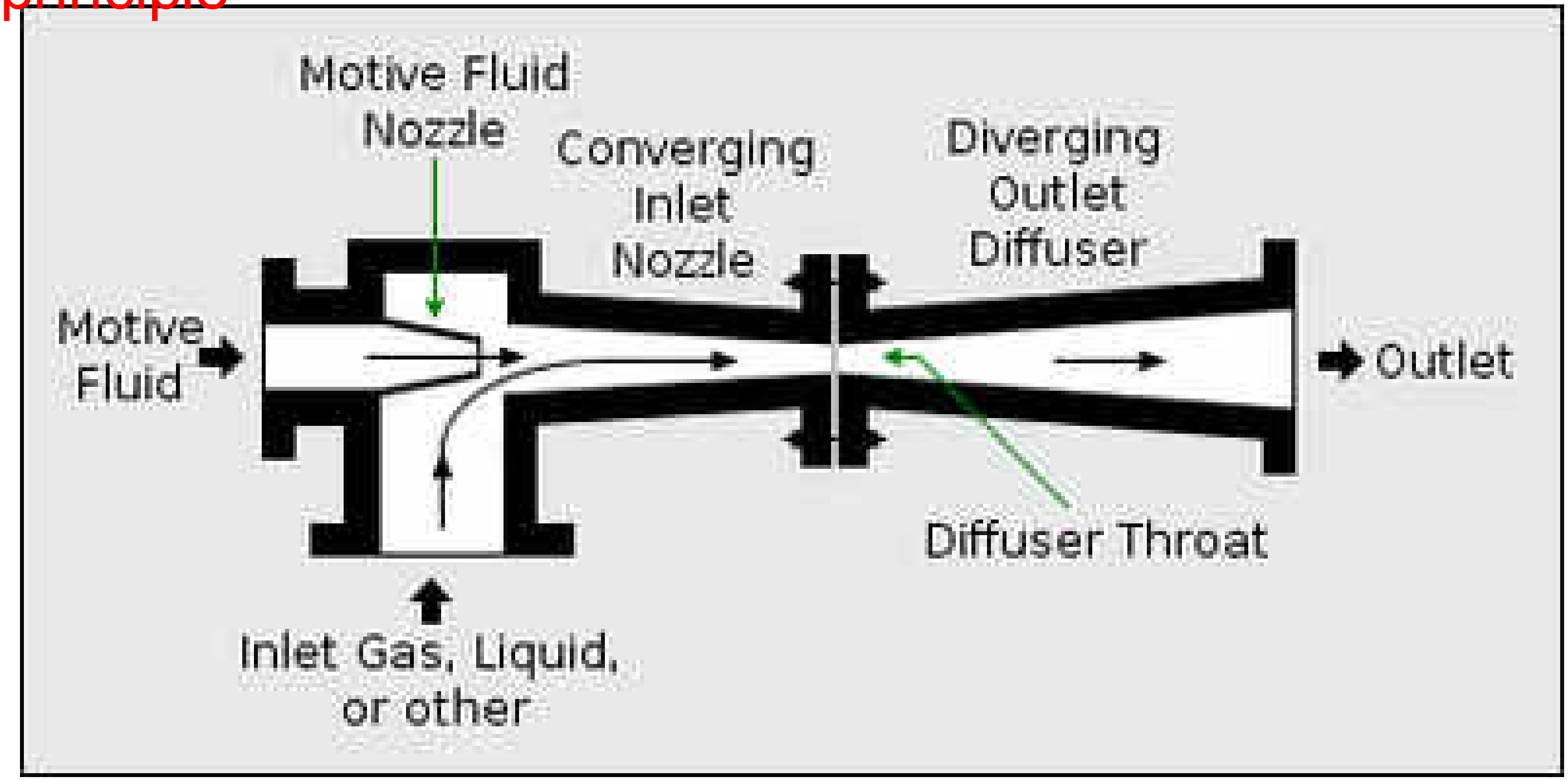
- The adjacent diagram depicts a typical modern ejector.
- It consists of a motive fluid inlet nozzle and a converging-diverging outlet nozzle.
- Water, air, steam, or any other fluid at high pressure provides the motive force at the inlet



- Fluid under high pressure is converted into a high-velocity jet at the throat of the convergent-divergent nozzle which creates a low pressure at that point. The low pressure draws the suction fluid into the convergent-divergent nozzle where it mixes with the motive fluid.

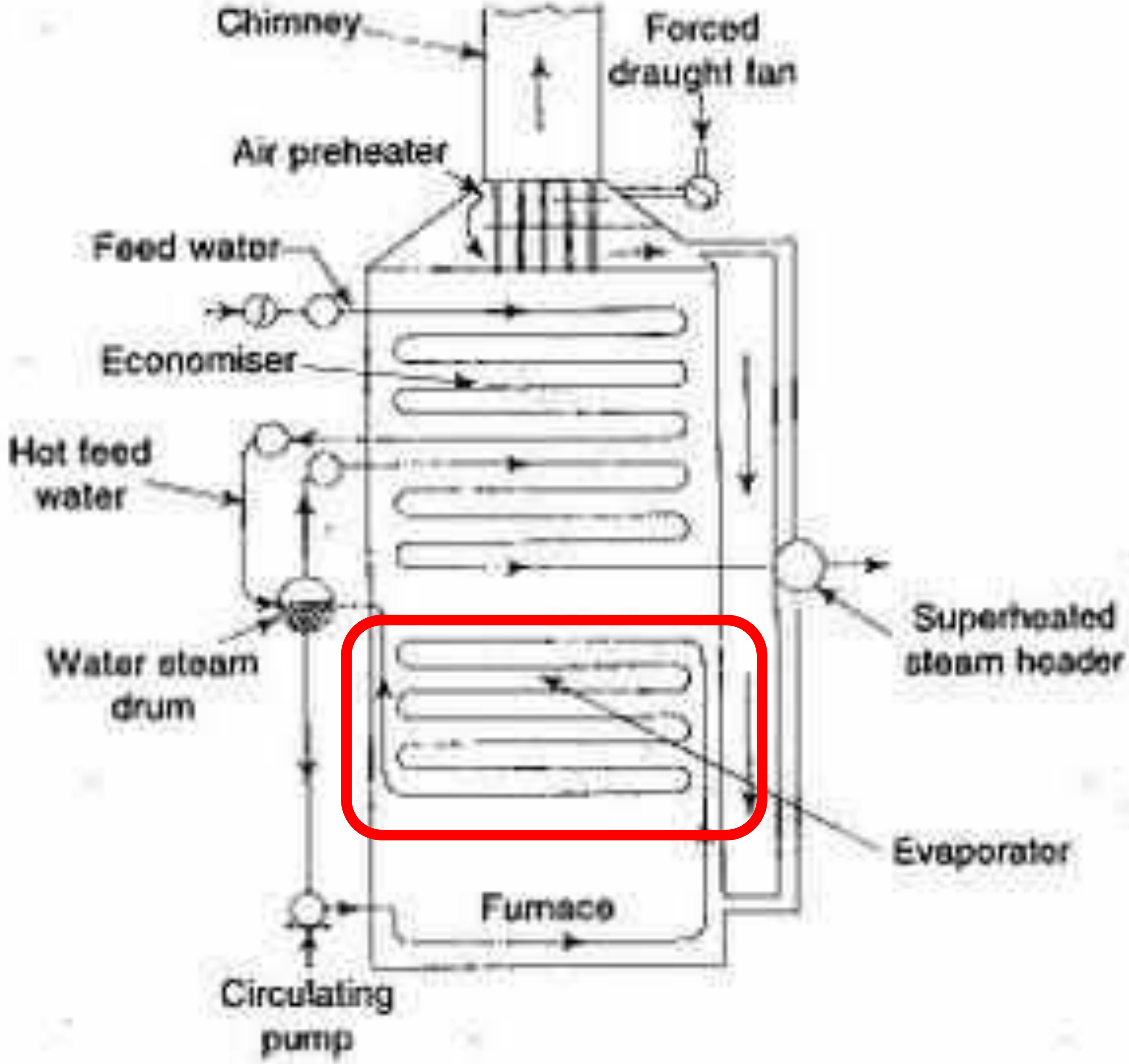


- the pressure energy of the inlet motive fluid is converted to kinetic energy in the form of velocity head at the throat of the convergent-divergent nozzle.
- As the mixed fluid then expands in the divergent diffuser, the kinetic energy is converted back to pressure energy at the diffuser outlet in accordance with Bernoulli's principle



Accessories of high pressure boiler

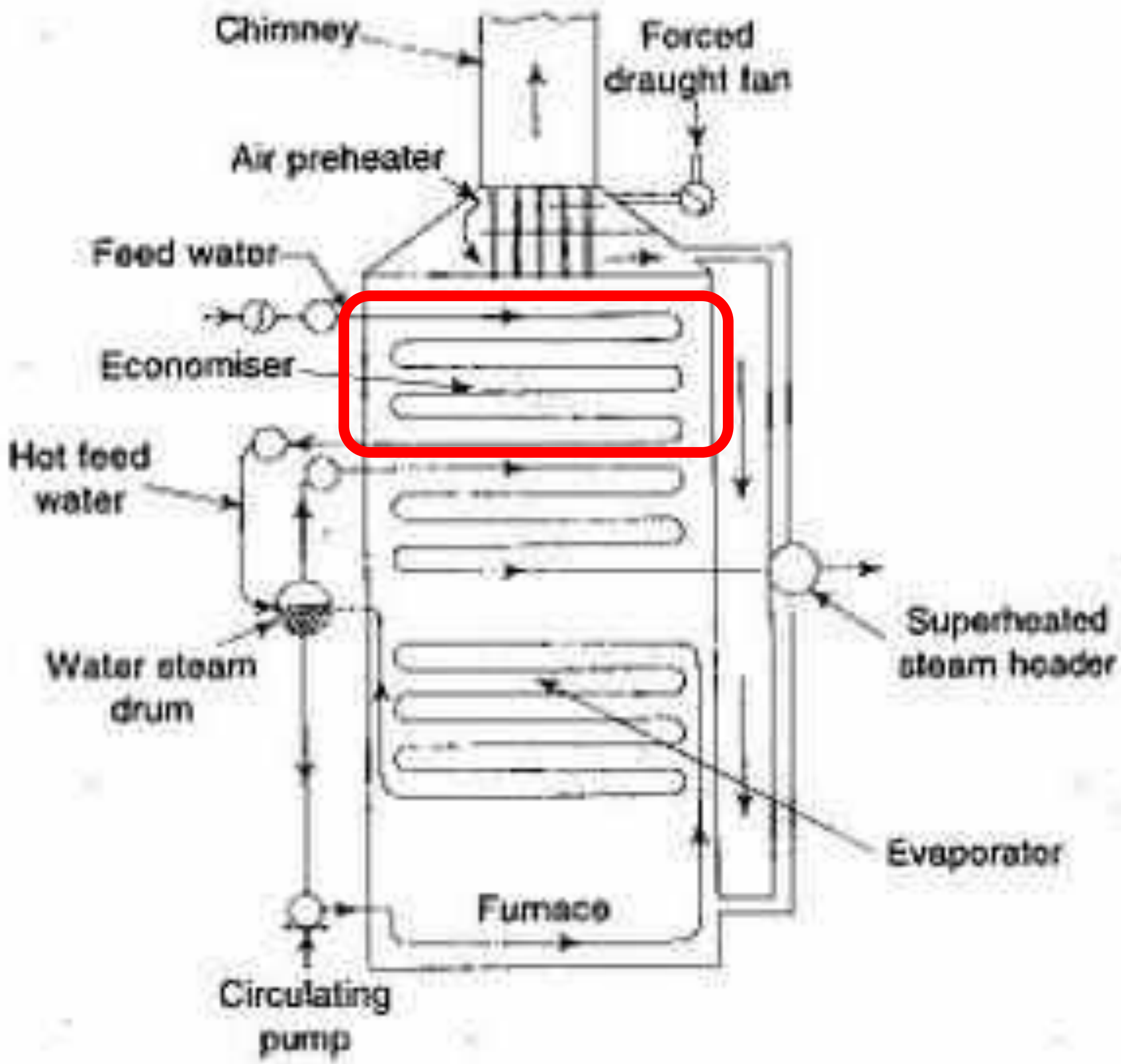
- 3. **Evaporator:** Evaporator is used in high pressure boiler which is placed after the air in the way of flue gases water are tube.
- Hence evaporator is a unit which consumes the energy of flue gases in boiler.
- Its main function is to convert the water to steam add much to the boiler efficiency.



High pressure Lamont boiler

Accessories of high pressure boiler

- 4. **Economiser:** An economiser is a device in which the waste heat of the flue gases is utilized for heating the feed water.
- Economiser is very important part of the boiler, with the help the economiser the efficiency of the boiler increased and the evaporative capacity of the boiler is increased. Economiser are of two type Independent type and Integral type.
- Video of economiser



High pressure Lamont boiler

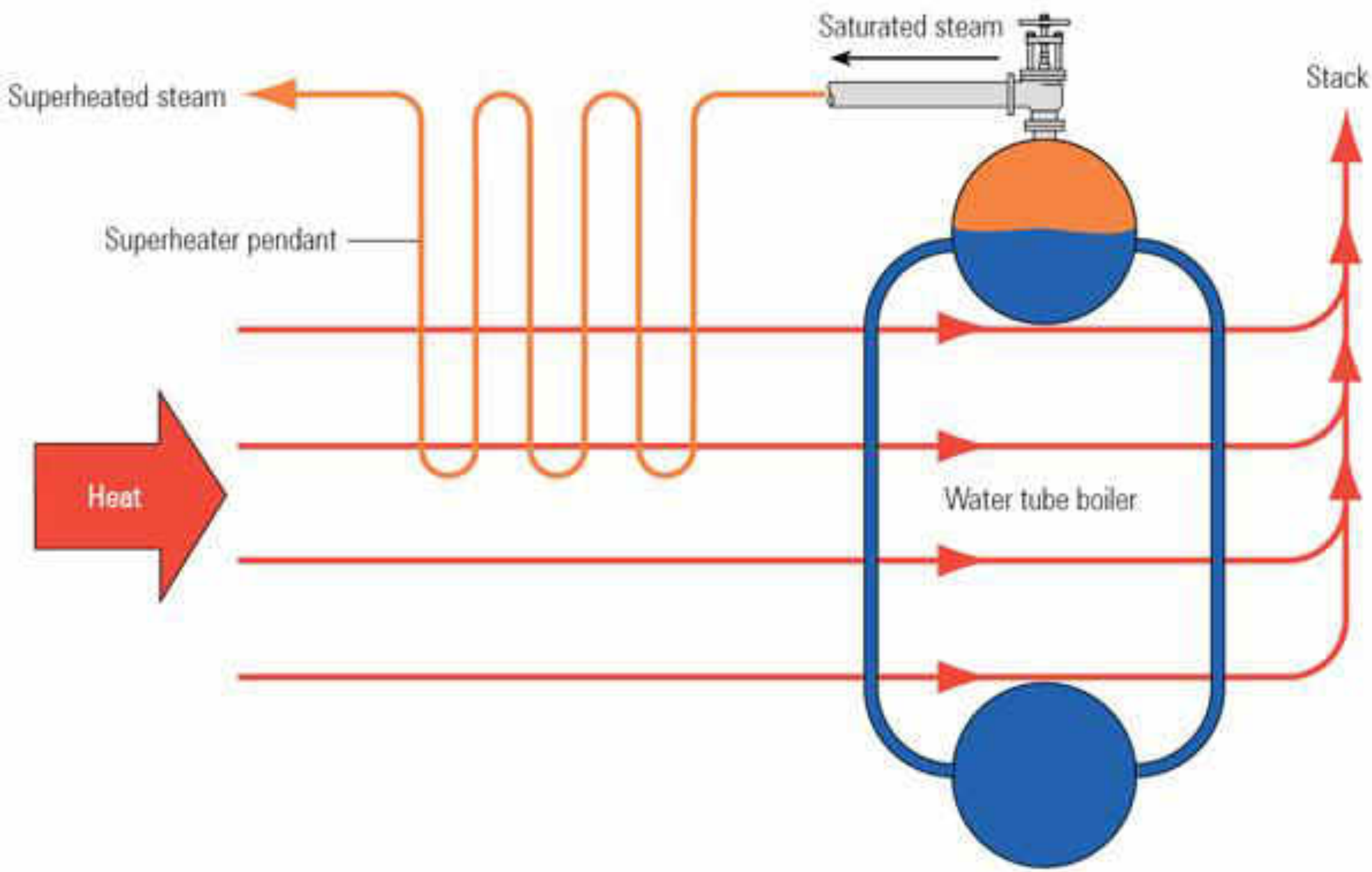
Accessories of high pressure boiler

- 5. **Super heater:** The function of a super heater is to increase the temperature of the steam above its saturation point.
- Whatever type of boiler is used, steam will leave the water at its surface and pass into the steam space.
- Steam formed above the water surface in a shell boiler is always saturated and cannot become superheated in the boiler shell, as it is constantly in contact with the water surface.

- If superheated steam is required, the saturated steam must pass through a super heater.
- This is simply a heat exchanger where additional heat is added to the saturated steam.

  In water-tube boilers, the super heater may be an additional pendant suspended in the furnace area where the hot gases will provide the degree of superheat required (see Figure 3.4.4).

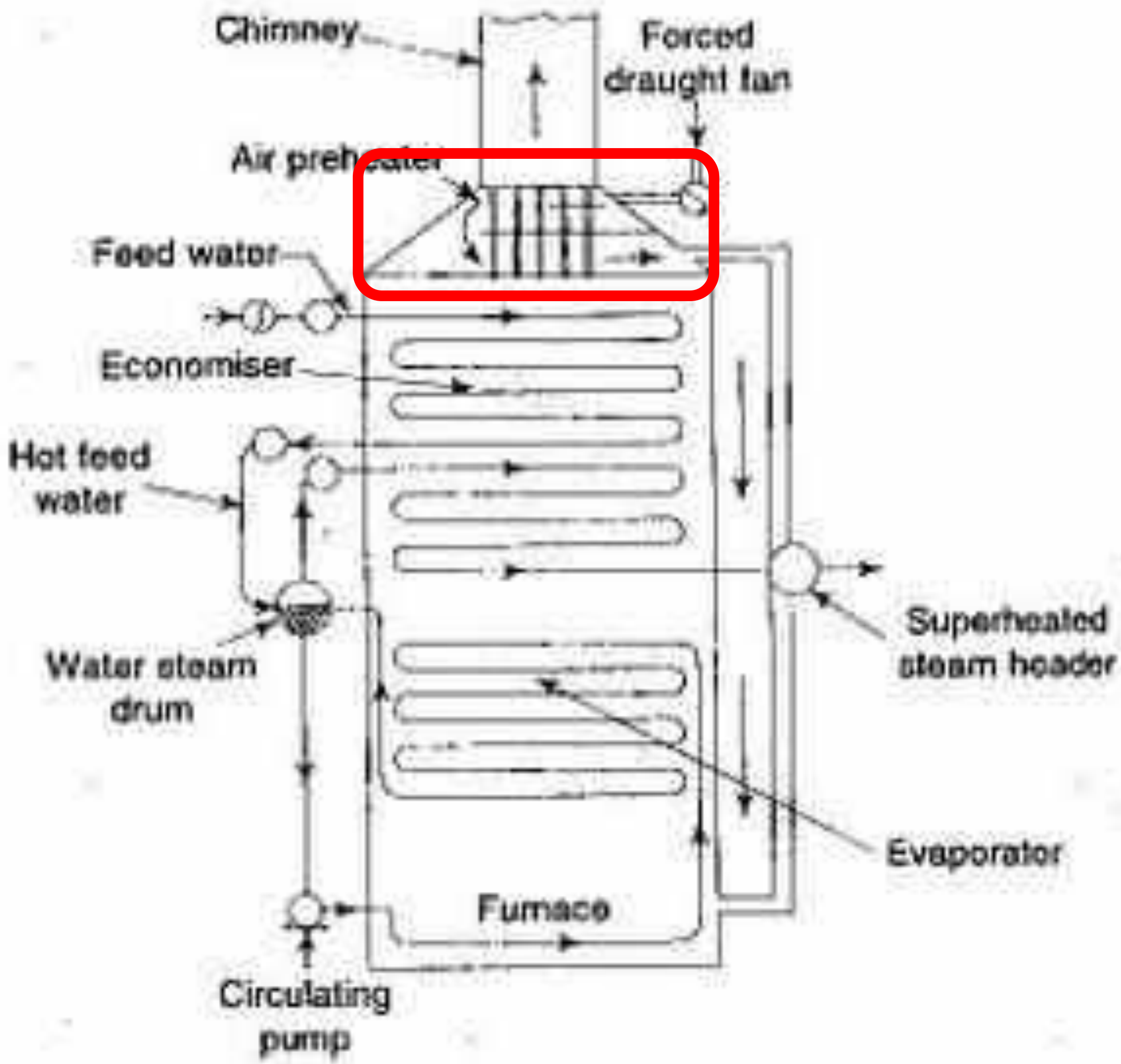
Fig. 3.4.4
A water tube boiler with a super heater



Super heater

Accessories of high pressure boiler

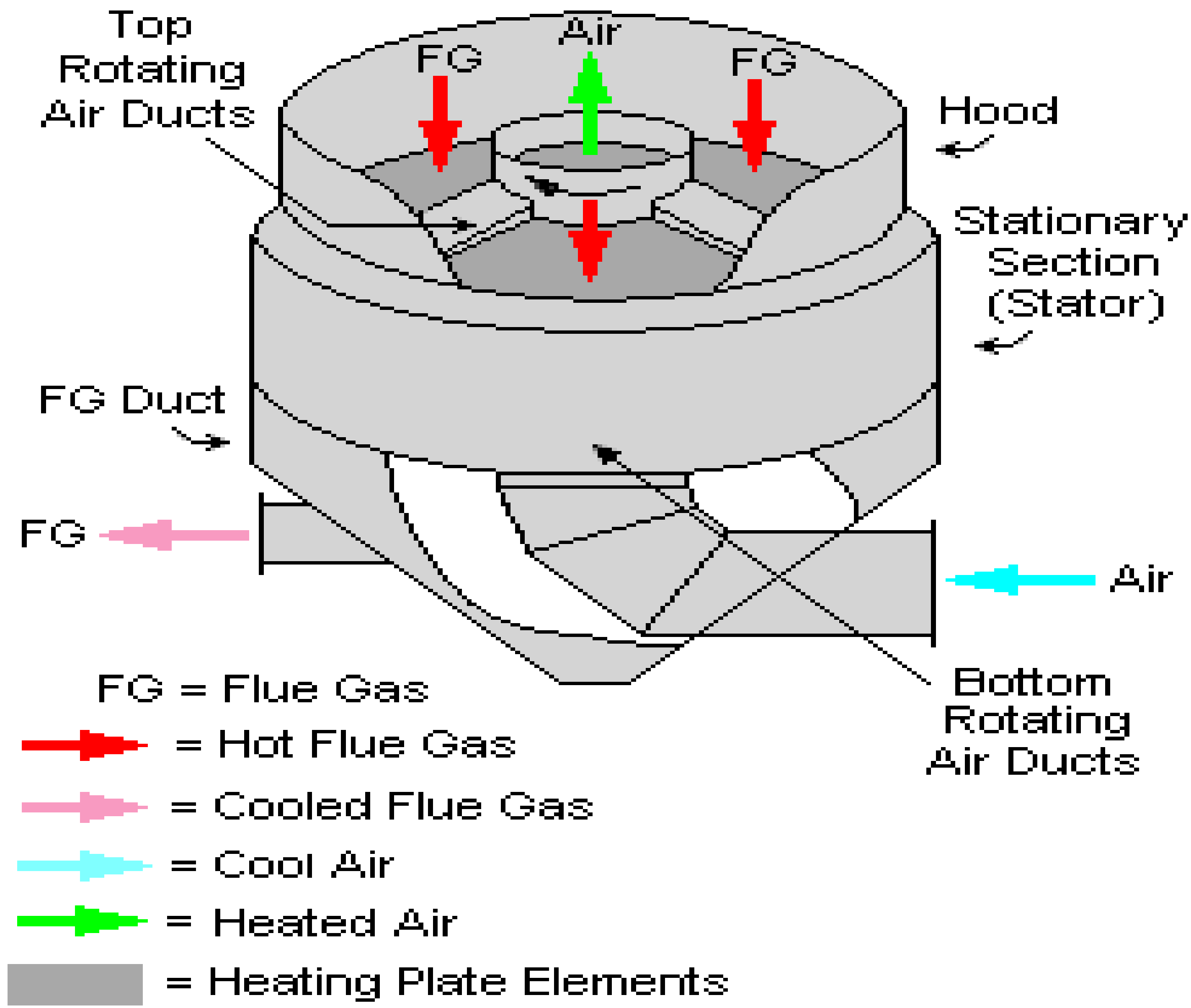
- 6. **Air-pre heater:** The function of air pre heater is to increase the temperature of air before is enters the furnace.
- It is generally placed after the economiser.
- So that flue gases pass through the economiser and then to air preheat.
- Usually, there are three types of pre-heater are Tubular type, Plate type and Regenerative type.



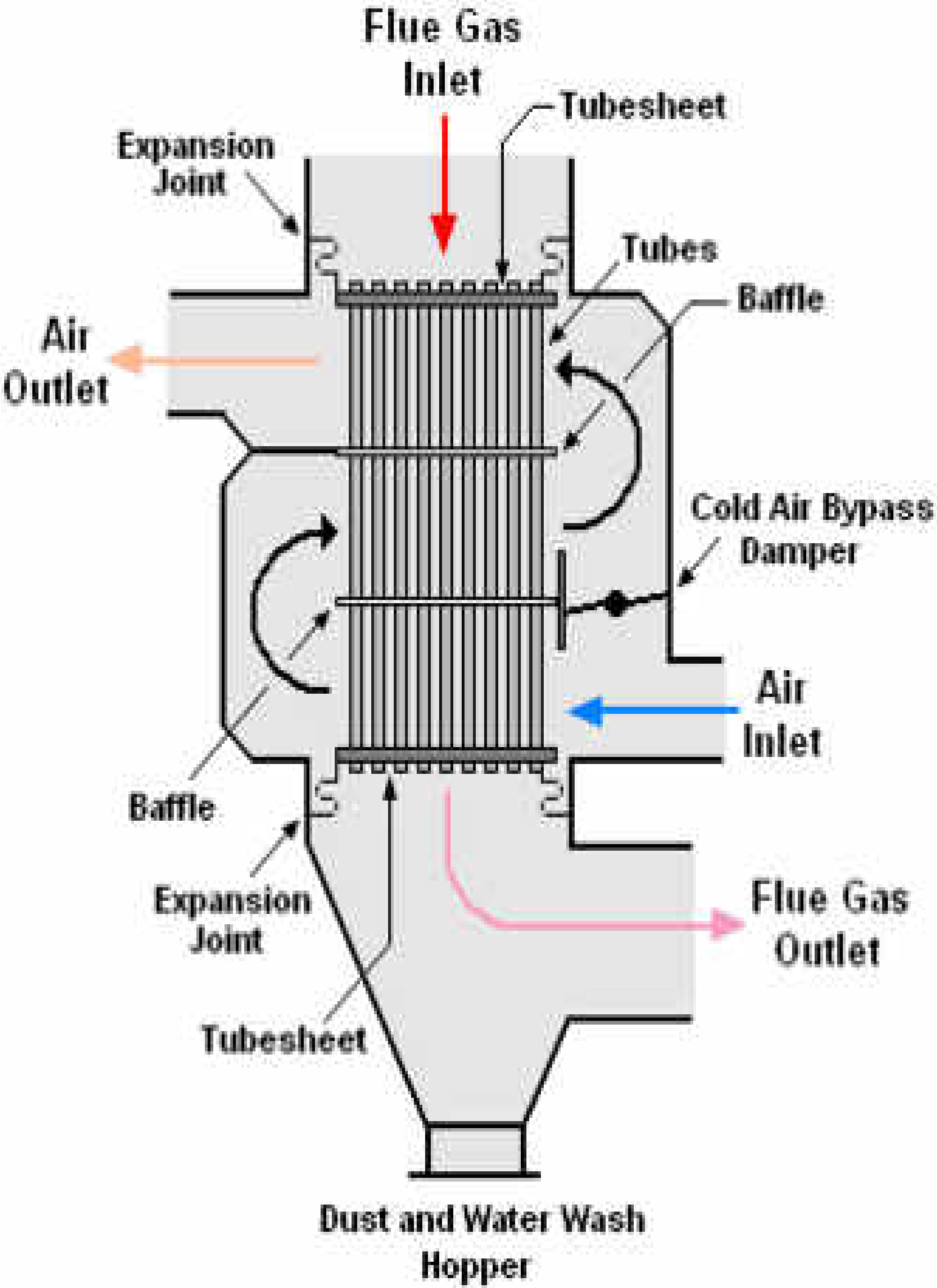
High pressure Lamont boiler

Air pre heater

- **Stationary-plate regenerative air preheater**
- Instead the air ducts in the preheater are rotated so as to alternatively expose sections of the heating plate elements to the upflowing cool air.[1][2][3]
- As indicated in the adjacent drawing, there are rotating inlet air ducts at the bottom of the stationary plates similar to the rotating outlet air ducts at the top of the stationary plates.



Typical Stationary Plate Air Preheater



Tubular type

A bundle of vertical tubes through which the flue gas flows downward (see adjacent diagram) and exchanges heat with ambient air flowing horizontally across the exterior of the tubes.

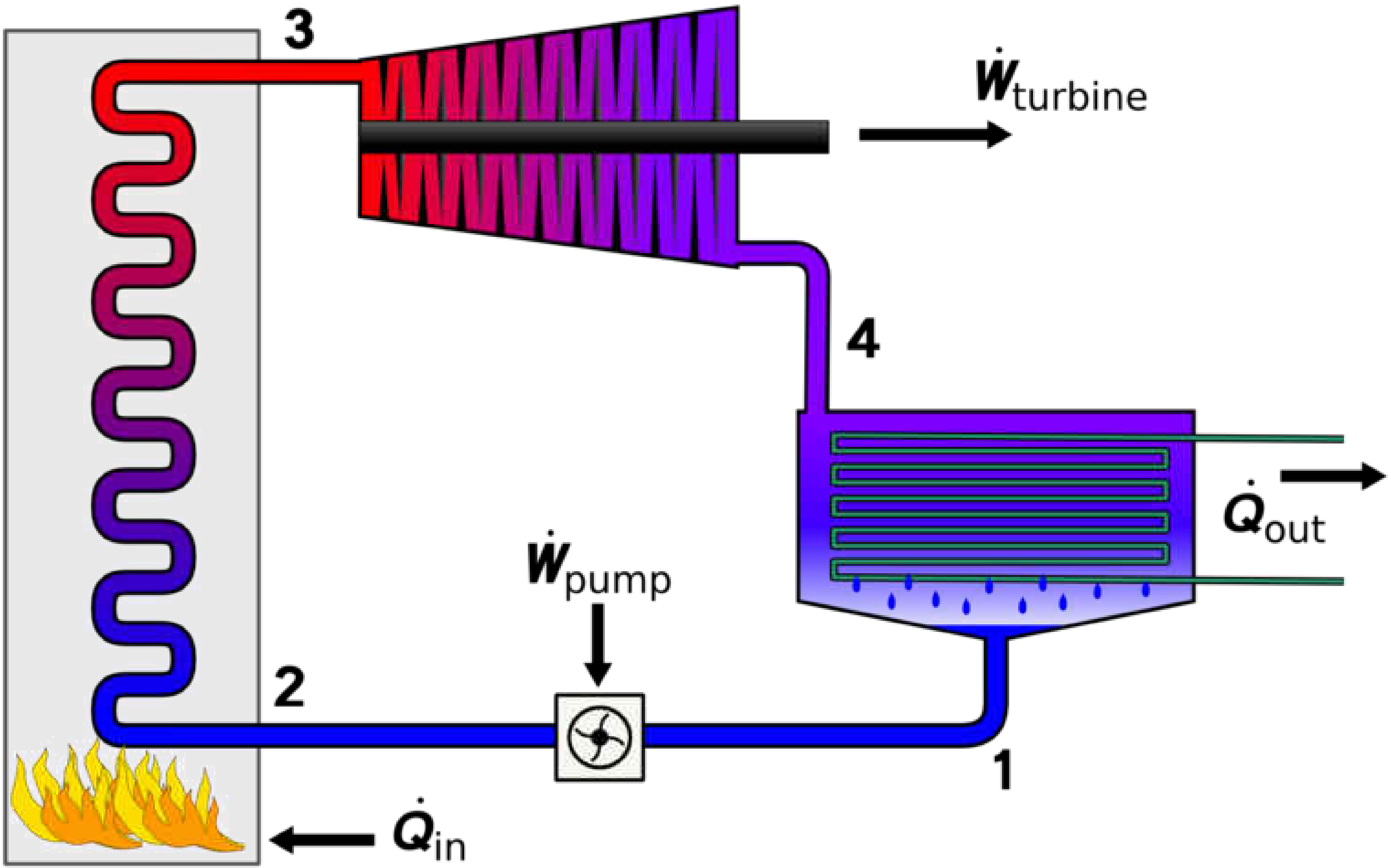
. Baffles are usually provided so that the air flows across the tubes a number of times. For example, as shown in the adjacent diagram, the air flow across the tubes three times and is referred to as 3-pass tubular air pre heater.

The same as (1) above except that the flue gas flows upward rather than downward.

A bundle of horizontal tubes through which the air flows and exchanges heat with the hot flue gas flowing downward across the tube bundle

- . In some designs, there may be three separate horizontal tube bundles one above the other.
- The air enters the lower tube bundle from the right-hand side, exits on the left-hand side and then enters the middle tube bundle on the left-hand side and exits on the right-hand side.
- Finally, the air enters the upper tube bundle on the right-hand side and exits on the left-hand side.
- In essence, such a design is similar to the 3-pass design of (1) above except that the air is in the tubes rather than outside the tubes.

Rankine cycle



Rankine cycle

- There are four processes in the Rankine cycle. These states are identified by numbers (in brown) in the above Ts diagram.

Process 1-2: The working fluid is pumped from low to high pressure.

As the fluid is a liquid at this stage the pump requires little input energy.

- **Process 2-3**: The high pressure liquid enters a boiler where it is heated at constant pressure by an external heat source to become a dry saturated vapors.

- **Process 3-4**: The dry saturated vapor expands through a turbine, generating power. This decreases the temperature and pressure of the vapour, and some condensation may occur.
- **Process 4-1**: The wet vapour then enters a condenser where it is condensed at a constant pressure to become a saturated liquid.
- In an ideal Rankine cycle the pump and turbine would be isentropic, i.e., the pump and turbine would generate no entropy and hence maximize the net work output.
- Processes 1-2 and 3-4 would be represented by vertical lines on the T-S diagram and more closely resemble that of the Carnot cycle.

Draught

- This difference of pressure for to maintaining the constant flow of air and discharging the gases through the chimney to atmosphere is known as draught.

Draught

- A fuel-heated boiler must provide air to oxidize its fuel. Early boilers provided this stream of air, or *draught*, through the natural action of convection in a chimney connected to the exhaust of the combustion chamber.
- Since the heated flue gas is less dense than the ambient air surrounding the boiler, the flue gas rises in the chimney, pulling denser, fresh air into the combustion chamber.
- Most modern boilers depend on mechanical draught rather than natural draught. This is because natural draught is subject to outside air conditions and temperature of flue gases leaving the furnace, as well as the chimney height.
- All these factors make proper draught hard to attain and therefore make mechanical draught equipment much more reliable and economical.
- Types of draught can also be divided into *induced draught*, where exhaust gases are pulled out of the boiler; *forced draught*, where fresh air is pushed into the boiler; and *balanced draught*, where both effects are employed

Classification of draught

Draught

- (a) Natural draught : In this the pressure difference is created naturally without using any positive displacement device.
- (b Artificial draught: Artificial draught is created using some external assistance causing forced displacement of gases. It can be created either by using mechanical devices or steam. Artificial draught can be further of induced, forced or combination of two types.
  - Mechanical draught
    - Induced draught
    - Forced draught
    - Balanced draught
  - Steam draught
    - Induced draught
    - Forced draught

Draught

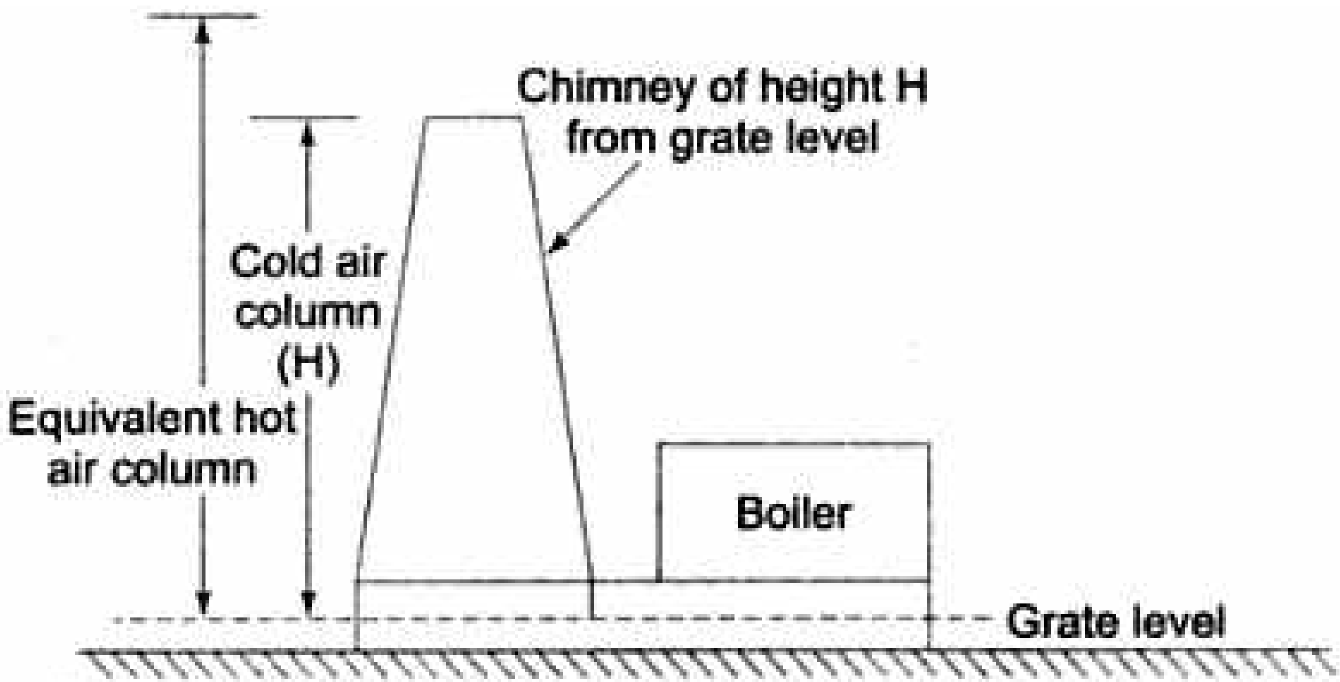
- Natural draught through the use of a chimney is a type of induced draught; mechanical draught can be induced, forced or balanced.
- There are two types of mechanical induced draught. The first is through use of a steam jet. The steam jet oriented in the direction of flue gas flow induces flue gasses into the stack and allows for a greater flue gas velocity increasing the overall draught in the furnace.
- This method was common on steam driven locomotives which could not have tall chimneys. The second method is by simply using an induced draught fan (ID fan) which removes flue gases from the furnace and forces the exhaust gas up the stack. Almost all induced draught furnaces operate with a slightly negative pressure.
- Mechanical forced draught is provided by means of a fan forcing air into the combustion chamber. Air is often passed through an air heater; which, as the name suggests, heats the air going into the furnace in order to increase the overall efficiency of the boiler. Dampers are used to control the quantity of air admitted to the furnace. Forced draught furnaces usually have a positive pressure.

Draught

- Balanced draught is obtained through use of both induced and forced draught. This is more common with larger boilers where the flue gases have to travel a long distance through many boiler passes.
- The induced draught fan works in conjunction with the forced draught fan allowing the furnace pressure to be maintained slightly below atmospheric.

- **Natural draft:**
- When air or flue gases flow due to the difference in density of the hot flue gases and cooler ambient gases.
- The difference in density creates a pressure differential that moves the hotter flue gases into the cooler surroundings.[

It is produced employing chimney. The natural draught is produced by a chimney due to the fact that the hot gases inside the chimney are lighter than the outside cold air i.e. density difference of hot gases inside chimney and cold atmospheric air. Thus in a boiler unit the combustion products (hot) rise from fuel bed through chimney, and are replaced by fresh air (cold) entering the grate. It means that amount of draught produced by a chimney depends upon flue-gas temperature. Intensity of draught produced by chimney also depends upon height of chimney. Draught produced by a taller chimney is large as the difference in weight between the column of air inside and that of air outside increases with height. Generally draught is less than 12 kgf/m^2 in chimneys.

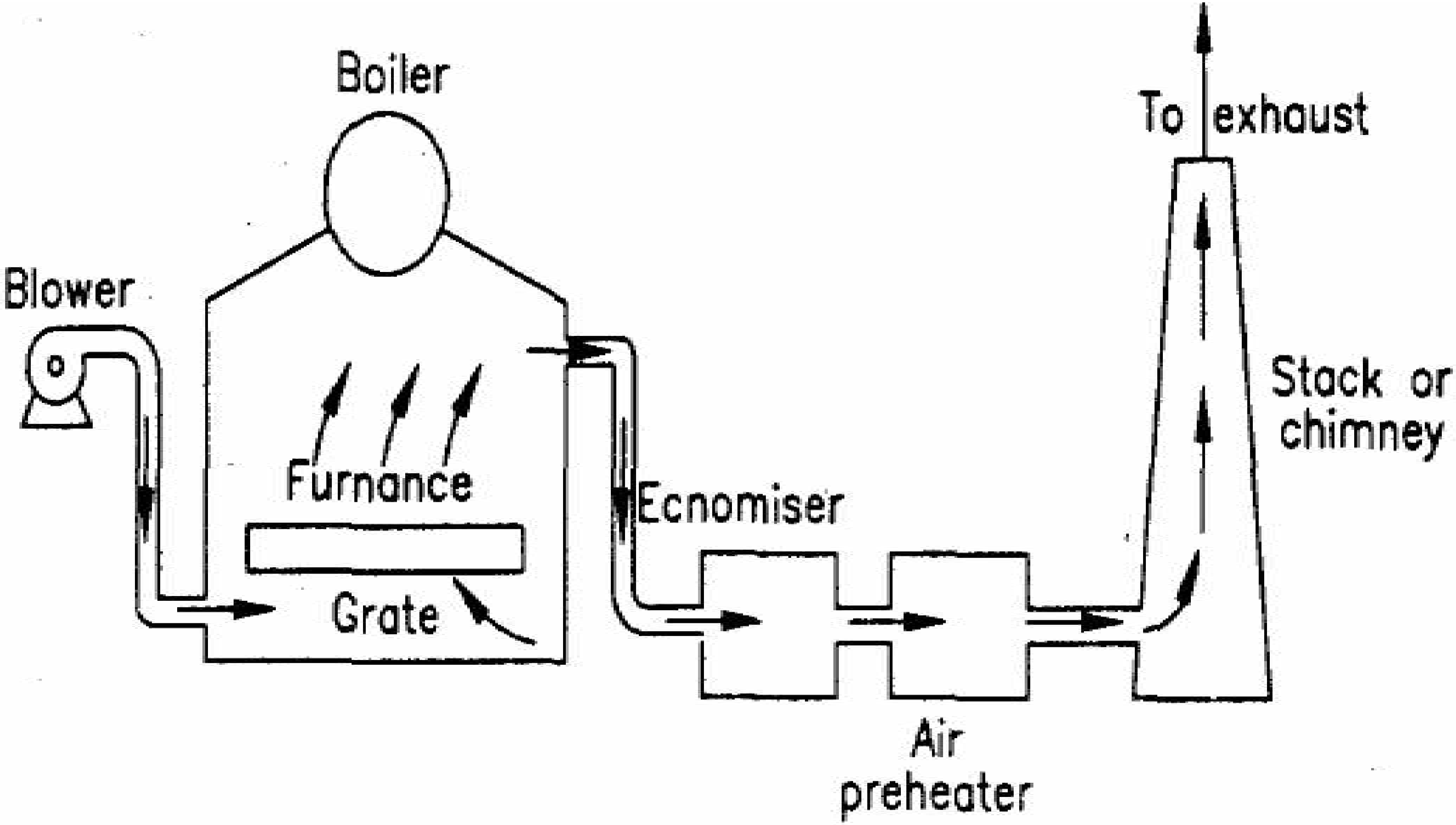


- ADVANTAGES :
- (1) It does not require any external power for producing the draught.
- (2) The capital investment is less. The maintenance cost is nil as there is no mechanical part.
- (3) Chimney keeps the flue gases at a high place in the atmosphere which prevents the contamination of atmosphere.
- (4) It has long life.

Forced draught

- In a forced draught system, a blower is installed near the base of the boiler.
- This draught system is known as positive draught system or forced draught system because the pressure of air throughout the system is above atmospheric pressure and air is forced to flow through the system.
- The arrangement of the system is shown in figure.
- A stack or chimney is also used in this system as shown in figure but it is not much significant for producing draught.

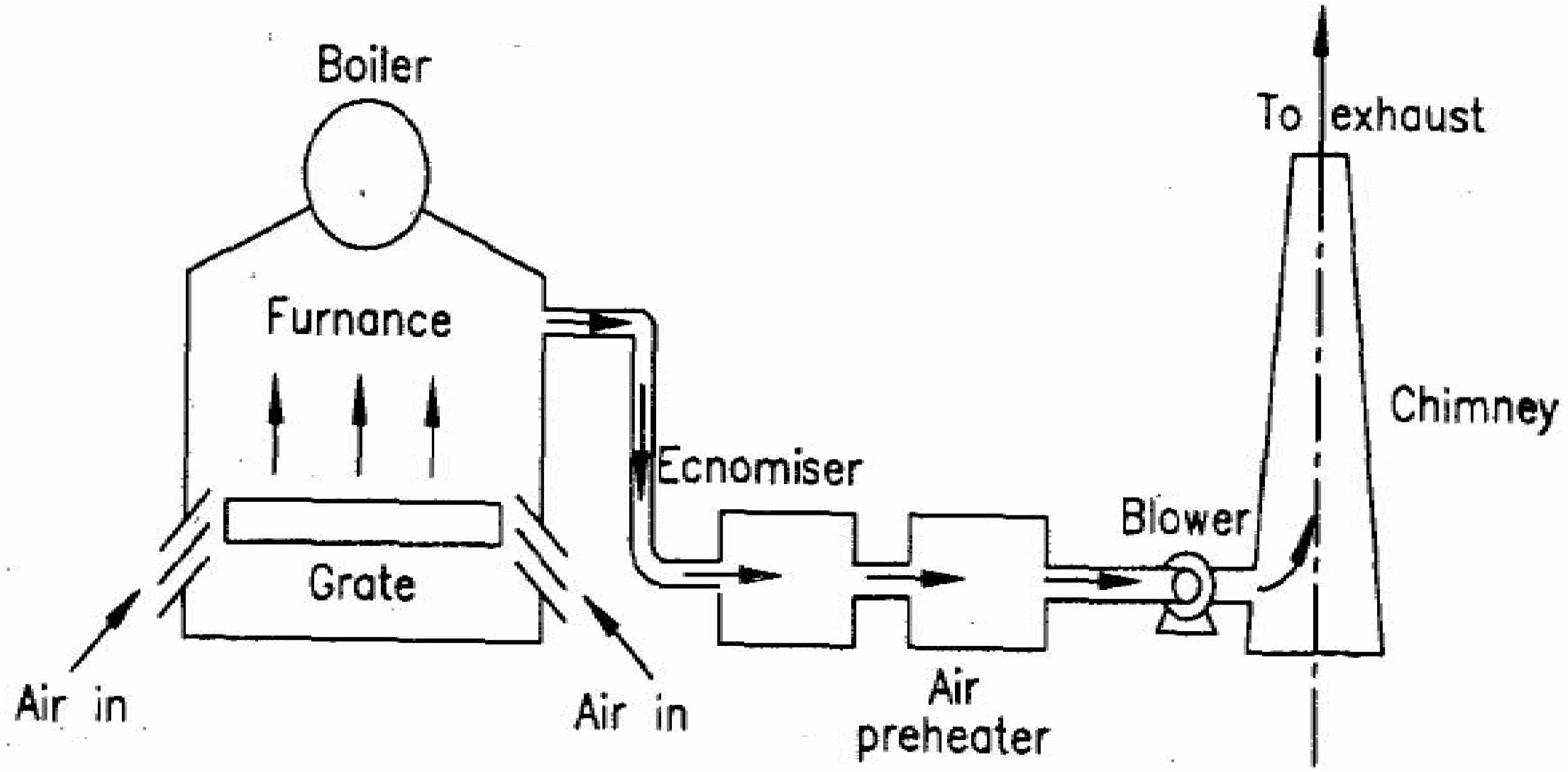
Forced draught



induced draught

- In this system, the blower is located near the base of the chimney instead of near the grate.
- The air is sucked in the system by reducing the pressure through the system below atmosphere.
- The action of the induced draught is similar to the action of the chimney.
- The draught produced is independent of the temperature of the hot gases therefore the gases may be discharged as cold as possible after recovering as much heat as possible in air pre-heater and economizer.

induced draught



induced draught

- This draught is used generally when economizer and air pre –heater are incorporated in the system.
  The fan should be located at such a place that the temperature of the gas handled by the fan is lowest.
- The chimney is also used in this system and its function is similar as mentioned in forced draught but total draught produced in induced draught system is the sum of the draughts produced by the fan and chimney.
- The arrangement of the system is shown in Figure.

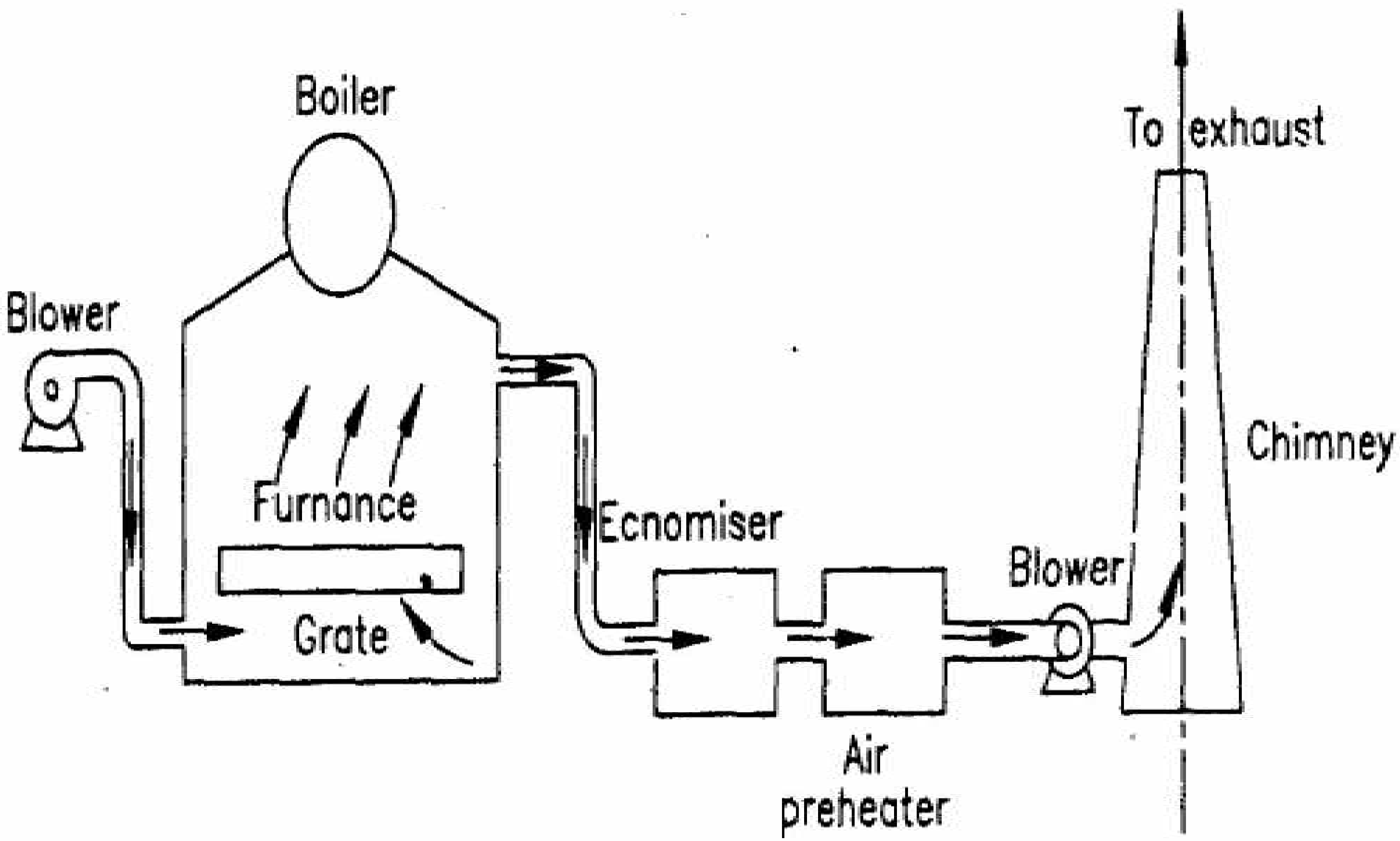
Balanced draught

- The balanced draught is a combination of forced and induced draught.
- If the forced draught is used alone, then the furnace cannot be opened either for firing or inspection because the high pressure air inside the furnace will try to blow out suddenly and there is every chance of blowing out the fire completely and furnace stops.

Balanced draught

- If the induced draught is used alone, then also furnace cannot be opened either for firing or inspection because the cold air will try to rush into the furnace as the pressure inside the furnace is below atmospheric pressure.
- To overcome both the difficulties mentioned above either using forced draught or induced draught alone, a balanced draught is always preferred.

Balanced draught



comparision

S. No.	Forced draught	Induced draught
1.	The fan is placed before the fire grate.	The fan is placed after the fire grate.
2.	The pressure inside the furnace is above the atmospheric pressure.	The pressure inside the furnace is below the atmospheric pressure.
3.	It forces fresh air into the combustion chamber.	It sucks hot gases from the combustion chamber, and forces them into the chimney.
4.	It requires less power as the fan has to handle cold air only. Moreover, volume of air handled is less because of low temperature of the cold air.	It requires more power as the fan has to handle hot air and flue gases. Moreover, volume of air and gases is more because of high temperature of the air and gases.
5.	The flow of air through grate and furnace is more uniform.	The flow of air through grate and furnace is less uniform.
6.	As the leakages are outward, therefore there is a serious danger of blow out when the fire doors are opened and the fan is working.	As the leakages are inward, therefore there is no danger of blow out. But if the fire doors are opened and the fan is working there will be a heavy air infiltration.

Vapour process

- Constant pressure process
- The generation of steam in boiler is an example of constant pressure process.
- In this process, the pressure of steam before and after the process is constant.
- We have already discussed heating of wet steam is done at constant pressure in order to convert it in to dry saturated steam.
- We also know that the superheating is done at a constant pressure.

Fig. 3.10.

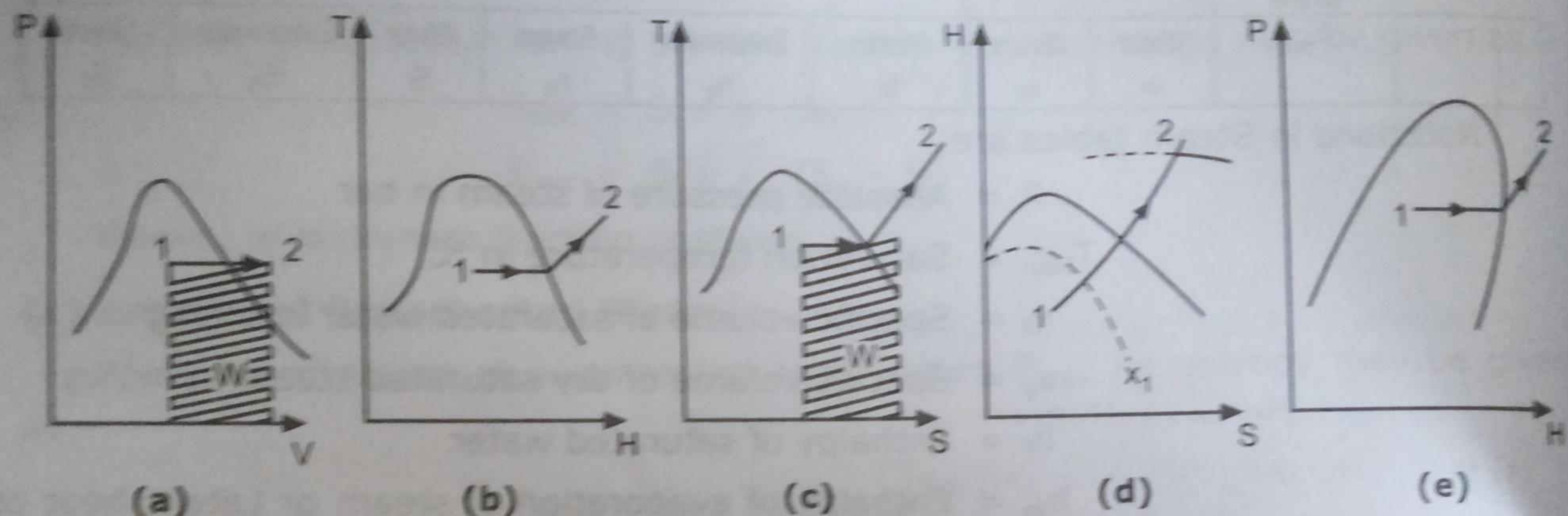


Fig. 3.10: Representation of constant pressure process 1-2

(a) Work done:

The work done during reversible constant flow process at constant pressure is given as,

$$W_{1-2} = \int_1^2 P_1\, dv = P_1 (v_2 - v_1)$$

where, v_1 = Specific volume at point 1 and v_2 = Specific volume at point 2.

(b) Heat transferred:

From first law, the heat transferred during non-flow process is given by,

$$\partial q = du + Pdv$$

$$\therefore \quad q_{1-2} = (u_2 - u_1) + P (v_2 - v_1)$$

$$\therefore \quad q_{1-2} = (u_2 - u_1) + P_2v_2 - P_1v_1 = h_2 - h_1$$

Thus, the heat transferred at constant pressure is equal to change in enthalpy.

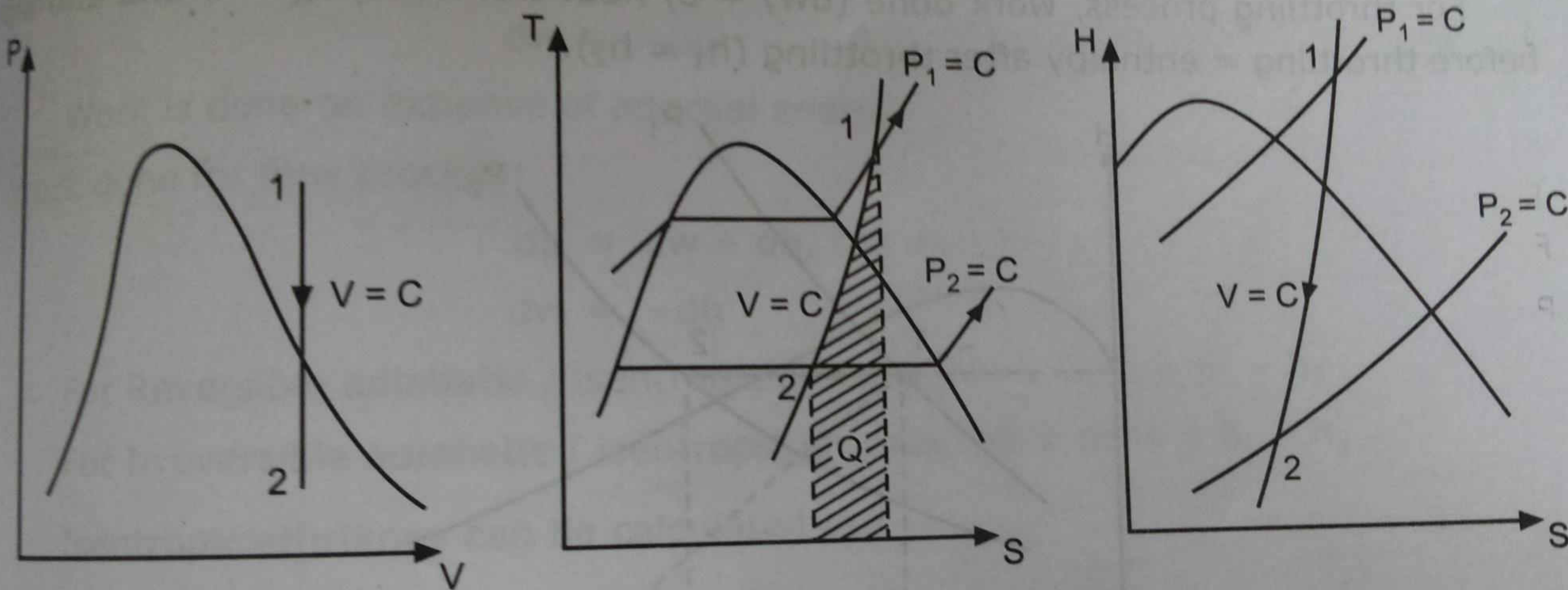
(c) Change in internal energy:

The change in internal energy is given by,

$$u_2 - u_1 = (h_2 - h_1) - (P_2v_2 - P_1v_1)$$

Constant volume process

- The heating or cooling of the steam in a closed vessel is an example of constant volume process.
- In this process, the volume (mass)of the steam before and after the process is constant.
- It may be noted that, in this process, no work is done.



(a)

(b)

(c)

Fig. 3.11: Representation of constant volume process 1-2

(a) Work done:

As the volume remains constant, Work done = 0.

(b) Heat transferred:

From First law,

$$\partial q = du + Pdv$$

Since

$$dv = 0$$

$$q_{1-2} = u_2 - u_1$$

$\therefore$

$$q_{1-2} = (h_2 - P_2v_2) + (h_1 - P_1v_1)$$

The values of h_1 and h_2 can be found corresponding to initial and condition. Thus, heat transferred at constant volume is equal to change in in energy.

3. Constant Enthalpy Process:

It is known as throttling process which involves the passing of high pressure through a narrow constriction resulting in,

1. Reduction in pressure and temperature.
2. Increase in volume.
3. Increase in entropy.
4. Constant enthalpy.

4. Constant enthalpy.

In this process, enthalpy remains constant and process is adiabatic as there heat flow. Throttle valve is used to throttle the steam.

Let steam throttles from pressure P_1 to pressure P_2. Due to throttling, wet gets converted into superheated steam as shown in Fig. 3.12.

For throttling process, work done (dw) = 0, heat transfer (dq) = 0 and en before throttling = enthalpy after throttling ($h_1 = h_2$).

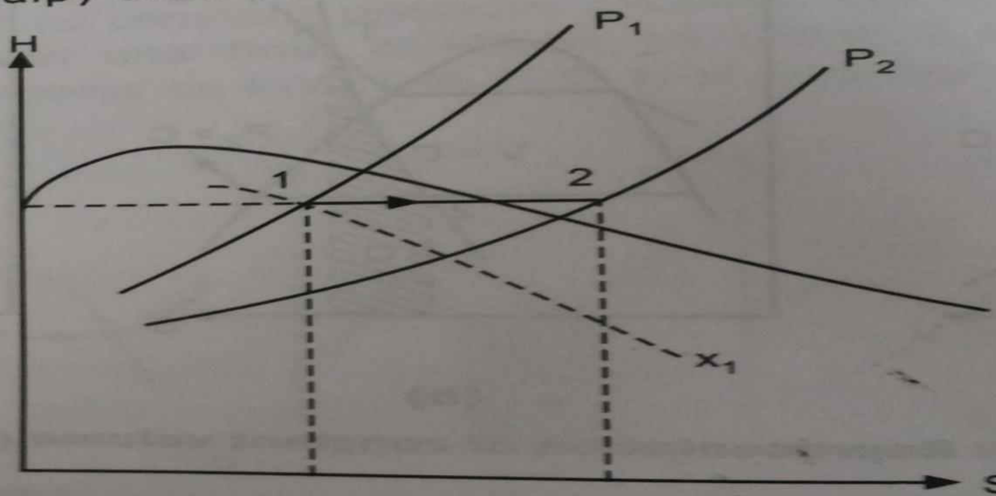


Fig. 3.12: Representation of constant enthalpy process 1-2

4. Constant Entropy Process:

Assume process 1-2, adiabatic expansion of steam. Initial condition of steam is superheated and final condition is wet. In reversible adiabatic or isentropic process, no heat transfer takes place and entropy remains constant.

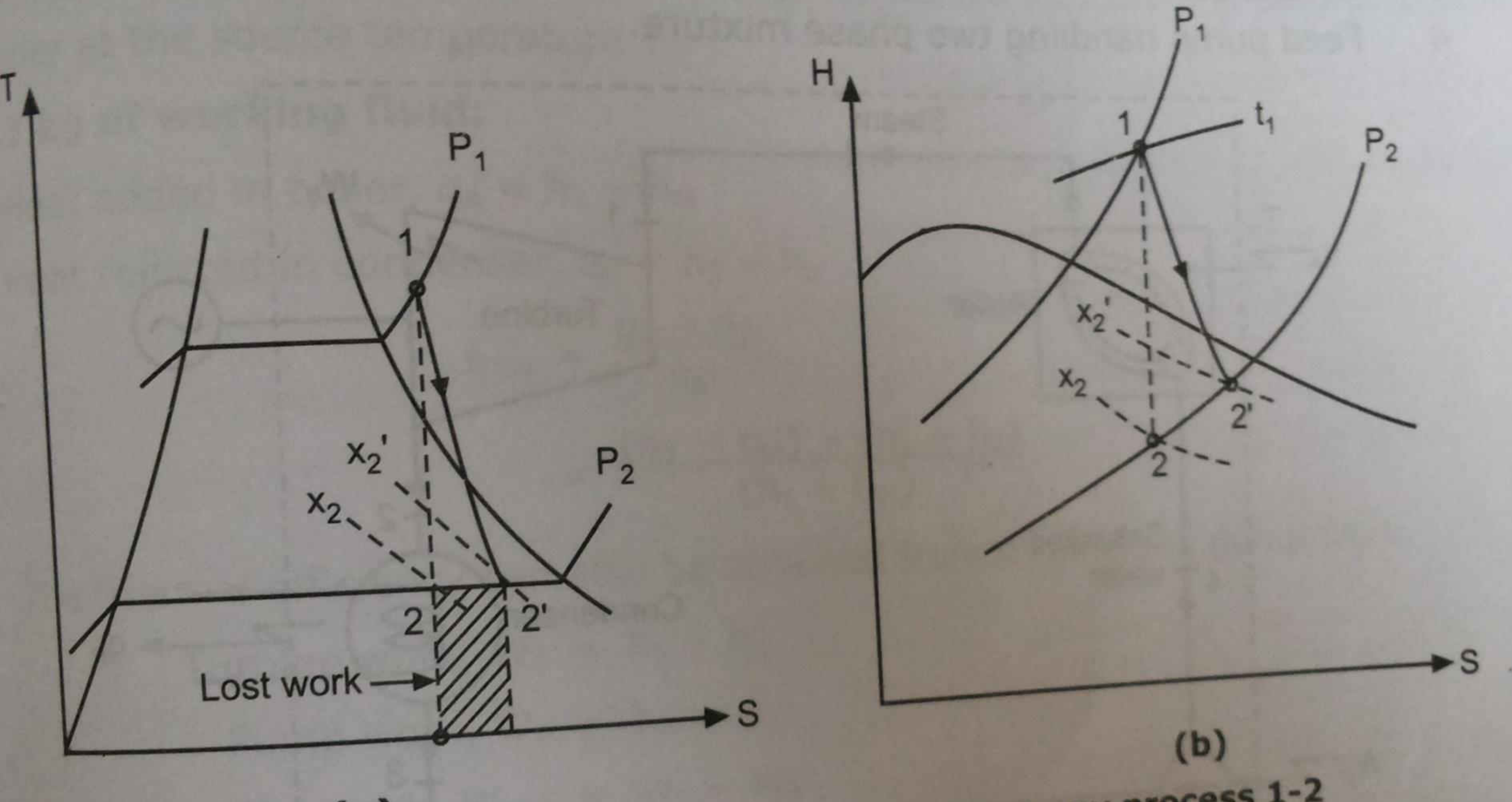


Fig. 3.13: Representation of constant entropy process 1-2

...ntropy process 1-2

(a) Work done:

From first law,

$$dq = du + dw, \text{ but } dq = 0$$

$$\therefore \quad dw = -du$$

Work is done on expense of internal energy.

Work done for flow process:

$$dq = dw + dh, \text{ but } dq = 0$$

$$\therefore \quad dw = -dh$$

For Reversible adiabatic / isentropic process, Work done = $h_1 - h_2$

For Irreversible adiabatic / isentropic process, Work done = $h_1 - h_2'$

Isentropic efficiency can be calculated as,

$$\eta_{isentropic} = \frac{h_1 - h_2'}{h_1 - h_2}$$

Work lost is shown by shaded area.

(b) Heat transferred:

For isentropic/adiabatic process, heat transfer = 0

8

Air Standard Cycles

Table of Contents

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- The Three Thermodynamic Analysis of IC Engines are
  I. **Ideal Gas Cycle (Air Standard Cycle)**
     - Idealized processes
     - Idealize working Fluid
  II. **Fuel-Air Cycle**
     - Idealized Processes
     - Accurate Working Fluid Model
  III. **Actual Engine Cycle**
     - Accurate Models of Processes
     - Accurate Working Fluid Model

Introduction

- The operating cycle of an IC engine can be broken down into a sequence of separate processes
  - Intake, Compression, Combustion, Expansion and Exhaust.
- Actual IC Engine does not operate on thermodynamic cycle that ideal are operated on open cycle.
- The accurate analysis of IC engine processes is very complicated, to understand it well, it is advantageous to analyze the performance of an Idealized closed cycle

Air Standard Cycles

Air-Standard cycle differs from the actual by the following

1. The gas mixture in the cylinder is treated as air for the entire cycle, and property values of air are used in the analysis.

2. The real open cycle is changed into a closed cycle by assuming that the gases being exhausted are fed back into the intake system.

3. The combustion process is replaced with a heat addition term Q_{in} of equal energy value

Air standard cycles

4. The open exhaust process, which carries a large amount of enthalpy is denoted by Q_{out} of the equal energy value
5. Actual engine processes are approximated with ideal processes
   a. The almost-constant-pressure intake and exhaust strokes are assumed to be constant pressure.
   b. Compression strokes and expansion strokes are approximated by isentropic processes

Air Standard Cycles Assumption

c. The combustion process is idealized by a constant-volume process (SI cycle), a constant-pressure process (CI cycle), or a combination of both (CI Dual cycle).

d. Exhaust blow down is approximated by a constant-volume process.

e. All processes are considered reversible.

Charles Law

- For an ideal gas, volume is directly proportional Kelvin's temperature, if moles of gas and pressure are constant.
- $V \propto T$ If n and P are constant
- Decreased temperature ⟹ Decreased volume
- Increased temperature ⟹ Increased volume

Gay Lussac's Law

- The pressure of an ideal gas is directly proportional to the Kelvin temperature of the gas if the volume and moles of gas are constant
- $P \propto T$ if n and V are constant
- Decreased temperature → Decreased pressure
- Increased temperature → Increased pressure

Ideal Gas Law

- Combining Boyle's and Charles' laws allows for developing a single equation:

$$\boxed{Pv = RT}$$

  - Where the constant of proportionality R is called the ***gas constant***.

- The gas constant R is different for each gas and is determined from:

$$\boxed{R = \frac{R_U}{M} \qquad [kJ/kg.K]}$$

  - Ru is the ***Universal gas constant*** and M is the ***molar mass (molecular weight)***

Polytropic Process

- During Expansion and compression process, pressure and volume are often related by:

$$\boxed{PV^{n} = C \quad or \;\; P = CV^{-n}}$$

  - Where C is a constant, and n is a value between 1.0 and 1.4.

- Assuming the combustion chamber is perfectly insulated (adiabatic), $n=k=1.4$. For an isothermal process, n=1.0.

Isentropic process

- For an ideal gas k is constant.

$$PV^k = C$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^k$$

- Using the equation of state for an ideal gas,

$$PV = mRT$$

- $$TV^{k-1} = C$$

$$TP^{\frac{1-k}{k}} = C$$

Isentropic work

- Work done during isentropic Process becomes:

$$\boxed{W = \int_1^2 P\,dV = \int_1^2 CV^{-n}\,dV = C\,\frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1} = \frac{P_2V_2 - P_1V_1}{1-n}}$$

- For an ideal gas this equation can be written as:

$$\boxed{W = \frac{mR(T_2 - T_1)}{1-k}}$$

$$P = \rho R T$$

$$dh = c_P\,dT$$

$$du = c_v\,dT$$

Specific heat constants

- When analyzing what occurs within engines during the *operating cycle* *and* *exhaust flow*, the following air property values are used:

$$C_P = 1.108 \quad [kJ/kg-K]$$

$$C_V = 0.821 \quad [kJ/kg-K]$$

$$k = C_P / C_V = 1.108/0.821 = 1.35$$

$$R = C_P - C_V = 0.287 \quad [kJ/kg-K]$$

Air properties

- For processes such as *inlet flow in superchargers, turbochargers, and carburetors, and air flow through the engine radiator,* the following air property values are used:

$$C_P = 1.005 \qquad \left[kJ/kg-K\right]$$

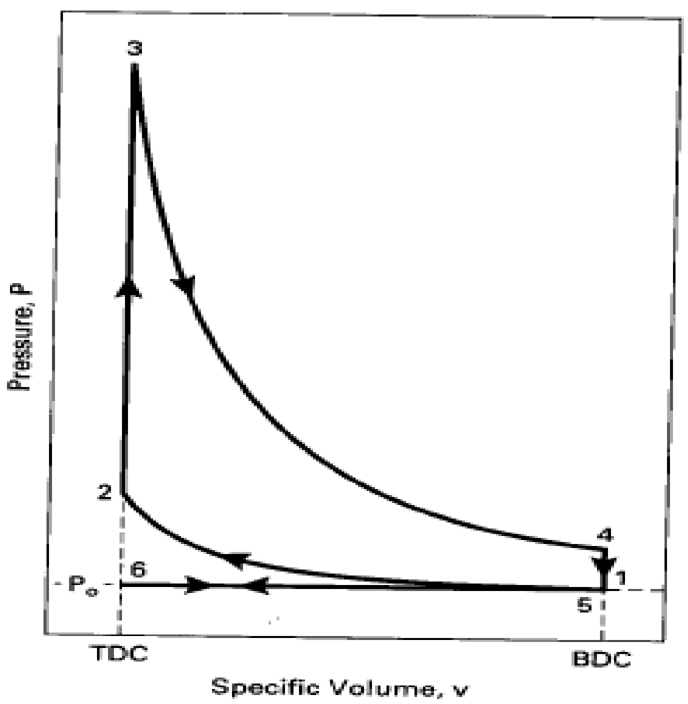
$$C_V = 0.718 \qquad \left[kJ/kg-K\right]$$

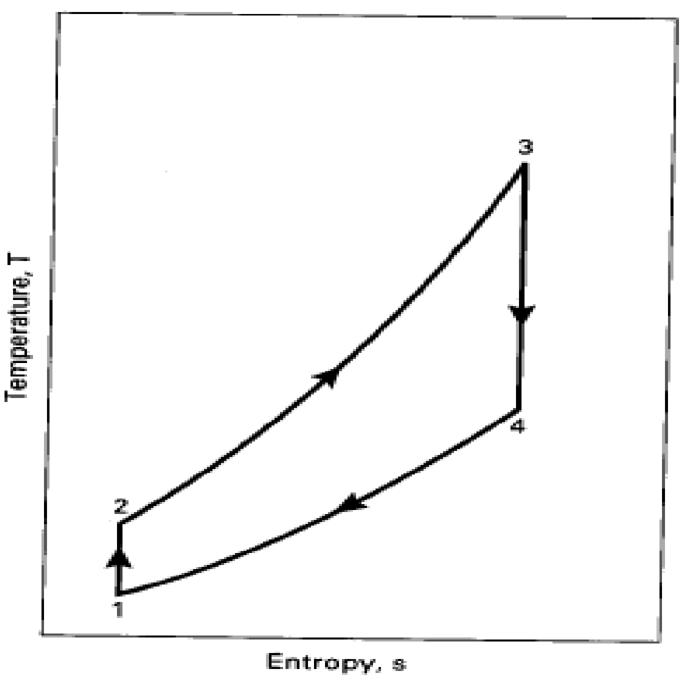
$$k = C_P / C_V = 1.005/0.718 = 1.4$$

$$R = C_P - C_V = 0.287 \qquad \left[kJ/kg-K\right]$$

Otto Cycle

- Point 6 to 1 - Intake
- Point 1 to 2 - Isentropic Compression
- Point 2 to 3 - Constant-Volume Heat Input
- Point 3 to 4 – Isentropic Expansion
- Point 4 to 5 – Blow Down
- Point 5 to 6 - Exhaust





Intake Stroke

- Starts with the piston at TDC
- Constant pressure process at the inlet pressure of one atmosphere.
- In real engine process 6-1 will be slightly less than atmospheric due to pressure losses in the inlet air flow.
- The temperature of the air during the inlet stroke is increased as the air passes through the hot intake manifold.

Compression Stroke

- It is an isentropic compression from BDC to TDC (process 1-2)
- In real engine, the beginning of the stroke is affected by the intake valve not being fully closed until slightly after BDC.
- The end of compression is affected by the firing of the spark plug before TDC.
- In addition to increase in pressure there is also increase in temperature due to compressive heating

Combustion Process

- It is a constant-volume heat input process 2-3 at TDC.
- In real engines combustion starts slightly bTDC, reaches its maximum speed near TDC, is terminated a little aTDC.
- Peak cycle pressure and temperature is reached at point 3 due to energy added to the air within the cylinder.

Power (Expansion) Stroke

- High pressure on the piston face forces the piston back towards BDC and produces the work and power output of the engine.

- The power stroke of the real engine cycle is approximated with an isentropic process in the Otto cycle.

- The beginning of the power stroke is affected by the last part of the combustion process.

- The end of the power stroke is affected by the exhaust valve being opened before BDC.

- Values of both the temperature and pressure within the cylinder decrease as volume increases from TDC to BDC.

Exhaust Blowdown

- Exhaust valve is opened near the end of the power stroke
- A large amount of exhaust gas is expelled from the cylinder, reducing the pressure to that of the exhaust manifold
- The exhaust valve is opened bBDC to allow for the finite time of blowdown to occur.
- The Otto cycle replaces the exhaust blowdown open-system process of the real cycle with ***a constant–volume pressure reduction***, closed-system process 4-5.
- Enthalpy loss during this process is replaced with heat rejection in the engine analysis.

Exhaust Stroke

- Occurs as the piston travels from BDC to TDC.

- Process 5-6 is the exhaust stroke that occurs at a constant pressure of one atmosphere due to the open exhaust valve.

Thermodynamic Analysis of Air-Standard Otto Cycle

- ***Process 6-1***-Constant-pressure intake of air at Po.
  - Intake valve open and exhaust valve closed:

$$\boxed{\begin{aligned} P_1 &= P_6 = P_o \\ w_{6-1} &= P_o\left(v_1 - v_6\right) \end{aligned}}$$

Process 1-2- Isentropic Compression Stroke

- All valves closed

$$T_2 = T_1\left(v_1/v_2\right)^{k-1} = T_1\left(V_1/V_2\right)^{k-1} = T_1\left(r_c\right)^{k-1}$$

$$P_2 = P_1 \quad /v_2\Big)^k = P_1\left(V_1/V_2\right)^k = P_1\left(r_c\right)^k$$

$$\Big(v_1 \quad q_{1-2} =$$

$$0$$

$$w_{1-2} = \frac{\left(P_2 v_2 - P_1 v_1\right)}{\left(1-k\right)} = \frac{R\left(T_2 - T_1\right)}{\left(1-k\right)}$$

$$= \left(u_1 - u_2\right) = c_v\left(T_1 - T_2\right)$$

Process 2-3- Constant– volume heat input (combustion).

- All valves closed

$v_3 = v_2 = v_{TDC}$ $w_{2-3} = 0$

$$Q_{2-3} = Q_{in} = m_f Q_{HV} \eta_C = m_m c_v (T_3 - T_2)$$

$$= (m_a + m_f) c_v (T_3 - T_2)$$

$$Q_{HV} \eta_C = (AF + 1) c_v (T_3 - T_2)$$

$$q_{2-3} = q_{in} = c_v (T_3 - T_2) = (u_3 - u_2)$$

$$T_3 = T_{\max}$$

$$P_3 = P_{\max}$$

Process 3-4: Isentropic power or expansion stroke.

- All valves closed:

$$q_{3-4} = 0$$

$$T_4 = T_3\left(v_3/v_4\right)^{k-1} = T_3\left(V_3/V_4\right)^{k-1} = T_3\left(1/r_C\right)^{k-1}$$

$$P_4 = P_3\left(v_3/v_4\right)^{k-1} = P_3\left(V_3/V_4\right)^{k-1} = P_3\left(1/r_C\right)^{k-1}$$

$$w_{3-4} = \frac{\left(P_4 v_4 - P_3 v_3\right)}{\left(1-k\right)} = \frac{R\left(T_4 - T_3\right)}{\left(1-k\right)}$$

$$= \left(u_3 - u_4\right) = c_v\left(T_3 - T_4\right)$$

Process 4-5: Constant-volume heat rejection (exhaust *blowdown*)

- Exhaust valve open and intake valve closed:

• $v_5 = v_4 = v_1 = v_{BDC}$ $w_{4-5} = 0$

$$Q_{4-5} = Q_{out} = m_m c_v \left(T_5 - T_4\right) = m_m c_v \left(T_1 - T_4\right)$$

$$q_{4-5} = q_{out} = c_v \left(T_5 - T_4\right) = \left(u_5 - u_4\right) = c_v \left(T_1 - T_4\right)$$

Process 5-6: Constant-pressure exhaust stroke at Po

- Exhaust valve open and intake valve closed:

$$P_5 = P_6 = P_o$$

$$w_{5-6} = P_o\left(v_6 - v_5\right) = P_o\left(v_6 - v_1\right)$$

Thermal efficiency of Otto Cycle

$$\left(\eta_t\right)_{OTTO} = \frac{\left|w_{net}\right|}{\left|q_{in}\right|} = 1 - \frac{\left|q_{out}\right|}{\left|q_{in}\right|}$$

$$= 1 - \frac{c_v\left(T_4 - T_1\right)}{c_v\left(T_3 - T_2\right)}$$

$$= 1 - \frac{\left(T_4 - T_1\right)}{\left(T_3 - T_2\right)}$$

Thermal efficiency of Otto Cycle

- For isentropic compression and expansion strokes and recognizing that $\boldsymbol{v_1 = v_4}$ ***and*** $\boldsymbol{v_2 = v_3}$

$$\boxed{(T_2/T_1) = (v_1/v_2)^{k-1} = (v_4/v_3)^{k-1} = (T_3/T_4)}$$

- Rearranging the temperature terms gives:

$$\boxed{(T_4/T_1) = (T_3/T_2)}$$

- Rearranging the above equation gives:

$$\boxed{(\eta_t)_{OTTO} = 1 - \left(\frac{T_1}{T_2}\right)\left\{\frac{\left(\frac{T_4}{T_1} - 1\right)}{\left(\frac{T_3}{T_2} - 1\right)}\right\}}$$

Thermal efficiency of Otto Cycle

- Thus efficiency becomes:

$$\boxed{(\eta_t)_{OTTO} = 1 - (T_1/T_2)}$$

- Combining this with:

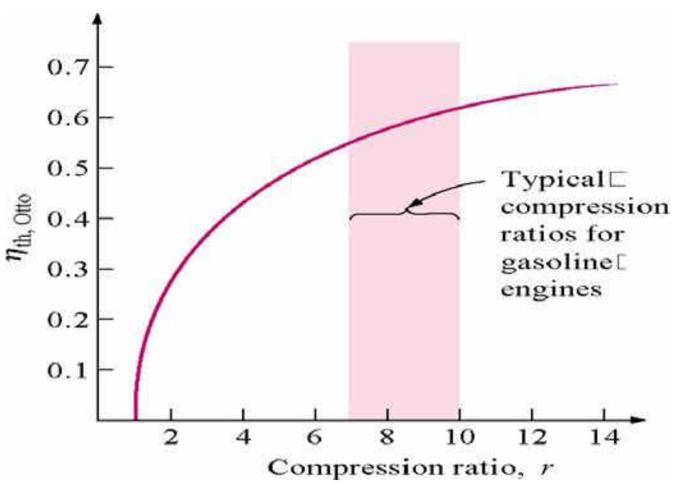
$$\boxed{T_2 = T_1(v_1/v_2)^{k-1} = T_1(V_1/V_2)^{k-1} = T_1(r_c)^{k-1}}$$

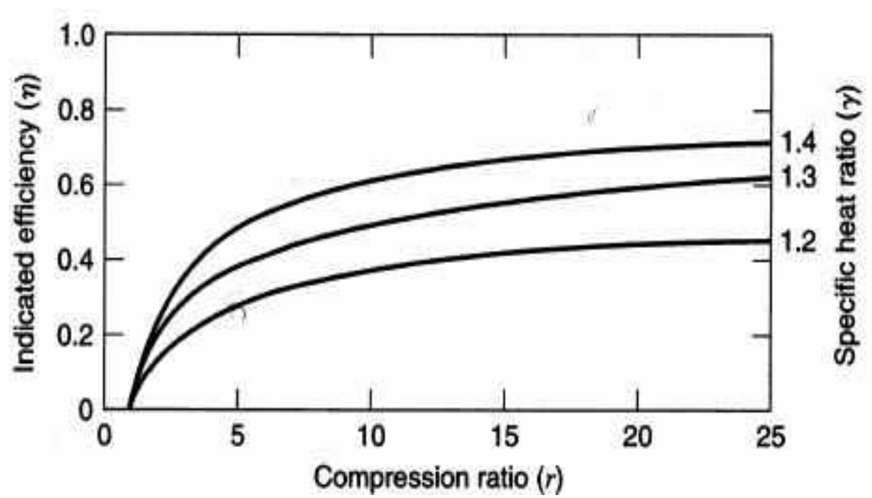
$$\boxed{(\eta_t)_{OTTO} = 1 - \left(1/(v_1/v_2)^{k-1}\right)}$$

$$\boxed{(\eta_t)_{OTTO} = 1 - (1/r_c)^{k-1}}$$

OTTO CYCLE

- Spark ignition engine compression ratio limited by
  - T_3 (Auto ignition)
  - P_3 (material strength),
- For r_c = 8 the efficiency is 56%
- Cylinder temperatures vary between 300K and 2000K so 1.2 < k < 1.4, k = 1.3 most representative





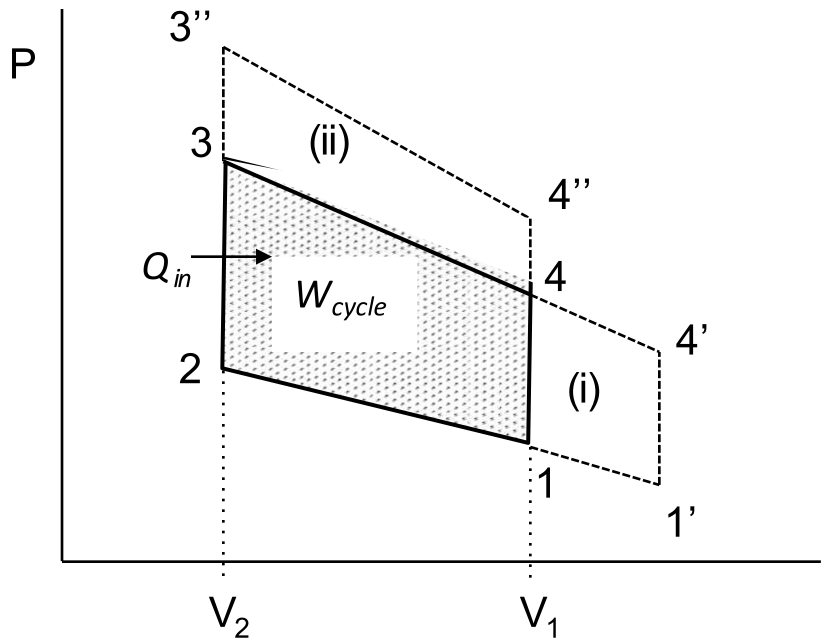
OTTO CYCLE

Factors Affecting Work per Cycle

- The indicted cycle work of an engine can be increased by either:
  - i) Increasing the *r* (1'→2)
  - ii) Increase *Qin* (2→3'')

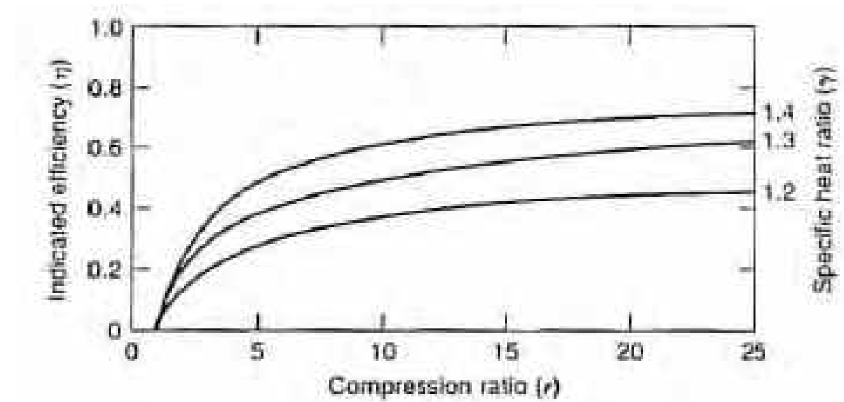
$$mep = \frac{W}{V_1 - V_2} = \frac{Q_{in}}{V_1}\left(\frac{r_c}{r_c - 1}\right)\eta_{th}$$

$$\frac{mep}{P_1} = \frac{W}{P_1\left(V_1 - V_2\right)} = \frac{Q_{in}}{P_1 V_1}\left(\frac{r_c}{r_c - 1}\right)\eta_{th}$$

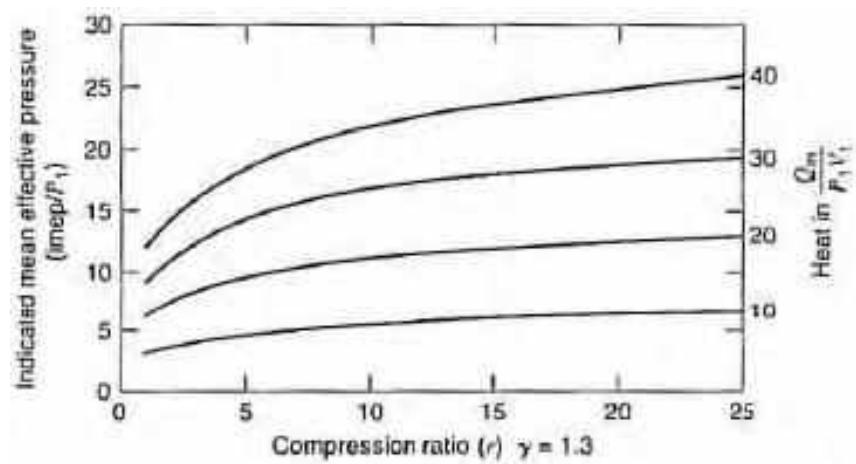


OTTO CYCLE

- The thermal efficiency of the Otto cycle depends only on compression ratio and specific heat ratio



- As expected the indicated mean effective pressure (imep) depends and compression ratio e heat addition ratio



OTTO CYCLE

$$mep = \eta_{th} \frac{Q_{in} \dfrac{p_1}{m R T_1}}{1 - \dfrac{1}{r_c}}$$

$$V_1 = m \frac{RT_1}{P_1}$$

- Non-dimensional mep with p_1 we get

$$\frac{mep}{p_1} = \eta_{th} \left[\frac{\square 1}{1 - \dfrac{1}{r_c}} \right] \left[\frac{q}{RT_1} \right]$$

$$R = c_v(\gamma - 1)$$

$$\frac{mep}{p_1} = \eta_{th} \frac{q}{c_v T_1} \frac{1}{\left[1 - \dfrac{1}{r_c} \right] [\gamma - 1]}$$

Mep/p_1 is a function of heat added, properties of air, namely, cv and ■ initial temperature, compression ratio

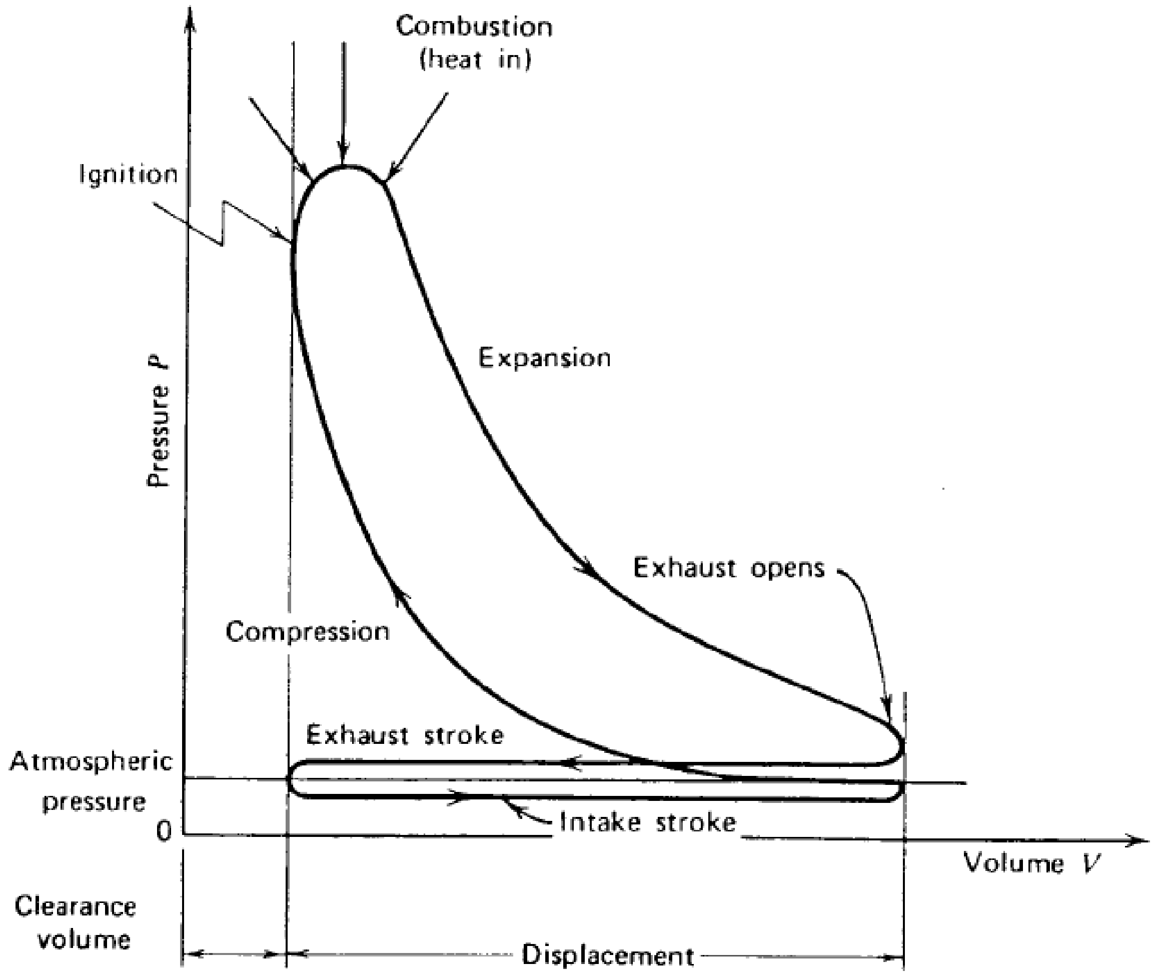
Determine the relation for mep/p3

$$\frac{mep}{p_3} = \frac{mep}{p_1} \frac{1}{r_c^{\gamma}} \frac{1}{\dfrac{q}{c_v T_1 r_c^{\gamma - 1}} + 1}$$

Closing Thoughts on the Otto Cycle

- The Otto-cycle efficiency serves as an upper limit to the efficiency of SI engines.
- In practice this efficiency is never achieved.
- This theoretical analysis is flawed in that it ignores friction and heat transfer.

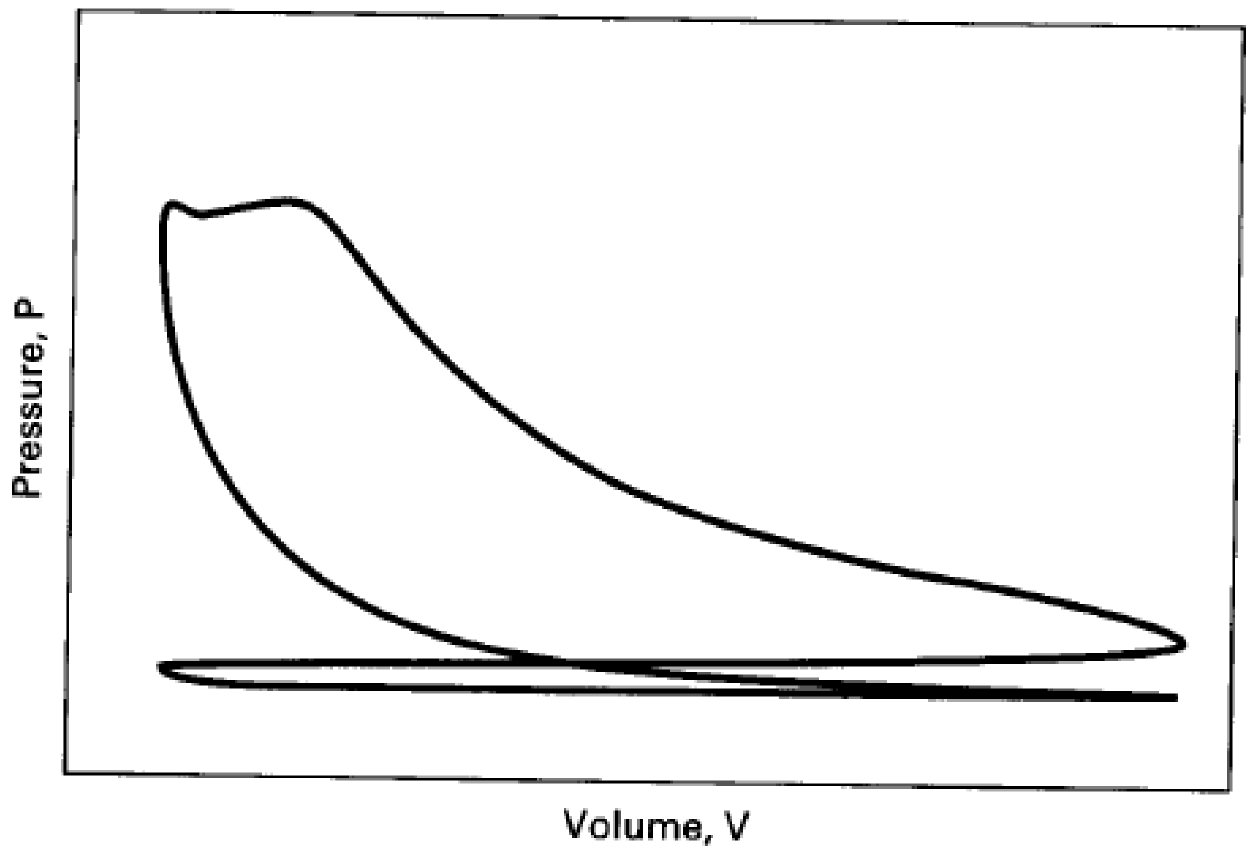
P-V Diagram of a SI Engine



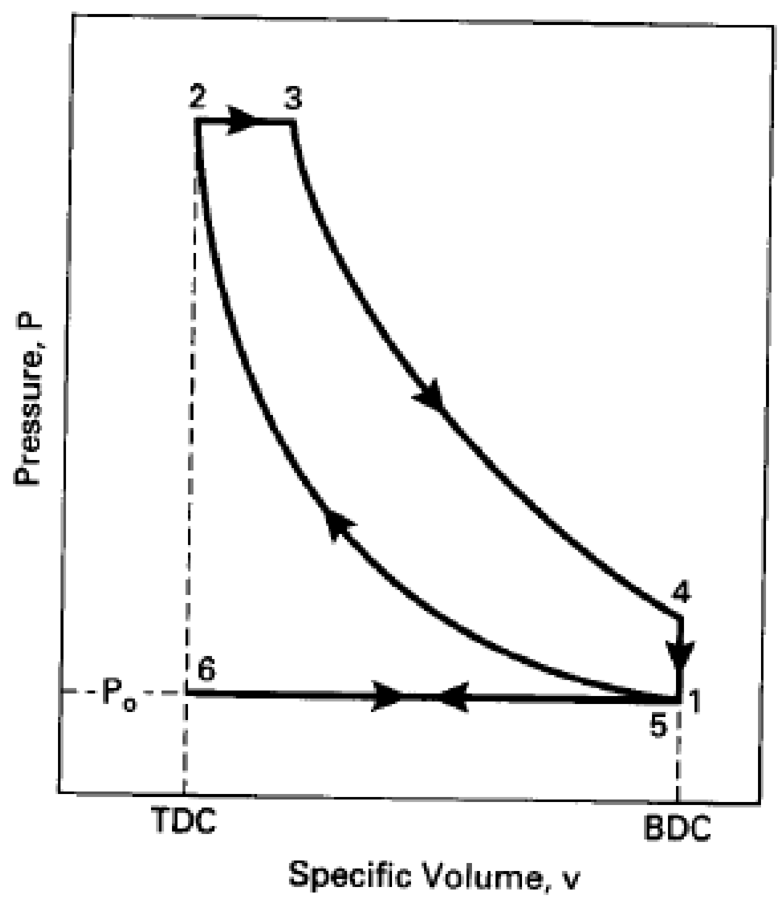
Diesel Cycle

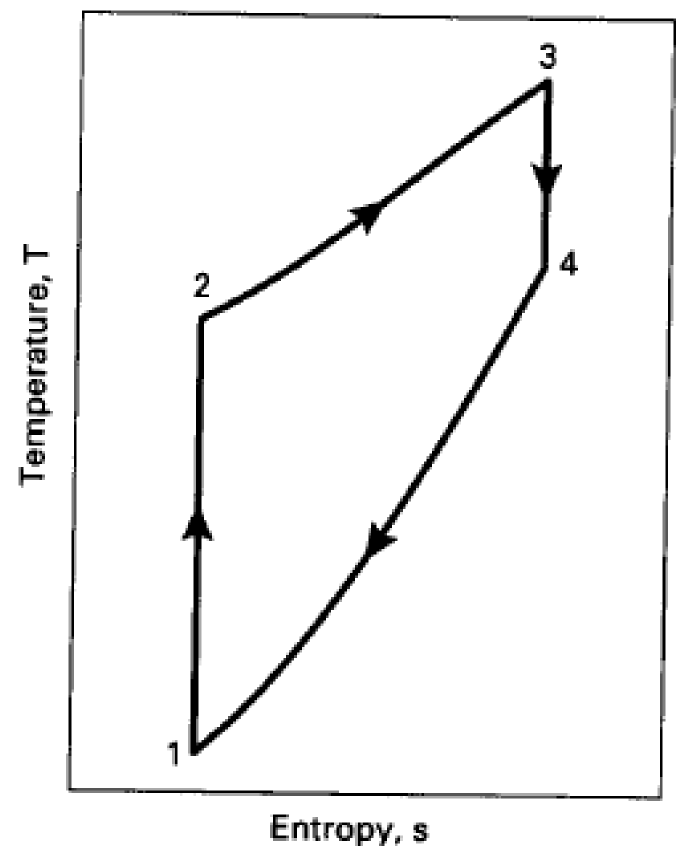
- Fuel is injected into the combustion chamber very late in the compression stroke
- Due to ignition delay and the finite time required to inject the fuel, combustion lasted into the expansion stroke
- This keeps the pressure at peak levels well past TDC.
- Combustion process is approximated as ***constant-pressure heat input*** in an air standard cycle

Indicator diagram of CI engine



Air-Standard diesel cycle





Thermodynamic Analysis of Air-Standard Diesel Cycle

- ***Process 6-1*** -Constant-pressure intake of air at Po.
- Intake valve open and exhaust valve closed:

$$\boxed{\begin{aligned} &P_1 = P_6 = P_o \\ &w_{6-1} = P_o\left(v_1 - v_6\right) \end{aligned}}$$

Process 1-2- Isentropic Compression Stroke

- All valves closed:

$$T_2 = T_1 \left(v_1 / v_2\right)^{k-1} = T_1 \left(V_1 / V_2\right)^{k-1} = T_1 \left(r_c\right)^{k-1}$$

$$P_2 = P_1 \quad / v_2)^{k} = P_1 \left(V_1 / V_2\right)^{k} = P_1 \left(r_c\right)^{k}$$

$$\left(v_1 \quad q_{1-2} =\right.$$

$$0$$

$$w_{1-2} = \frac{\left(P_2 v_2 - P_1 v_1\right)}{\left(1-k\right)} = \frac{R\left(T_2 - T_1\right)}{\left(1-k\right)}$$

$$= \left(u_1 - u_2\right) = C_V \left(T_1 - T_2\right)$$

Process 2-3- Constant – Pressure heat input (combustion).

- All valves closed:

$$Q_{2-3} = Q_{in} = m_f\, Q_{HV}\, \eta_C = m_m c_p \left(T_3 - T_2\right)$$
$$= \left(m_a + m_f\right) c_p \left(T_3 - T_2\right)$$
$$Q_{HV}\, \eta_C = \left(AF + 1\right) c_p \left(T_3 - T_2\right)$$
$$q_{2-3} = q_{in} = c_p \left(T_3 - T_2\right) = \left(h_3 - h_2\right)$$
$$w_{2-3} = q_{2-3} = \left(u_3 - u_2\right) = P_2 \left(v_3 - v_2\right)$$
$$T_3 = T_{\max}$$

Cut of ratio

Cut of Ratio (β): The change in volume that occurs during combustion, given as a ratio.

From ideal gas equation $P_2V_2/T_2 = P_3V_3/T_3$ $\Rightarrow$

$T_3/T_2 = V_3/V_2$, $P_3 = P_2$

$$\boxed{\beta = V_3/V_2 = v_3/v_2 = T_3/T_2}$$

Process 3-4: Isentropic power or expansion stroke.

- All valves closed:

$$q_{3-4} = 0$$

$$T_4 = T_3\left(v_3/v_4\right)^{k-1} = T_3\left(V_3/V_4\right)^{k-1}$$

$$P_4 = P_3\left(v_3/v_4\right)^{k} = P_3\left(V_3/V_4\right)^{k}$$

$$w_{3-4} = \frac{\left(P_4 v_4 - P_3 v_3\right)}{\left(1-k\right)} = \frac{R\left(T_4 - T_3\right)}{\left(1-k\right)}$$

$$= \left(u_3 - u_4\right) = c_v\left(T_3 - T_4\right)$$

Process 4-5: Constant-volume heat rejection (exhaust *blowdown)*

- Exhaust valve open and intake valve closed:

- $v_5 = v_4 = v_1 = v_{BDC}$ $w_{4-5} = 0$

$$Q_{4-5} = Q_{out} = m_m c_v \left(T_5 - T_4\right) = m_m c_v \left(T_1 - T_4\right)$$

$$q_{4-5} = q_{out} = c_v \left(T_5 - T_4\right) = \left(u_5 - u_4\right) = c_v \left(T_1 - T_4\right)$$

Process 5-6: Constant-pressure exhaust stroke at P_o

- Exhaust valve open and intake valve closed:

$$P_5 = P_6 = P_o$$

$$w_{5-6} = P_o\left(v_6 - v_5\right) = P_o\left(v_6 - v_1\right)$$

Thermal efficiency of diesel Cycle

$$\eta_{th,Diesel} = \frac{|w_{net}|}{|q_{in}|} = 1 - \frac{|q_{out}|}{|q_{in}|}$$

$$= 1 - \frac{c_v(T_4 - T_1)}{c_p(T_3 - T_2)}$$

$$= 1 - \frac{(T_4 - T_1)}{k(T_3 - T_2)}$$

$$= 1 - \frac{T_1(T_4 / T_1 - 1)}{T_2 k(T_3 / T_2 - 1)}$$

$$\frac{P_2V_2}{T_2} = \frac{P_3V_3}{T_3} \rightarrow \frac{T_3}{T_2} = \frac{V_3}{V_2} = \beta$$

From combined gas law

$$T_3 = T_2\beta$$

Thermal efficiency of diesel Cycle

From Isentropic compression

$$T_2 = T_1 (r_c)^{k-1}$$

Substituting isentropic expression into expression obtained from combined law we arrive at

$$T_3 = T_1 (r_c)^{k-1} \beta$$

From Isentropic Expansion

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{k-1} \rightarrow T_4 = T_3 \left(\frac{V_3}{V_2}\frac{V_2}{V_4}\right)^{k-1} \rightarrow T_4 = T_3 \left(\beta \frac{1}{r_c}\right)^{k-1}$$

$$T_4 = T_1 (r_c)^{k-1} \beta \left(\beta \frac{1}{r_c}\right)^{k-1}$$

$$T_4 = T_1 (\beta)^k$$

Thermal efficiency of diesel Cycle

- With rearrangement, this can be shown to equal:

$$(\eta_t)_{Diesel} = 1 - \left(\frac{1}{r_c}\right)^{k-1} \left[\frac{\beta^k - 1}{k(\beta - 1)}\right]$$

- Where:

$$\boxed{\begin{aligned} r_c &= Compression\ ratio \\ k &= C_P / C_V \\ \beta &= cutoff\ \ ratio \end{aligned}}$$

Note the term in the square bracket is always larger than unity so for the same compression ratio, r_c, the Diesel cycle has a *lower* thermal efficiency than the Otto cycle

Note: CI needs higher r_c compared to SI to ignite fuel

Thermal Efficiency - Diesel cycle

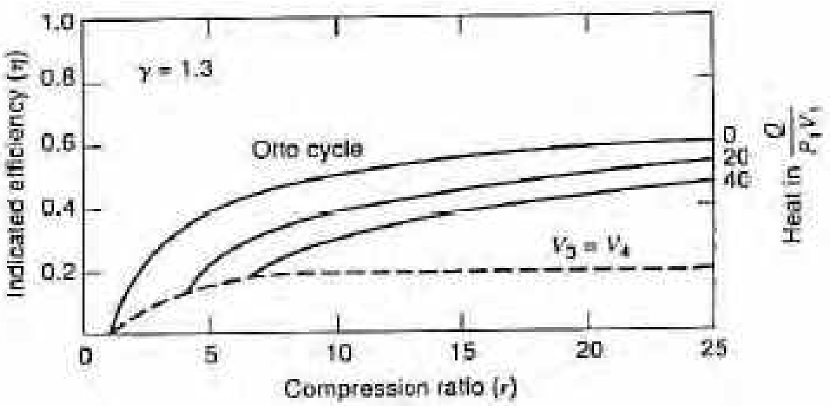
The cut-off ratio is not a natural choice of independent variable, more suitable parameter is the heat input, the two are related by:

$$\beta = 1 - \frac{k-1}{k}\left(\frac{Q_{in}}{P_1V_1}\right)\frac{1}{r_c^{\,k-1}}$$

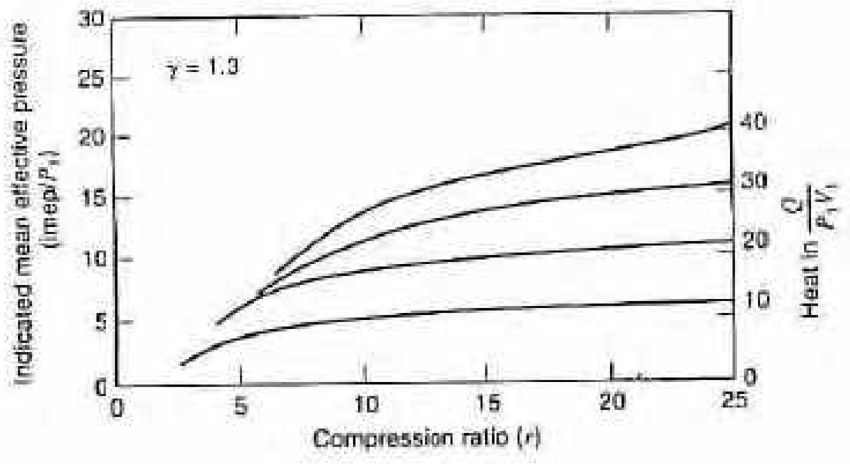
Substituting the cutoff ratio equation into the thermal efficiency equation for diesel cycle, it is possible to plot some curves

Thermal Efficiency - Diesel cycle

- Higher efficiency is obtained by adding less heat per cycle, Q_{in}



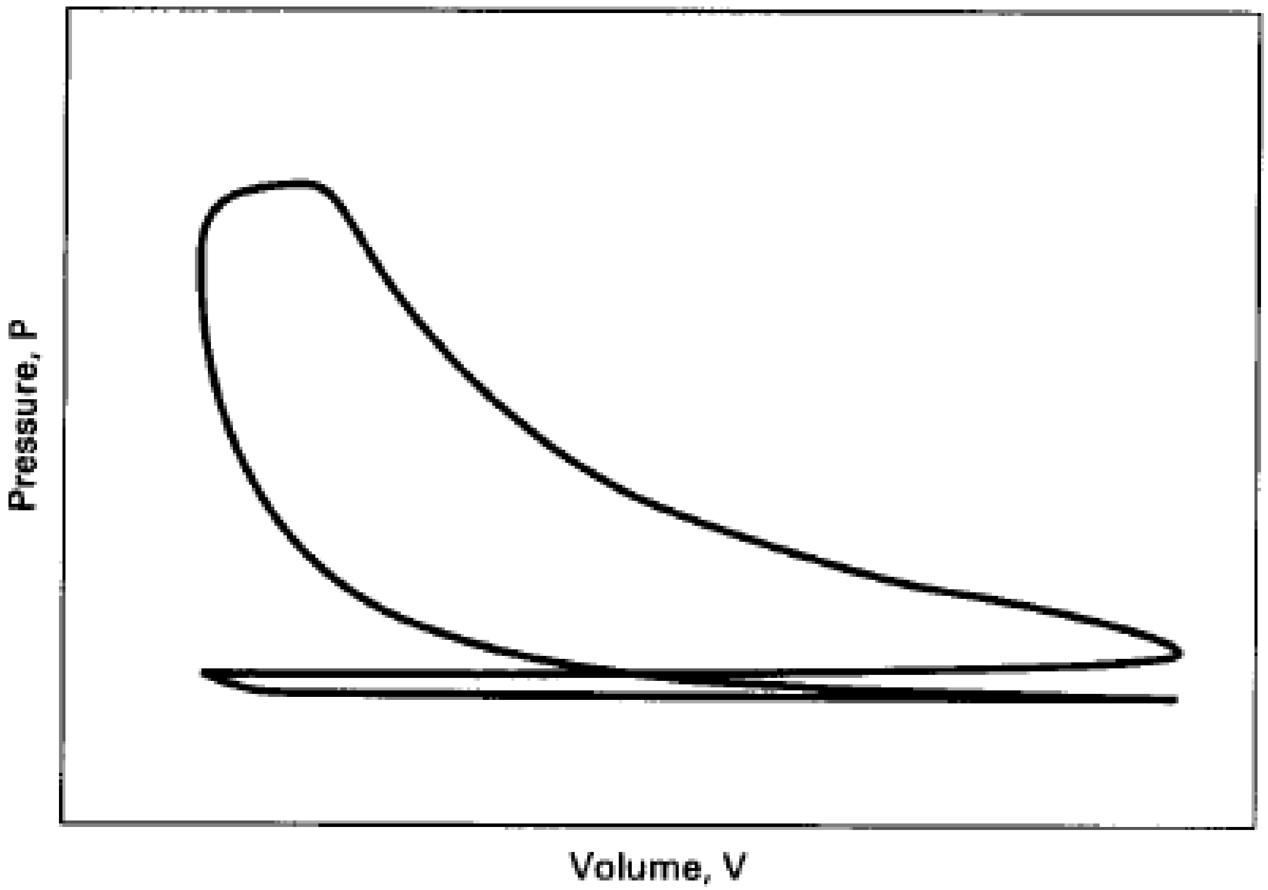
- In diesel engine the fuel cutoff ratio depends on the output maximum for maximum output



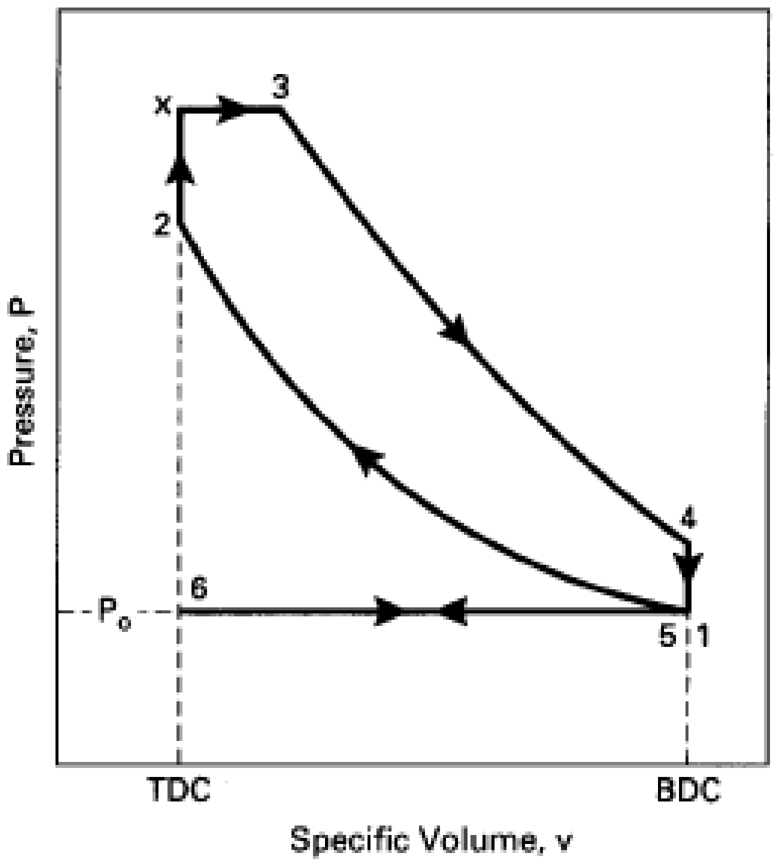
DUAL CYCLE

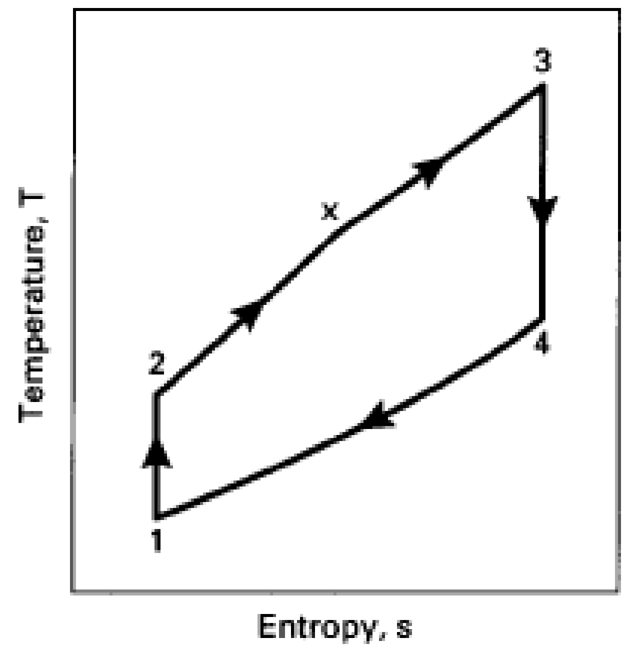
- Fuel is injected earlier in the compression stroke, around 20° bTDC
- Some of the combustion occurs almost at constant volume at TDC similar to Otto Cycle
- The fuel is being injected at TDC, and combustion of this fuel keeps the pressure high into the expansion stroke.
- The heat input process of combustion is approximated by a dual process of ***constant volume*** followed by ***constant pressure***

Indicator diagram of Dual cycleengine



Air Standard Dual cycle





Thermodynamic Analysis of Air-Standard Dual Cycle

- The analysis of an air-standard Dual cycle is the same as that of the Diesel cycle except for the heat input process (combustion) 2-x-3

- *Process 2-x: Constant-volume heat input (first part of combustion)*
- All valves closed:

$$V_x = V_2 = V_{TDC}$$

$$w_{2-x} = 0$$

$$Q_{2-x} = m_m c_v (T_x - T_2) = (m_a + m_f) c_v (T_x - T_2)$$

$$q_{2-x} = c_v (T_x - T_2) = (u_x - u_2)$$

$$P_x = P_{\max} = P_2 (T_X / T_2)$$

Thermodynamic Analysis of Air-Standard Dual Cycle

- Pressure ratio is defined as the rise in pressure during combustion, given as a ratio:

$$\alpha = P_X / P_2 = P_3 / P_2 = T_X / T_2 = (1/r_c)^K (P_3 / P_1)$$

$P_2/P_1 = (V_1/V_2)^k = (r_c)^k$

- *Process x-3- Constant-Pressure heat input (Second part of combustion)*
- All valves are closed:

$$P_3 = P_X = P_{\max}$$
$$w_{2-x} = 0$$
$$Q_{x-3} = m_m c_p (T_3 - T_x) = (m_a + m_f) c_p (T_3 - T_x)$$
$$q_{x-3} = c_p (T_3 - T_x) = (h_3 - h_x)$$
$$w_{x-3} = q_{x-3} - (u_3 - u_x) = P_x (v_3 - v_x) = P_3 (v_3 - v_x)$$
$$T_3 = T_{\max}$$

Thermal efficiency of Dual Cycle:

$$(\eta_t)_{DUAL} = \frac{|w_{net}|}{|q_{in}|} = 1 - \frac{|q_{out}|}{|q_{in}|}$$

$$= 1 - \frac{c_v(T_4 - T_1)}{[c_v(T_x - T_2) + c_p(T_3 - T_x)]}$$

$$= 1 - \frac{(T_4 - T_1)}{[(T_x - T_2) + k(T_3 - T_x)]}$$

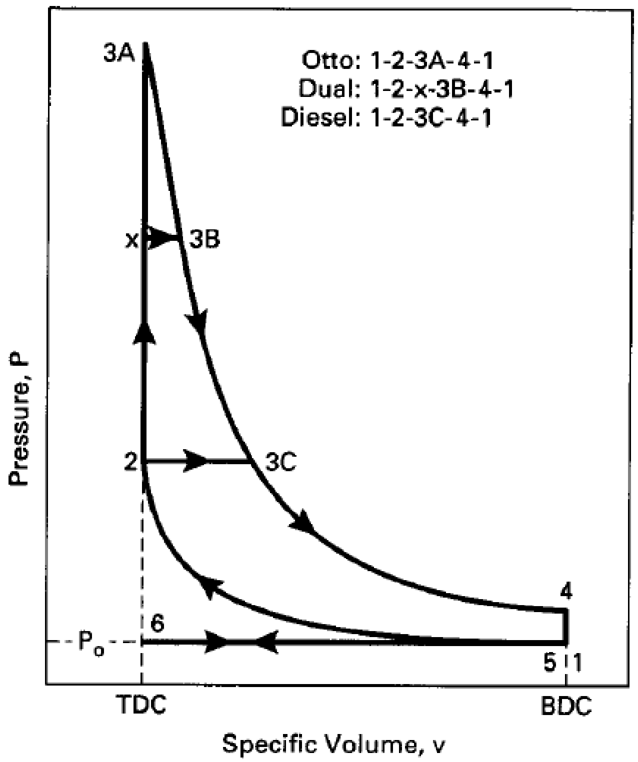
- This can rearranged to give:

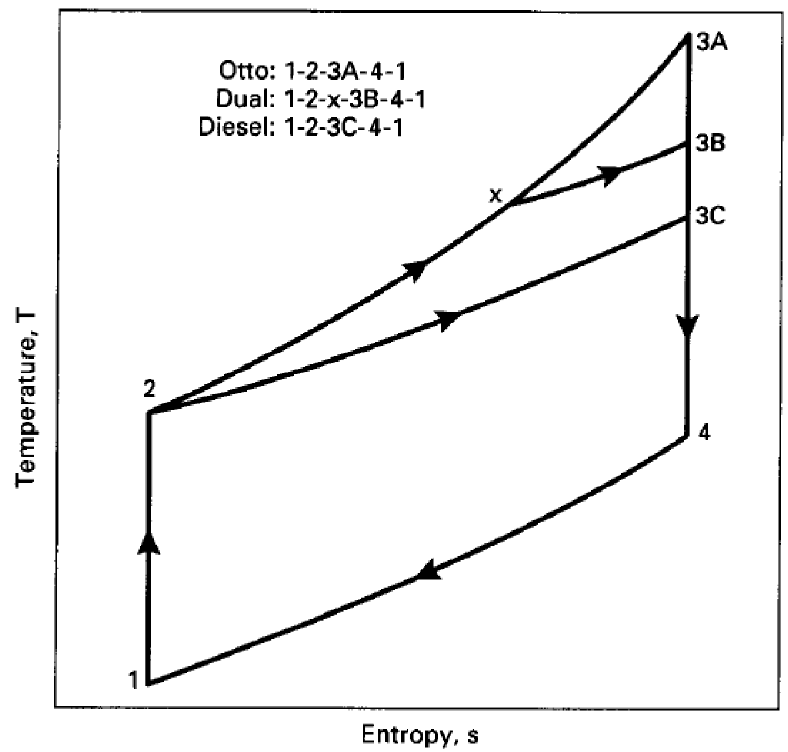
$$(\eta_t)_{DUAL} = 1 - \left(\frac{1}{r_c}\right)^{k-1} \left[\frac{\{\alpha\beta^k - 1\}}{\{k\alpha(\beta - 1) + \alpha - 1\}}\right]$$

Comparison of OTTO, DIESEL, and DUAL Cycles

- For the same inlet conditions, the same compression ratios and same heat removal:

$$(\eta_t)_{OTTO} > \boxed{(\eta_t)_{DUAL} > (\eta_t)_{DIESEL}}$$

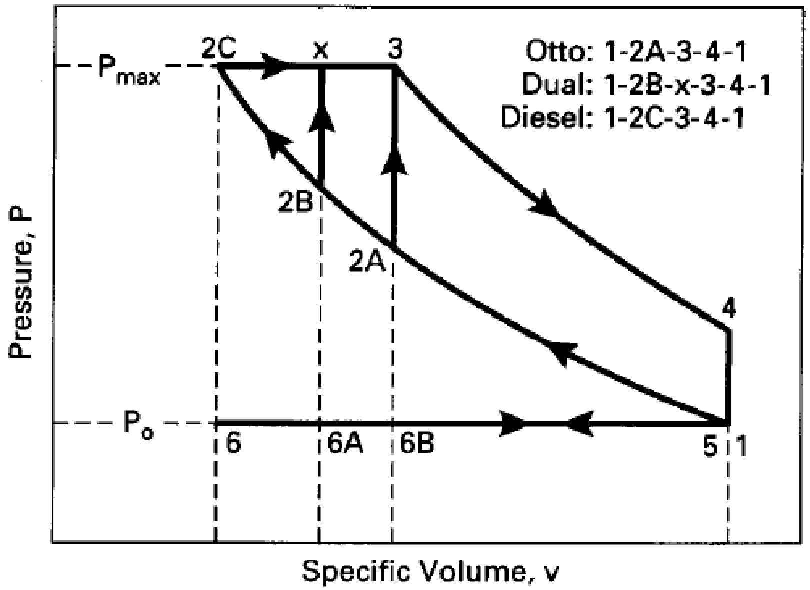


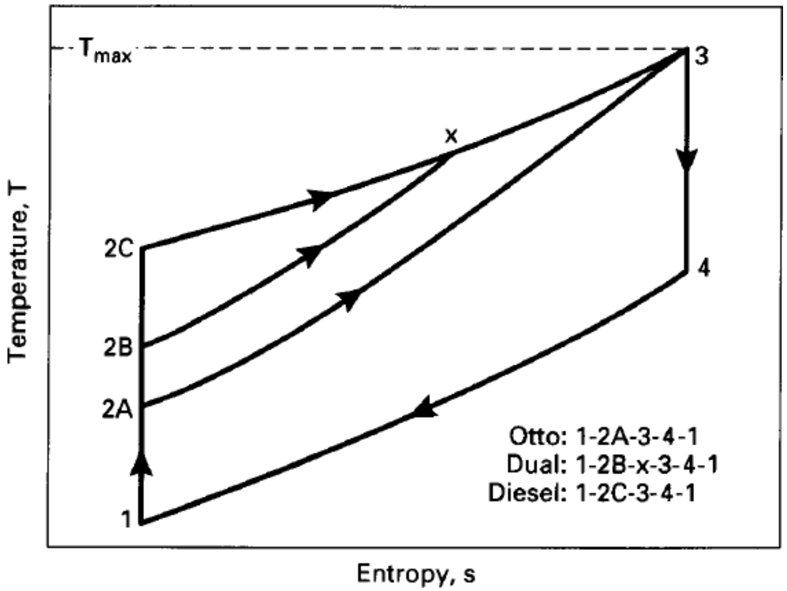


Comparison of OTTO, DIESEL, and DUAL Cycles

- For the same inlet conditions, the same peak pressure and same heat removal :

$$(\eta_t)_{DIESEL} > \boxed{(\eta_t)_{DUAL} > (\eta_t)_{OTTO}}$$

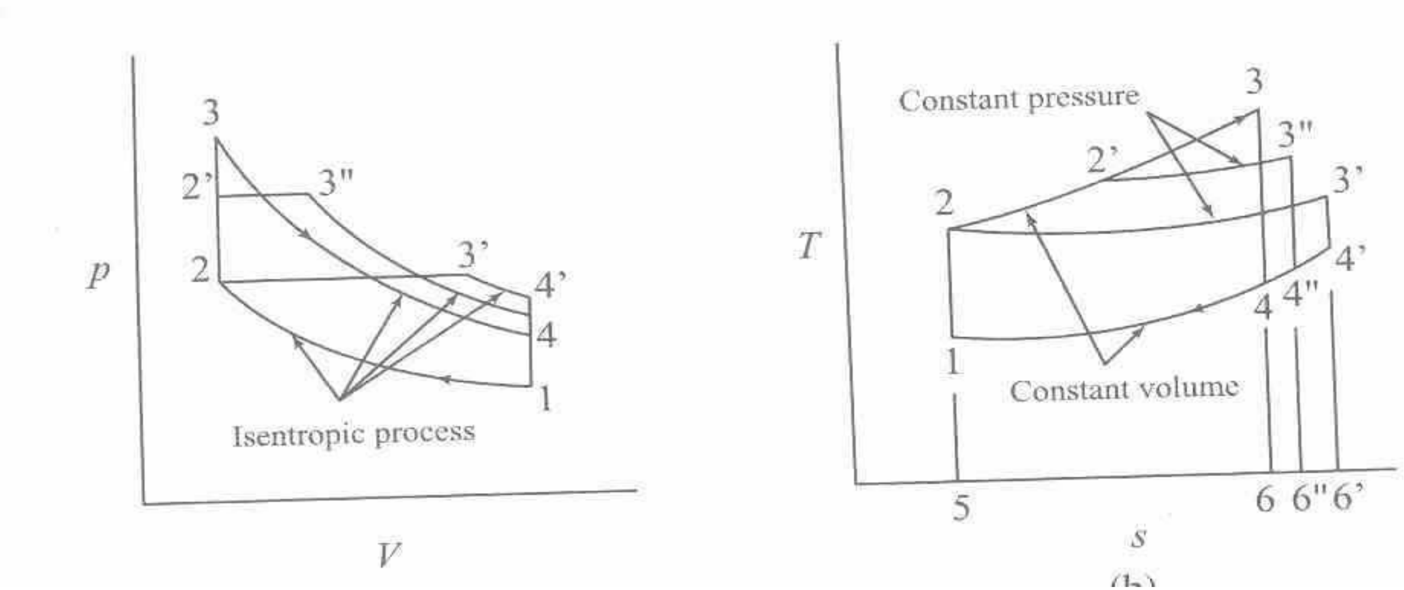




Comparison of OTTO,DIESEL, and DUAL Cycles

- For the same inlet conditions, the same compression ratios and same heat addition:

$$\boxed{\eta_{Otto} > \eta_{Dual} > \eta_{Diesel}}$$

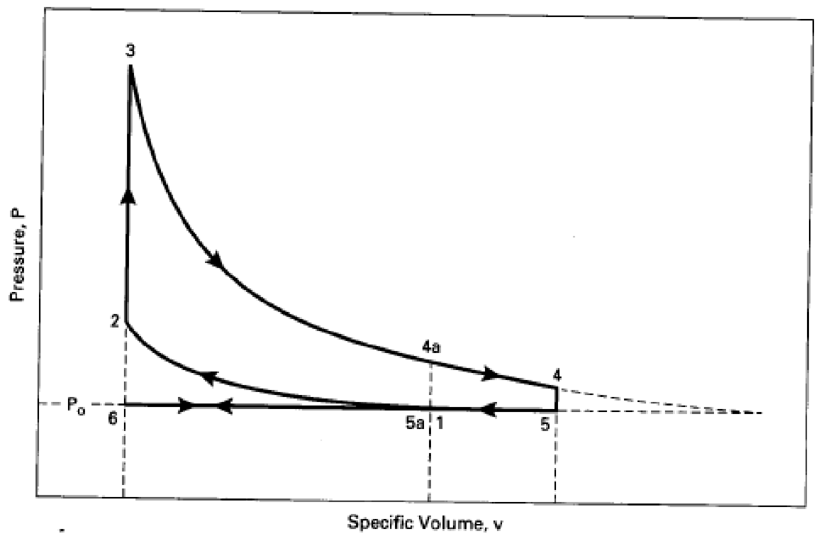


Atkinson Cycle

- Pressure in Otto and Diesel cycle is in order of three to five atmospheres when the exhaust valve opens

- If the exhaust valve is not opened until the gas in the cylinders allowed to expand down to atmospheric pressure a greater amount of work can be obtained in the expansion stroke

Air Standard Atkinson Cycle



- Such an air standard cycle is called an Atkinson Cycle or Over expanded Cycle
- Mechanical Linkage of some kind is needed to achieve this cycle

Miller Cycle

- Introduced by Ralph Miller in the 1940s
- It is a modified Otto Cycle that improves fuel efficiency by 10%-20%.
- It relies on a supercharger/turbocharger, and takes advantage of the supercharger's greater efficiency at low compression levels.
- This is applicable for the late closing cycle
- It uses unique time valve timing

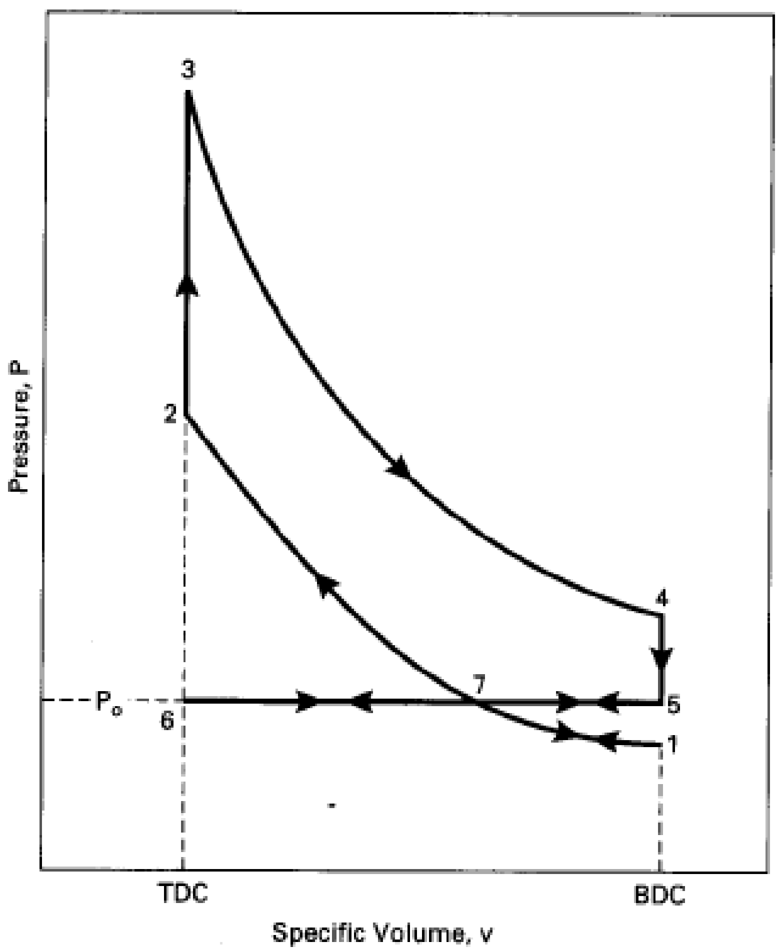
Miller Cycle

- Air intake in a Miller cycle is unthrottled

- Closing the intake valve at the proper time long before BDC controls the amount of air ingested into each cylinder

- Has an expansion ratio greater than the compression ratio.

Air standard Miller Cycle



- Cylinder pressure is reduced along process 7-1
- When the piston reaches BDC and starts back towards TDC

cylinder pressure increases during process 1-7

- Air-standard Miller cycle for unthrottled naturally aspirated four stroke cycle SI engine.

Air standard Miller Cycle

- If the engine has early intake valve closing, the cycle will be 6-7-1-7-2-3-4-5-7-6

- If the engine has late intake valve closing, the cycle will be 6-7- 5-7-2-3-4-5-7-6

- The work produced in the first part of the intake process 6-7 is canceled by part of the exhaust stroke 7-6, process 7-1 is canceled by process 1-7.

Air standard Miller Cycle

- The net indicated work is then the area within loop 7-2-3-4-5-7 so that there is essentially no pump work resulting in higher thermal efficiency

- The compression ratio is:

$$r_c = \frac{V_7}{V_2}$$

- The larger expansion ratio is:

$$r_e = \frac{V_4}{V_2} = \frac{V_4}{V_3}$$

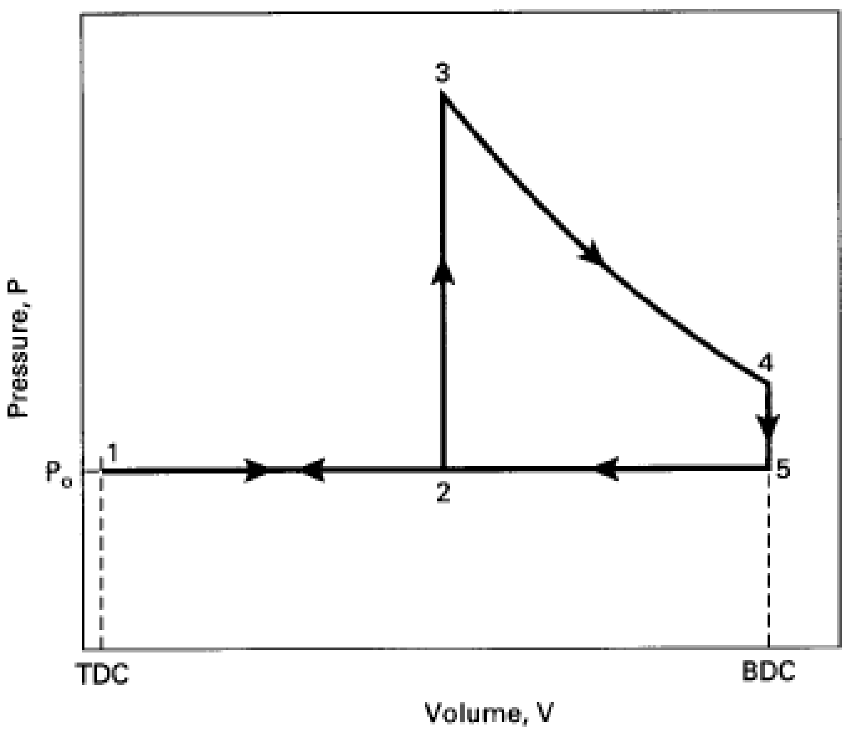
Lenoir Cycle

- The Lenoir cycle is approximated by the air-standard cycle with two strokes

- The first half of the first stroke was intake with air-fuel entering the cylinder at atmospheric pressure (process 1- 2)

- Halfway through the first stroke the intake valve was closed and the air-fuel mixture was ignited without any compression

Air standard approximationfor Lenoir Engine Cycle



There was essentially no clearance volume !

Air standard approximation for Lenoir Engine Cycle

- Combustion raised the temperature and pressure in the cylinder almost at constant volume (process 2-3)

- The second half of the first stroke then became the power or expansion process 3-4.

- Near BDC, the exhaust valve opened and blowdown occurred (4-5).

- This is followed by the exhaust stroke 5-1, completing the two stroke cycle.

Thermodynamic Analysis of Air-Standard Lenoir Cycle

- ***Process 2-3: Constant-volume heat input (combustion)***
- All valves closed:

$$P_2 = P_1 = P_0$$

$$v_3 = v_2$$

$$w_{2-3} = 0$$

$$q_{2-3} = q_{in} = c_v\left(T_3 - T_2\right) = \left(u_3 - u_2\right)$$

Process 3-4: Isentropic power or expansion stroke.

- All valves closed:

$$q_{3-4} = 0$$

$$T_4 = T_3 \left(\frac{v_3}{v_4} \right)^{k-1}$$

$$P4 = P_3 \left(\frac{v_3}{v_4} \right)^{k}$$

$$w_{3-4} = \frac{(P_4 v_4 - P_3 v_3)}{(1-k)} = \frac{R(T_4 - T_3)}{(1-k)}$$

$$= c_v (T_3 - T_4) = (u_3 - u_4)$$

Process 4-5: Constant-volume heat rejection (exhaust *blowdown)*

- Exhaust valve open and intake valve closed

$$v_5 = v_4 = v_{BDC}$$
$$w_{4-5} = 0$$
$$q_{4-5} = q_{out} = c_v\left(T_5 - T_4\right) = \left(u_5 - u_4\right)$$

Process 4-5: Constant-volume heat rejection (exhaust blowdown)

$$P_5 = P_2 = P_1 = P_0$$
$$w_{5-2} = P_0\left(v_2 - v_5\right)$$
$$q_{5-2} = q_{out} = c_p\left(T_2 - T_5\right) = \left(h_2 - h_5\right)$$

9

Air Compressors

Air Compressors

COMPRESSOR – *A device which takes a definite quantity of fluid (usually gas, and most often air) and deliver it at a required pressure.*

Air Compressor – 1) Takes in atmospheric air,

2) Compresses it, and

3) Delivers it to a storage vessel (i.e. Reservoir).

Compression requires Work to be done <u>on the gas</u>,

$\Rightarrow$ Compressor must be driven by some sort of Prime Mover (i.e. Engine)

How they are different from pumps?

- Major difference is that compressors handles the gases and pumps handles the liquids.

- As gases are compressible, the compressor also reduces the volume of gas.

- Liquids are relatively incompressible; while some can be compressed

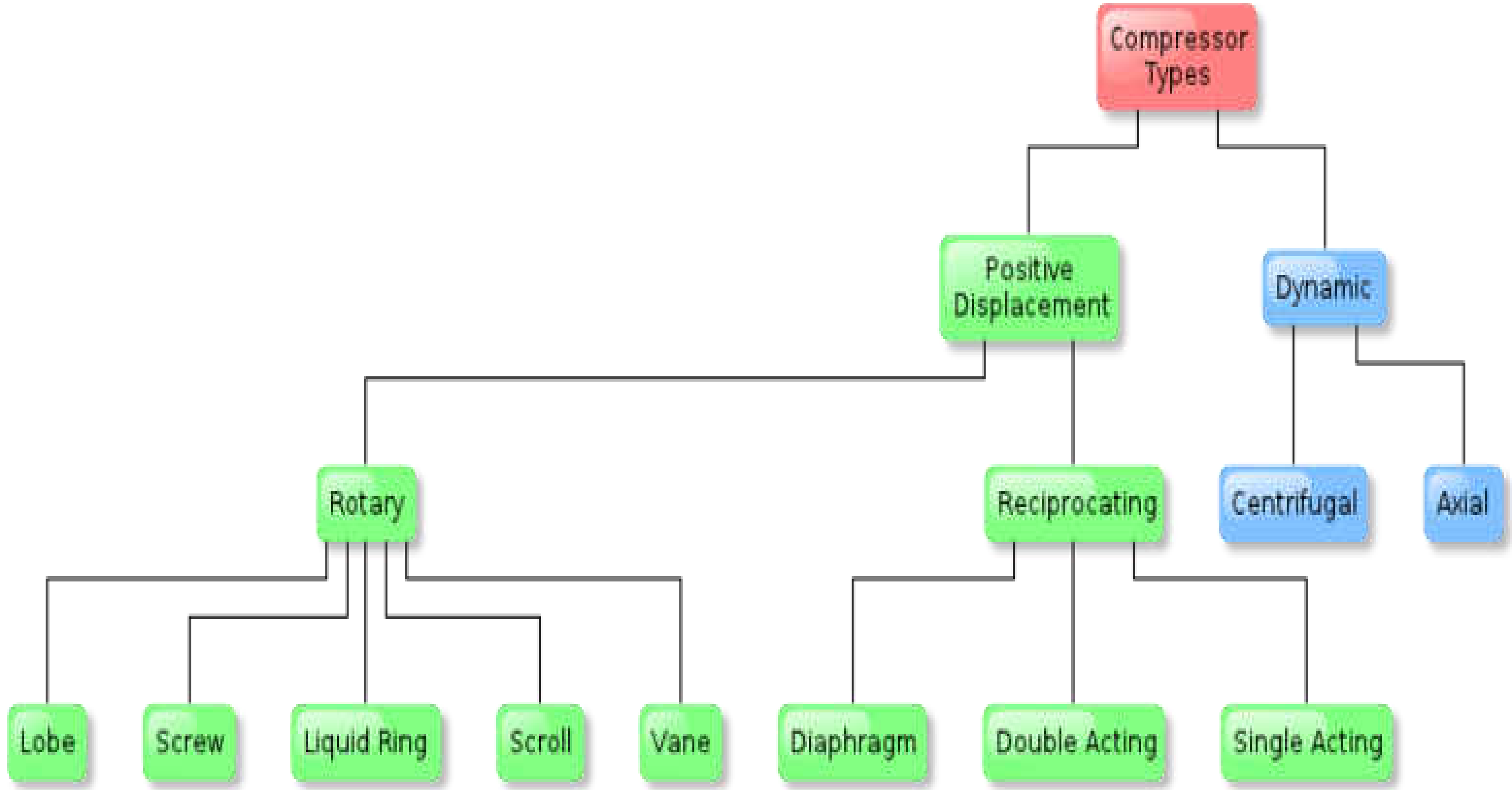
Applications

Compressors have many everyday uses, such as in :

- Air conditioners, (car, home)
- Pneumatic devices
- Home and industrial refrigeration
- Hydraulic compressors for industrial machines
- Air compressors for industrial manufacturing

Classification of Compressor

Compressor classification can be described by following flow chart:



Dynamic Compressors

The dynamic compressor is continuous flow compressor is characterized by rotating impeller to add velocity and thus pressure to fluid.

It is widely used in chemical and petroleum refinery industry for specific services.

There are two types of dynamic co

- Centrifugal Compressor
- Axial Flow Compressor

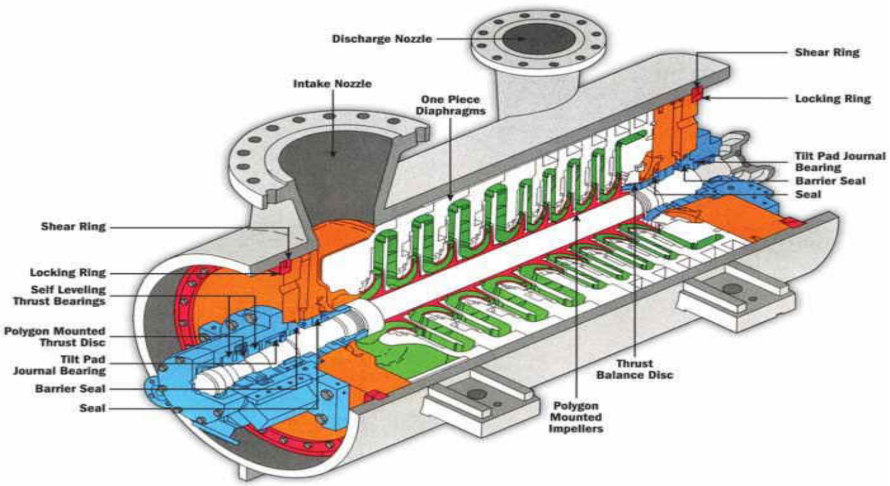


Figure 1. Major Components of Multistage Barrel-type Centrifugal Compressors (Dresser-Rand Co., Olean, NY)

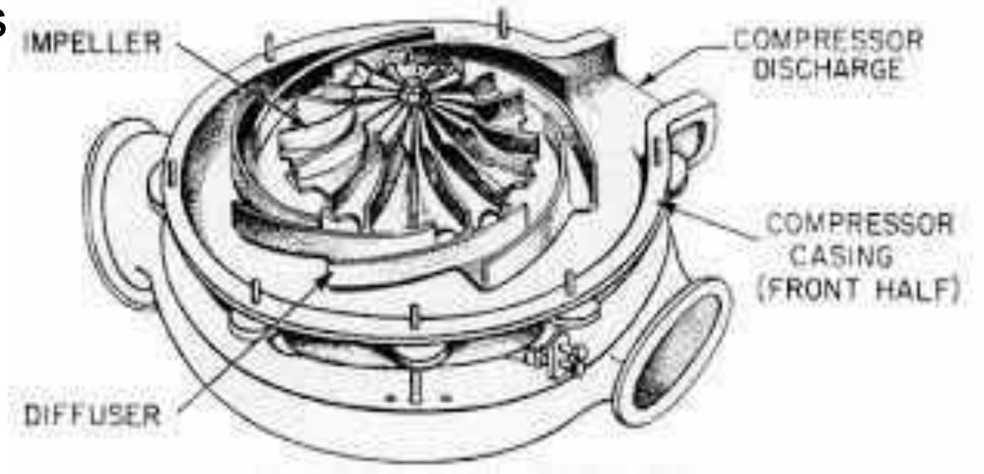
Centrifugal Compressors

•Achieves compression by applying inertial forces to the gas by means of rotating impellers.

•It is multiple stage ; each stage consists of an impeller as the rotating element and the stationary element, i.e. diffuser

• Fluid flow enters the impeller axially and discharged radially

• The gas next flows through a circular chamber (diffuser), where it loses velocity and increases press



Axial Flow Compressor

- Working fluid principally flows parallel to the axis of rotation.
- The energy level of air or gas flowing through it is increased by the action of the rotor blades which exert a torque on the fluid
- Have the benefits of high efficiency and large mass flow rate
- Require several rows of airfoils to achieve large pressure rises making them complex and expensive

Axial Flow Compressor

Positive displacement Compressor

Positive displacement compressors causes movement by trapping a fixed amount of air then forcing (displacing) that trapped volume into the discharge pipe.

It can be further classified according to the mechanism used to move air.

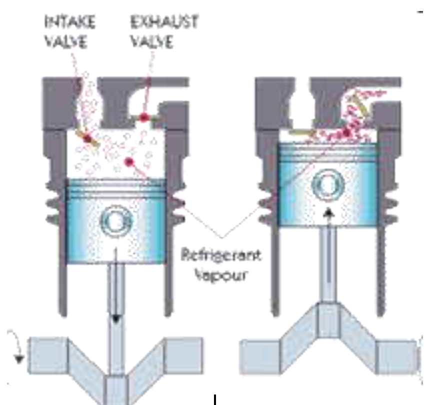
- Rotary Compressor
- Reciprocating compressor

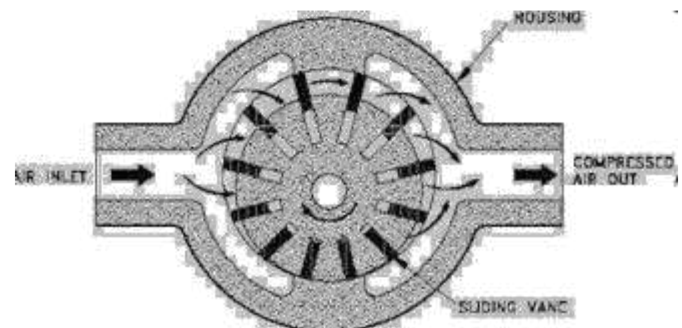
Positive displacement Compressor

Air Compressors

Reciprocating

Rotary





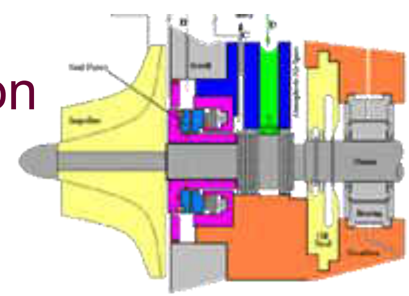
Centrifugal

No. of Stages for Compression

Single – stage
Multi - stage

No. of Sides of Piston in operation

Single – acting
Double - Acting



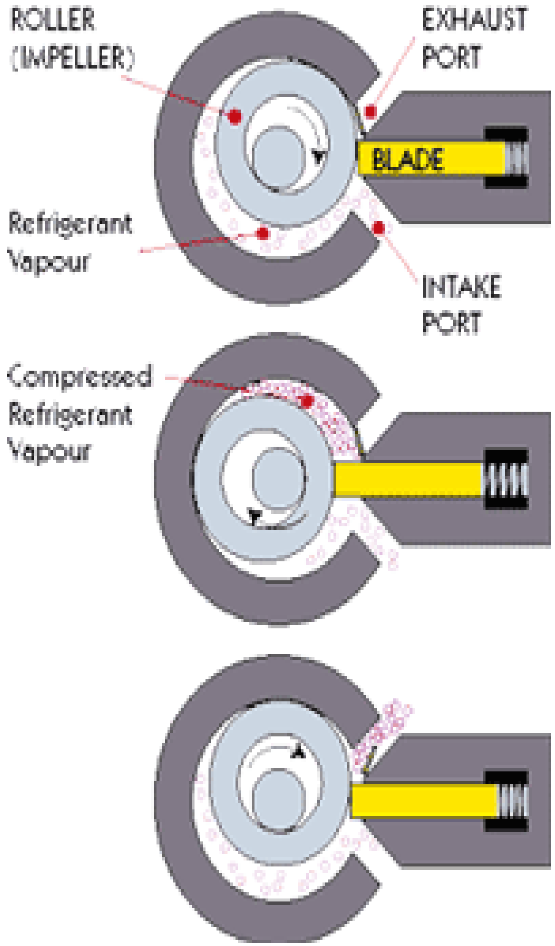
Rotary Compressor

•The gas is compressed by the rotating action of a roller inside a cylinder.

•The roller rotates off-centre around a shaft so that part of the roller is always in contact with the cylinder.

• Volume of the gas occupies is reduced and the refrigerant is compressed.

•High efficient as sucking and compressing refrigerant occur simultaneously.



Reciprocating Compressor

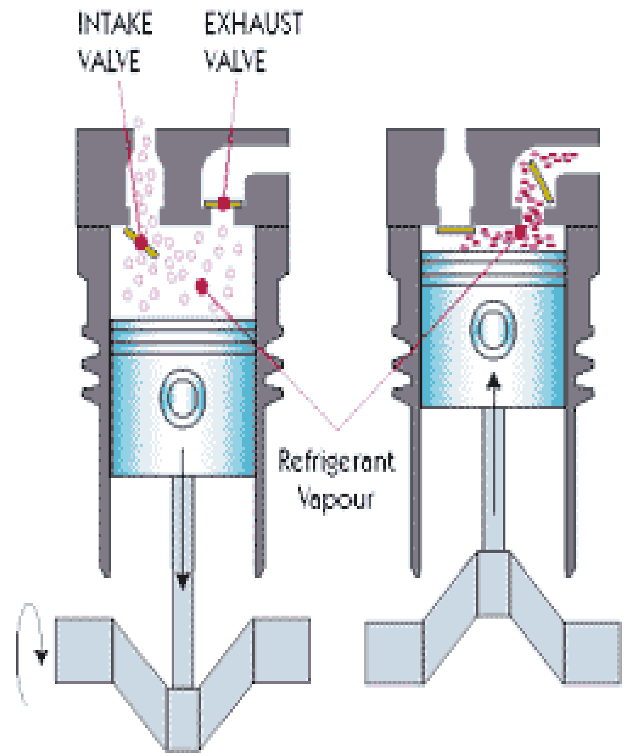
It is a positive-displacement compressor that

- Uses pistons driven by a crankshaft to deliver gases at high pressure.

•The intake gas enters the suction manifold, then flows into the compression cylinder

•It gets compressed by a piston driven in a reciprocating motion via a crankshaft,

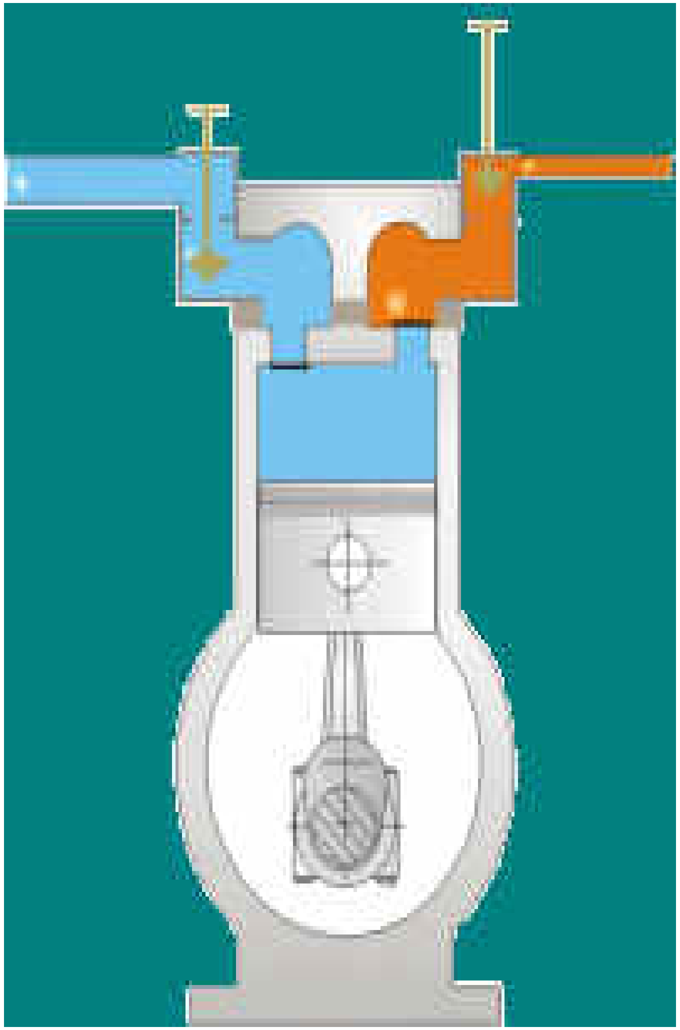
•Discharged at higher pressure



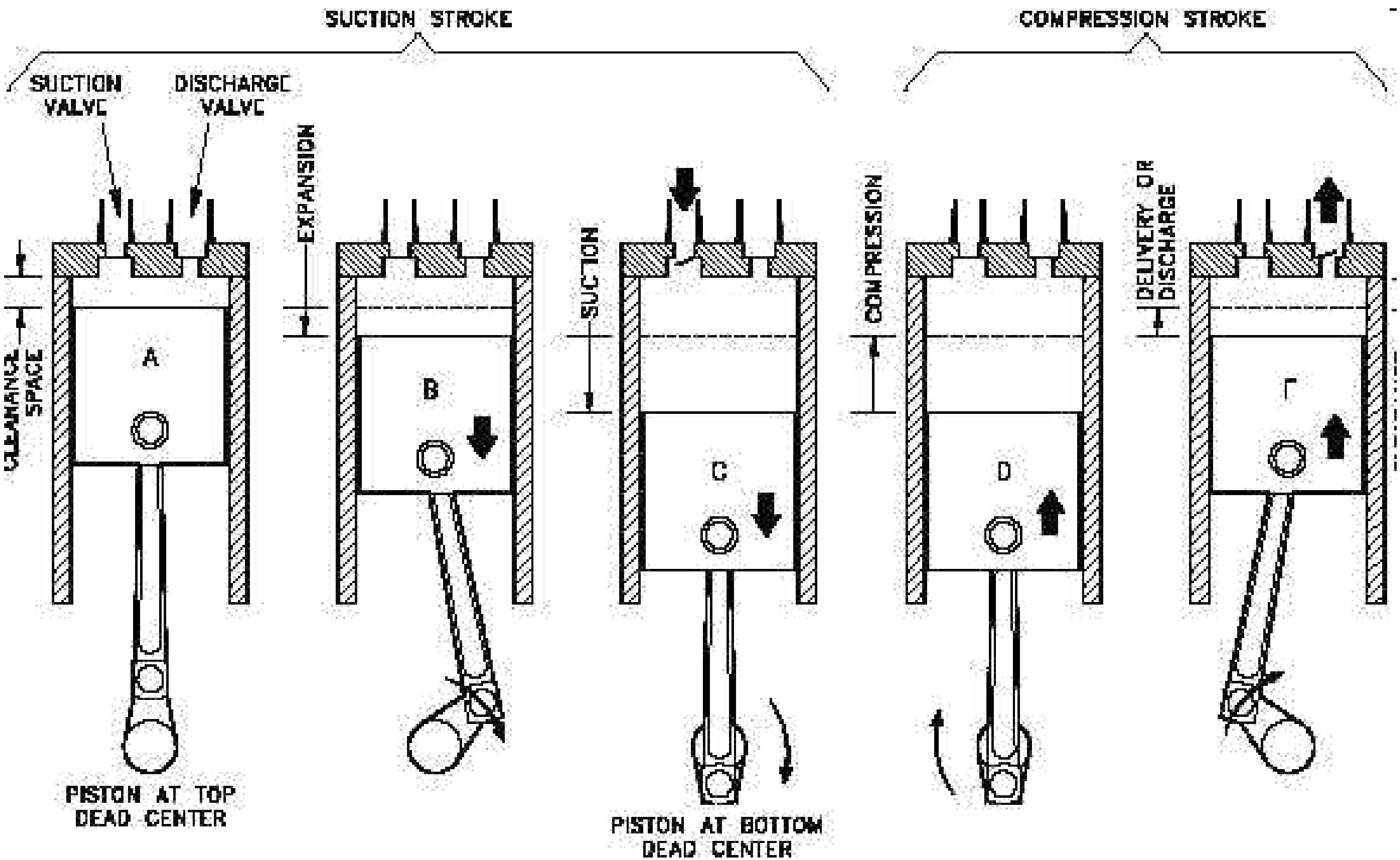
Reciprocating Compressor - Detailed Analysis

Principle of Operation

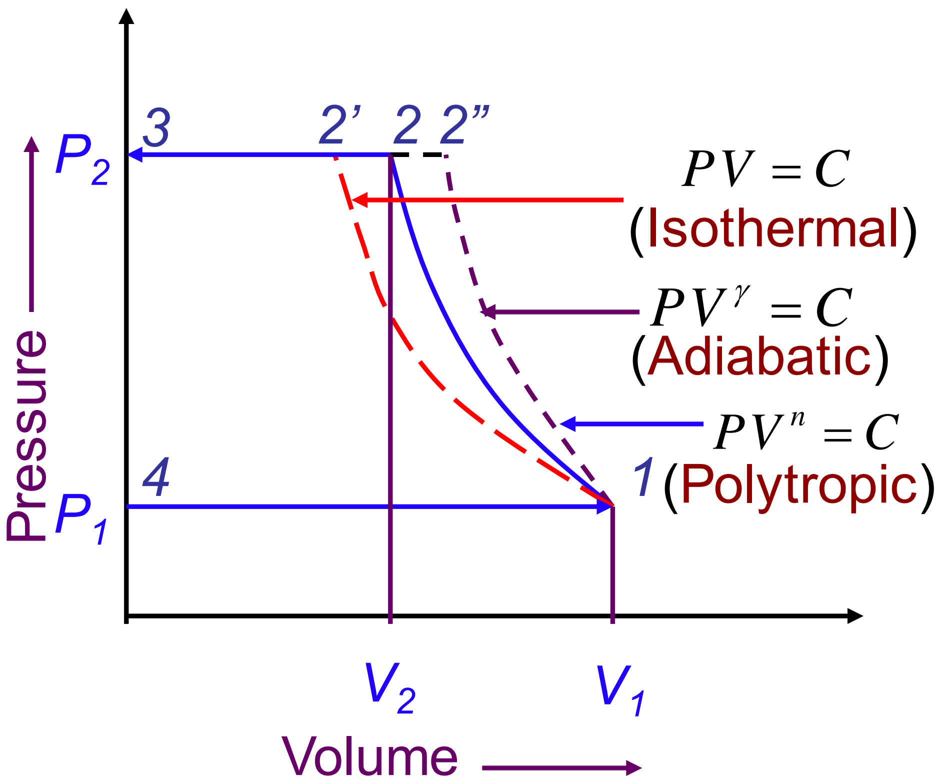
- Fig. shows single-acting piston actions in the cylinder of a reciprocating compressor.
- The piston is driven by a crank shaft via a connecting rod.
- At the top of the cylinder are a suction valve and a discharge valve.
- A reciprocating compressor usually has two, three, four, or six cylinders in it.



Reciprocating Compressor - Working



Reciprocating Compressor – Equation for Work



<u>*Operations*</u> *:* 4 – 1 : Volume V_1 of air aspirated into Compressor, at P_1 and T_1.

1 – 2 : Air compressed according to $PV^n = Const$. from P_1 to P_2.

$\longrightarrow$ Temp increase from T_1 to T_2.

2 – 3 : Compressed air at P_2 and V_2 with temperature T_2 is delivered.

Reciprocating Compressor – Equation for Work

During Compression, due to the excess temperature above surrounding, the air will exchange the heat to the surrounding.

$\Rightarrow$ Compression Index, n is always less than γ, the adiabatic index.

As Compressor is a work consuming device, every effort is desired to reduce t

Work done = Area under P-V curve

$\Rightarrow$ 1 – 2" : Adiabatic Compression = Max. Work.

$\Rightarrow$ 1 – 2 : Polytropic Compression

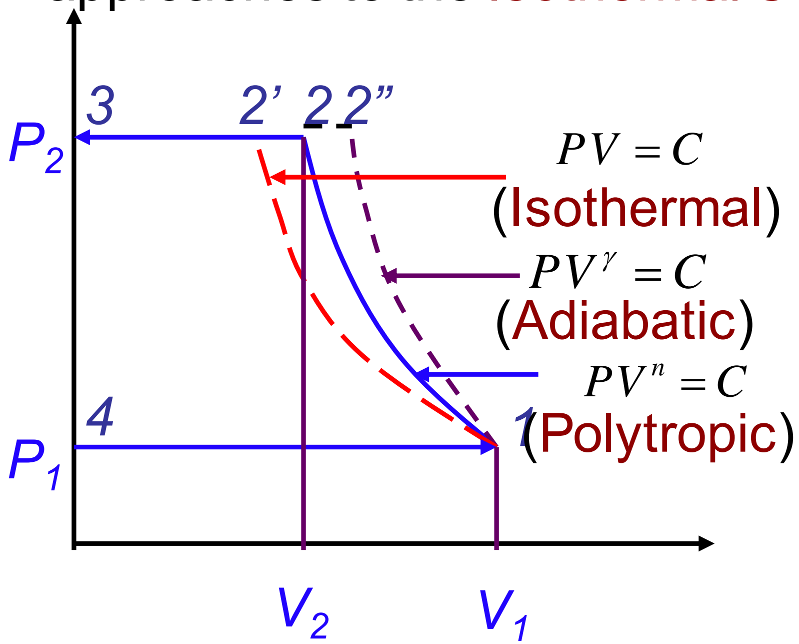
$\Rightarrow$ 1 – 2' : Isothermal Compression = Min. Work.

Reciprocating Compressor – Equation for Work

Thus, comparison between the Isothermal Work and the Actual Work is important.

$$\text{Isothermal Efficiency, } \eta_{iso} = \frac{\text{Isothermal Work}}{\text{Actual Work}}$$

Thus, more the Isothermal Efficiency, more the actual compression approaches to the Isothermal Compression.



Actual Work = W_{act} = Area 4-1-2-3-4

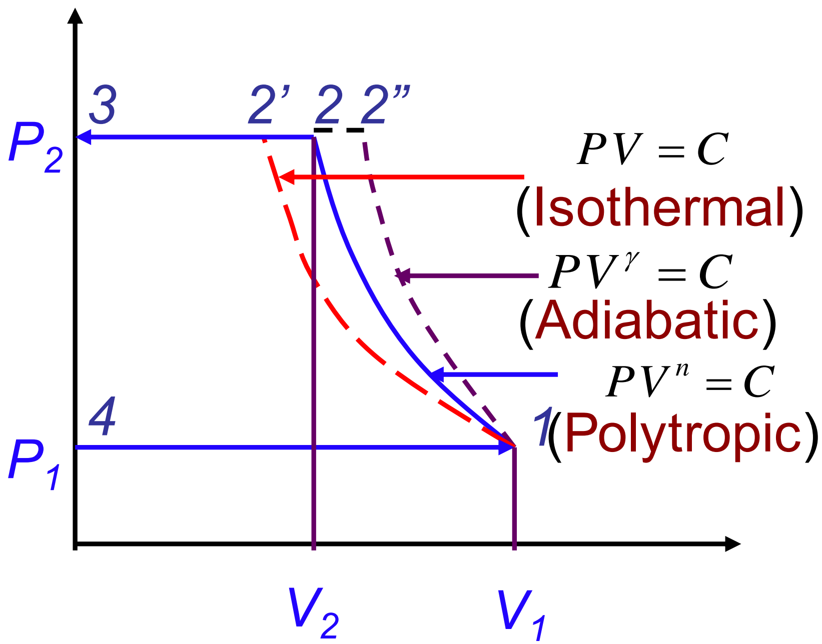
W_{act} = Area (4-1) – Area (1-2) – Area (2-3)

$$= P_1V_1 - \frac{P_2V_2 - P_1V_1}{n-1} - P_2V_2$$

$$= \left(P_1V_1 - P_2V_2\right) - \left(\frac{P_2V_2 - P_1V_1}{n-1}\right)$$

$$= \left(P_1V_1 - P_2V_2\right) + \left(\frac{P_1V_1 - P_2V_2}{n-1}\right)$$

Reciprocating Compressor – Equation for Work



$$W_{iso} = \left(1 + \frac{1}{n-1}\right)\left(P_1V_1 - P_2V_2\right)$$

$$= \left(\frac{n}{n-1}\right)\left(P_1V_1 - P_2V_2\right)$$

$$= \left(\frac{n}{n-1}\right) P_1V_1 \left(1 - \frac{P_2V_2}{P_1V_1}\right)$$

Now, $P_1V_1^{\,n} = P_2V_2^{\,n}$

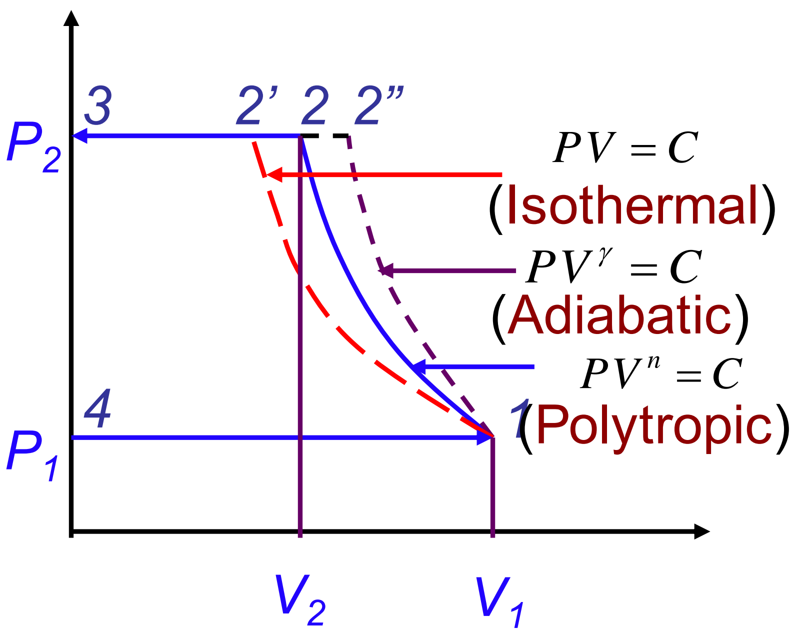
$$\Rightarrow \quad \frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{1/n}$$

$$W_{iso} = \left(\frac{n}{n-1}\right) P_1V_1 \left\{1 - \frac{P_2}{P_1}\left(\frac{P_1}{P_2}\right)^{1/n}\right\}$$

Reciprocating Compressor – Equation for Work

$$W_{iso} = \left(\frac{n}{n-1}\right) P_1V_1 \left\{1 - \frac{P_2}{P_1}\left(\frac{P_1}{P_2}\right)^{1/n}\right\}$$

$$= \left(\frac{n}{n-1}\right) P_1V_1 \left\{1 - \frac{P_2}{P_1}\left(\frac{P_2}{P_1}\right)^{-1/n}\right\}$$



$$W_{iso} = \left(\frac{n}{n-1}\right) P_1V_1 \left\{1 - \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right\}$$

$$W_{iso} = \left(\frac{n}{n-1}\right) mRT_1 \left\{1 - \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right\}$$

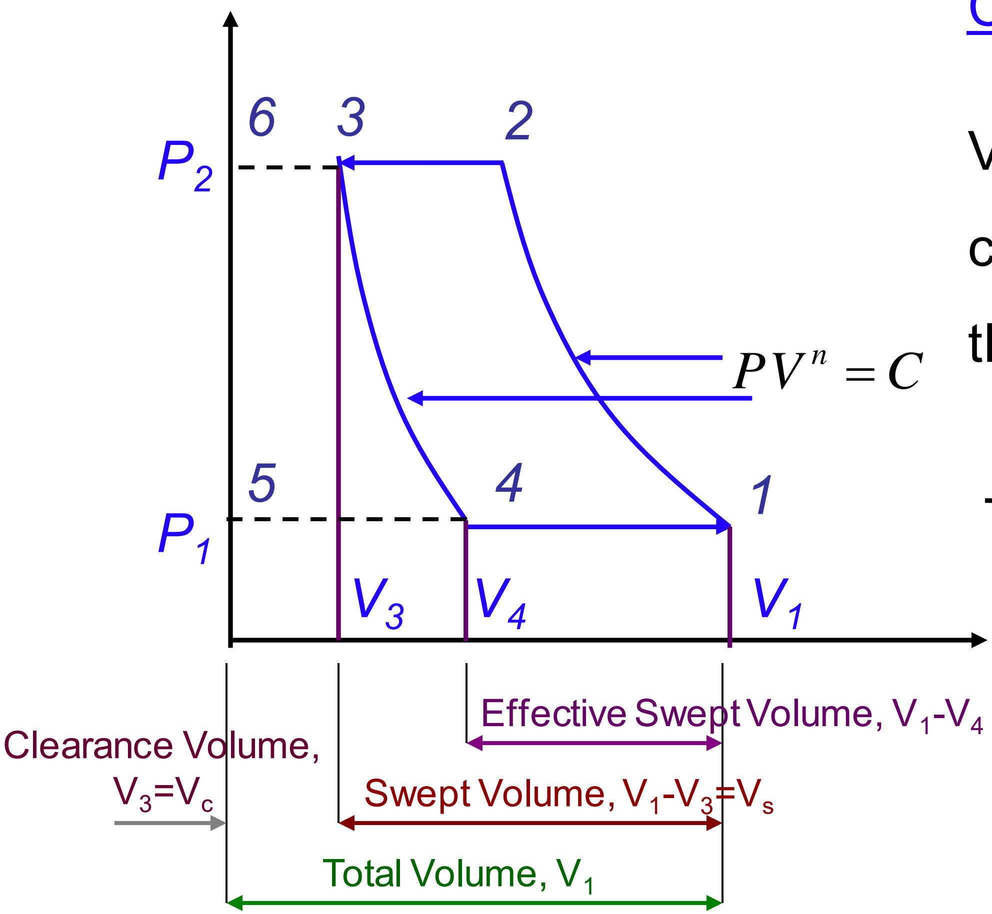
Delivery Temperature,

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

The solution of this equation is always *negative*

This shows that Work is done *ON* the Compres

Reciprocating Compressor – Equation for Work



Clearance Volume :

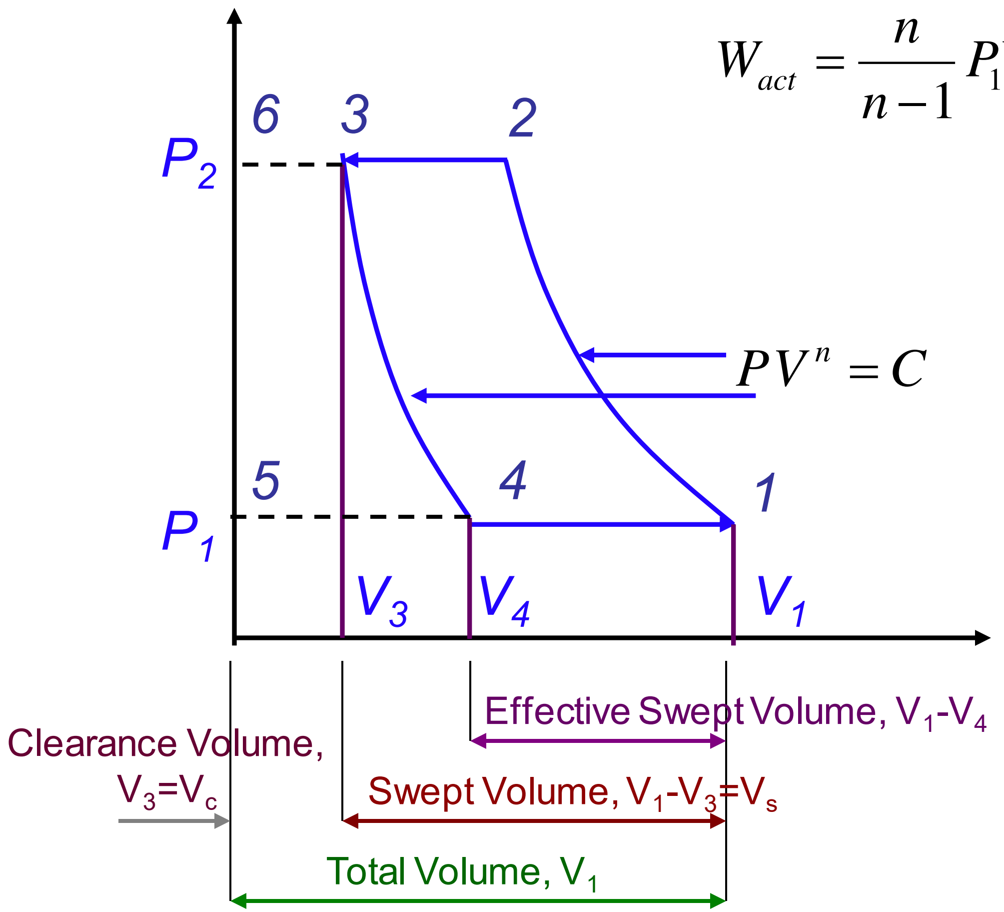
Volume that remains inside the cylinder after the piston reaches the end of its inward stroke.

Thus, *Effective Stroke Volume* = $V_1 - V$

Actual Work = W_{act} = Area 1-2-3-4

W_{act} = Area (5-1-2-6) – Area (5-4-3-6)

Reciprocating Compressor – Equation for Work



$$W_{act} = \frac{n}{n-1} P_1 V_1 \left\{ 1 - \left(\frac{P_2}{P_1} \right)^{\frac{m-1}{n}} \right\} - \frac{n}{n-1} P_4 V_4 \left\{ 1 - \left(\frac{P_3}{P_4} \right)^{\frac{m-1}{n}} \right\}$$

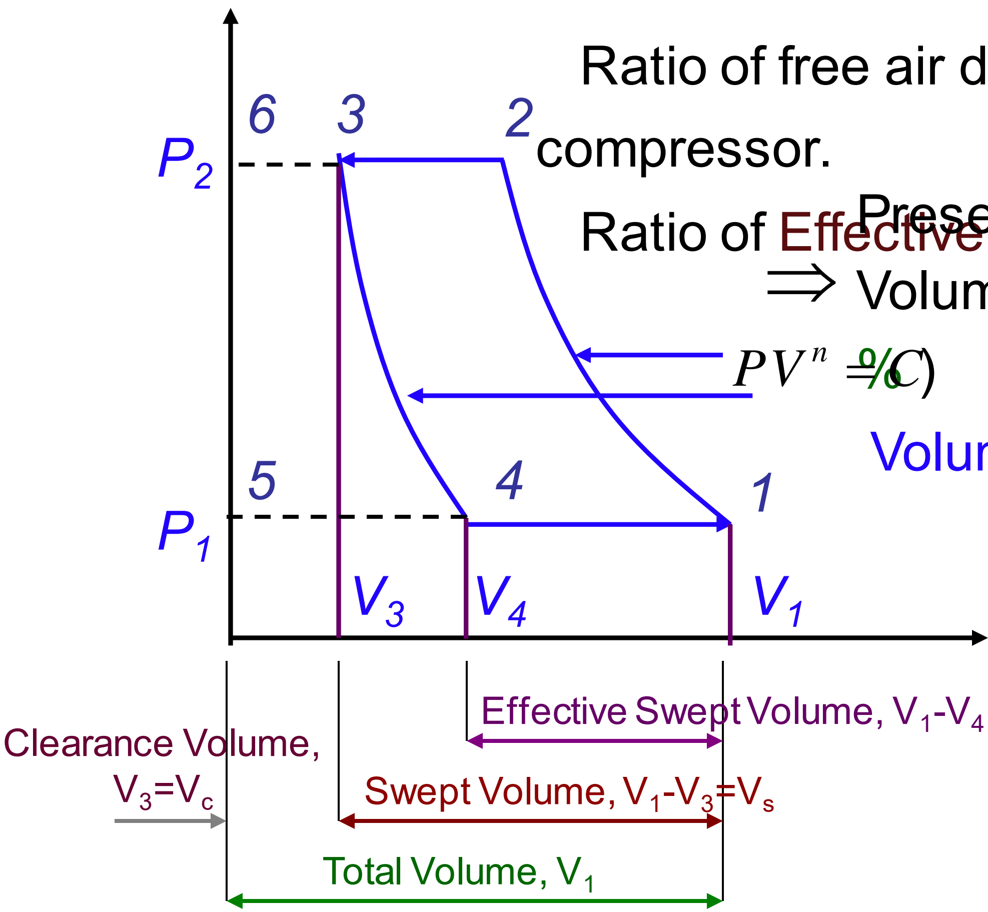
$$\Rightarrow \quad W_{act} = \frac{n}{n-1} P_1 V_1 \left\{ 1 - \left(\frac{P_2}{P_1} \right)^{\frac{m-1}{n}} \right\}$$

$$- \frac{n}{n-1} P_1 V_4 \left\{ 1 - \left(\frac{P_2}{P_1} \right)^{\frac{m-1}{n}} \right\}$$

$$\boxed{W_{act} = \left(\frac{n}{n-1} \right) P_1 \left(V_1 - V_4 \right) \left\{ 1 - \frac{P_2}{P_1} \left(\frac{P_1}{P_2} \right)^{1/n} \right\}}$$

But, $P_4 = P_1$ and $P_3 = P_2$

Reciprocating Compressor – Volumetric Efficiency



Volumetric Efficiency :

Ratio of free air delivered to the displacement of the compressor.

Ratio of Effective Swept Volume to Swept Volume.

Presence of Clearance Volume

$\Rightarrow$ Volumetric Efficiency less than 1. (60 – 85 %)

$$\text{Volumetric Efficiency} = \frac{\text{Effective Swept Volume}}{\text{Swept Volume}}$$

$$= \frac{V_1 - V_4}{V_1 - V_3}$$

$$\text{Clearance Ratio} = \frac{\text{Clearance Volume}}{\text{Swept Volume}}$$

$$= \frac{V_c}{V_s} = \gamma \text{ (4 – 10 %)}$$

Reciprocating Compressor – Volumetric Efficiency

$\uparrow$ Pr. Ratio $\Rightarrow$ $\uparrow$ Effect of Clearance Volume

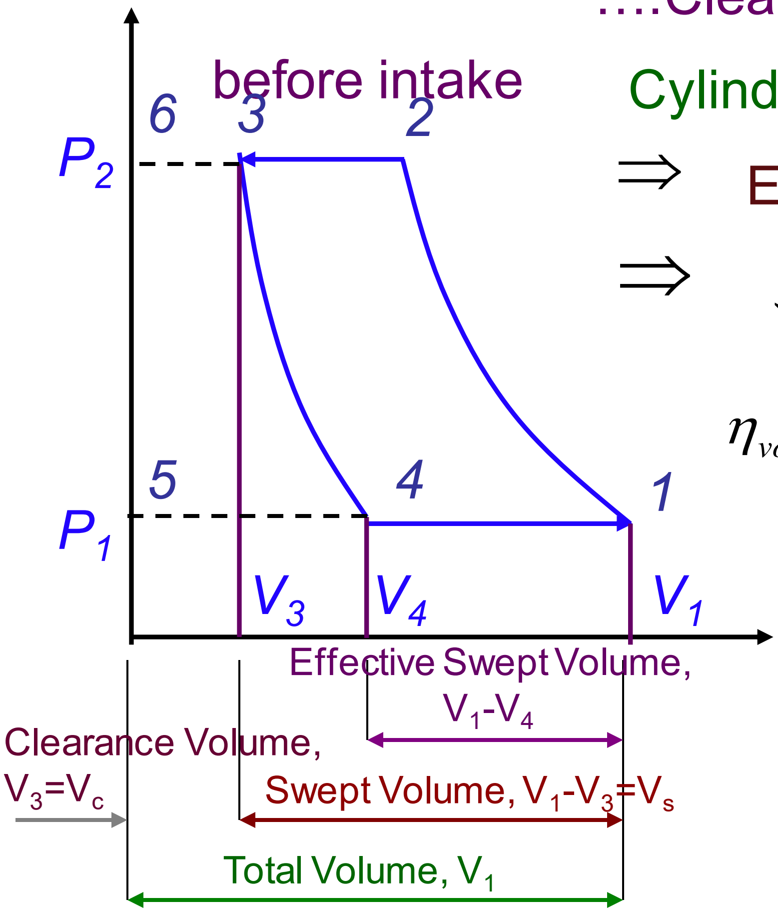
….Clearance air expansion through greater volume

before intake

Cylinder bore and stroke is fixed.

$\Rightarrow$ Effective Swept Volume (V1 – V4) $\downarrow$ with $\uparrow$ Pr. Ra

$\Rightarrow$ $\downarrow$ Volumetric Efficiency



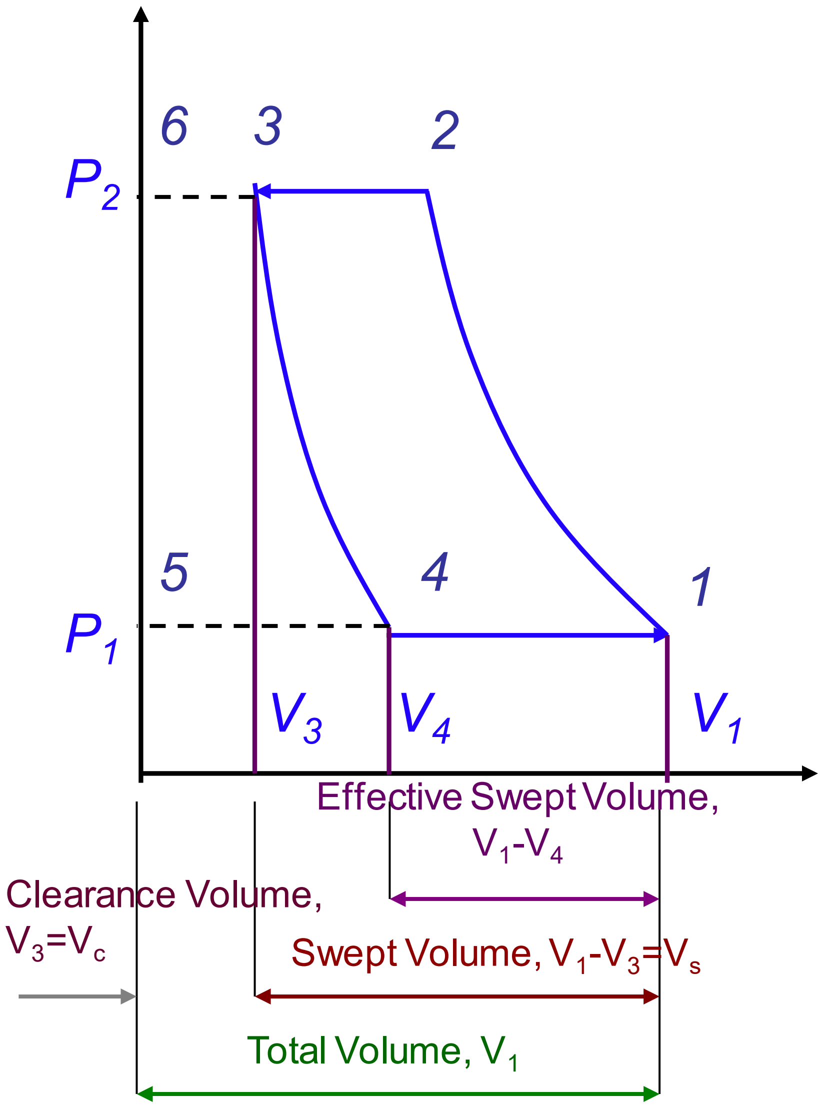
$$\eta_{vol} = \frac{V_1 - V_4}{V_1 - V_3}$$

$$= \frac{(V_1 - V_3) + (V_3 - V_4)}{(V_1 - V_3)} = 1 + \frac{V_3}{(V_1 - V_3)} - \frac{V_4}{(V_1 - V_3)}$$

$$= 1 + \frac{V_3}{(V_1 - V_3)} - \frac{V_4}{(V_1 - V_3)} \bullet \frac{V_3}{V_3}$$

$$= 1 + \frac{V_3}{(V_1 - V_3)} - \frac{V_3}{(V_1 - V_3)} \bullet \frac{V_4}{V_3}$$

Reciprocating Compressor – Volumetric Efficiency



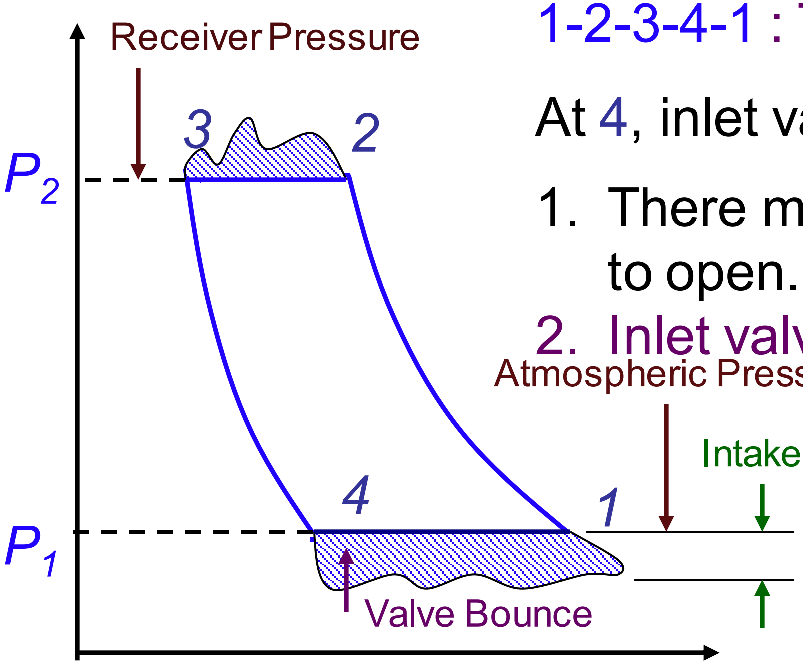
$$\eta_{vol} = 1 + \frac{V_3}{V_1 - V_3}\left(1 - \frac{V_3}{V_4}\right)$$

$$\eta_{vol} = 1 - \frac{V_3}{V_1 - V_3}\left(\frac{V_3}{V_4} - 1\right)$$

$$\eta_{vol} = 1 - \frac{V_3}{V_1 - V_3}\left(\left(\frac{P_3}{P_4}\right)^{1/n} - 1\right)$$

$$\eta_{vol} = 1 - \gamma \bullet \left(\left(\frac{P_3}{P_4}\right)^{1/n} - 1\right)$$

Reciprocating Compressor – Actual *P-V* Diagram



1-2-3-4-1 : Theoretical *P-V* Diagram.

At 4, inlet valve does not open due to :

1. There must be a pressure difference across the valve to open.
2. Inlet valve inertia.

Pr. Drop continues till sufficient level for valve to force its seat.

Some *valve bounce* is set (wavy lin

Eventually, the pressure sets down at a level *lower than* atmospheric pressure. This negative pressure difference is known as *Intake Depression.*

Pressure rise, followed by valve bounce and then pressure settles at a level *higher than* the delivery pressure level.

Air delivery to a tank / receiver, hence, generally known as *Receiver Pressure*.

Reciprocating Compressor – F.A.D.

Free Air Delivery (F.A.D.) : If the volume of the air compressor is reduced to atmospheric temperature and pressure, this volume of air is called FAD (m^3/min)

Delivered mass of air = intake mass of air

$$\frac{P_t V_t}{T_t} = \frac{P_1 (V_1 - V_4)}{T_1} = \frac{P_2 (V_2 - V_3)}{T_2}$$

f clearance volume is neglected

$$\frac{P_t V_t}{T_t} = \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

Where

$$P_t = 101.325 KN / m^2$$

$$T_t = 15^0 C = 288 K$$

Reciprocating Compressor – Multistage

High Pressure required by Single – Stage :

$\Rightarrow$

1. Requires heavy working parts.
2. Has to accommodate high pressure ratios.
3. Increased balancing problems.
4. High Torque fluctuations.
5. Requires heavy Flywheel installations.

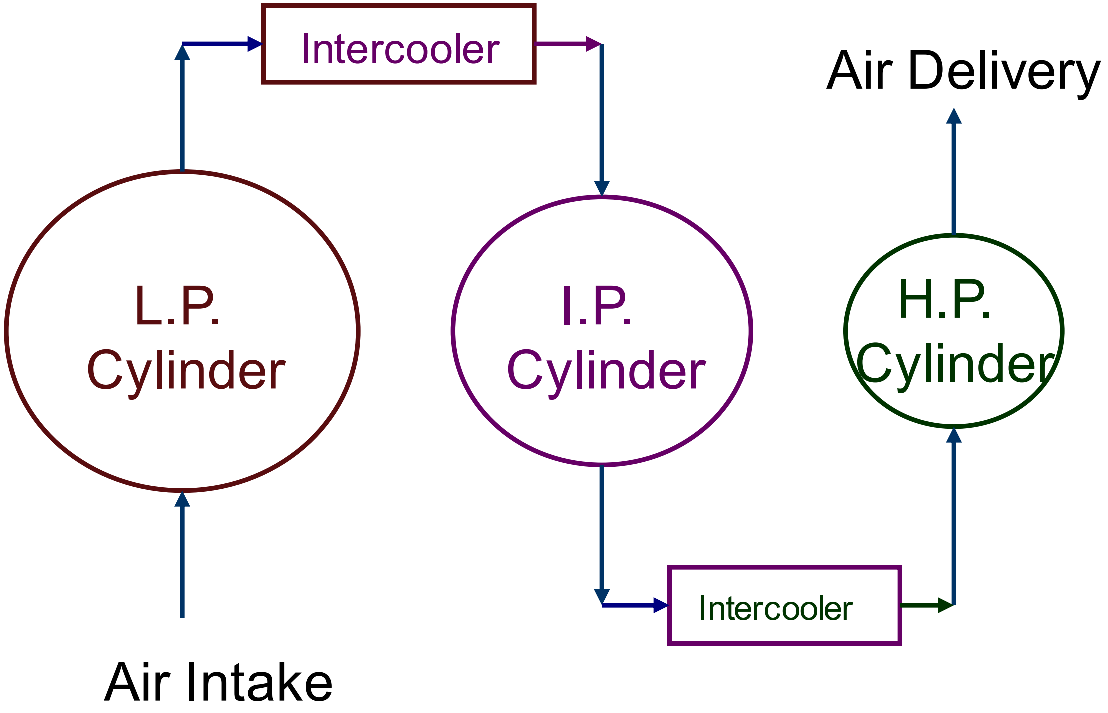
This demands for MULTI – STAGING…!!

Why multistage compressor?

- High temp rise leads into limitation for the maximum achievable pressure rise.
- Discharge temperature shall not exceed 150ºC and should not exceed 135^0C for hydrogen rich services
- A multistage compressor compresses air to the required pressure in multiple stages.
- Intercoolers are used in between each stage to removes heat and decrease the temperature of gas so that gas could be compressed to higher pressure without much rise in temperature

Reciprocating Compressor – Multistage

Series arrangement of cylinders, in which the compressed air from earlier cylinder (i.e. *discharge*) becomes the intake air for the next cylinder (i.e. *inlet*).



L.P. = Low Pressure

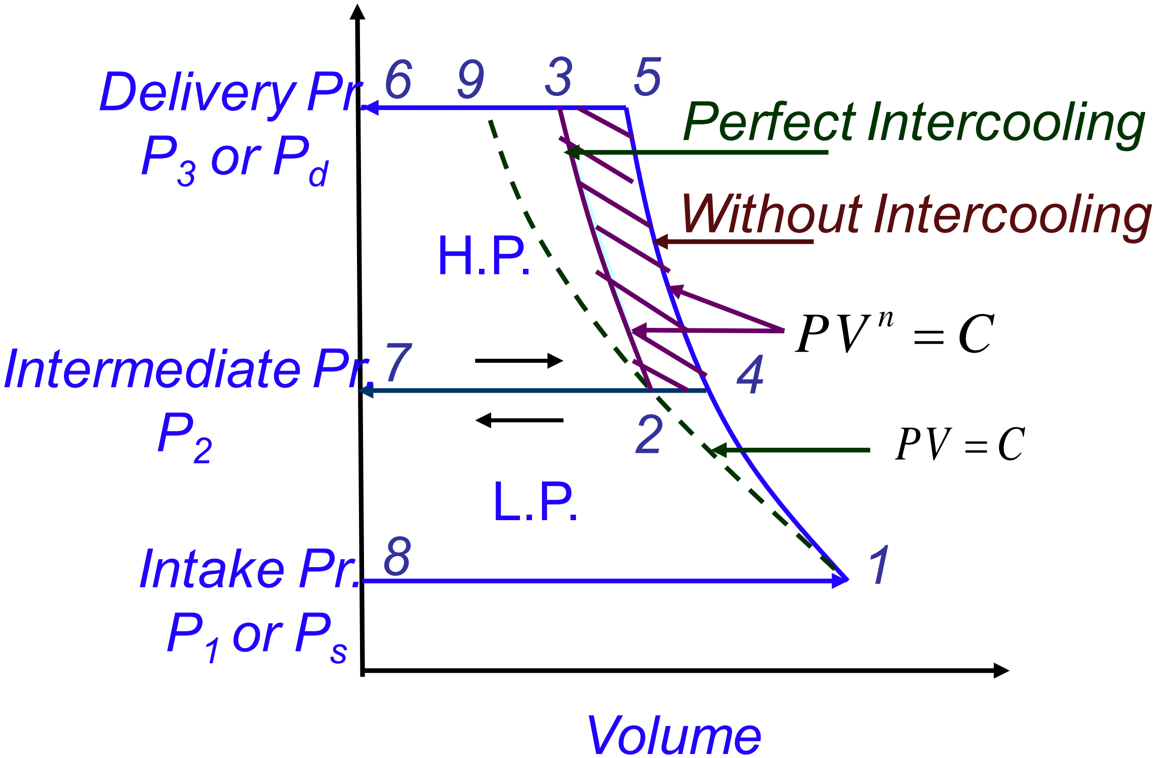
I.P. = Intermediate Pressure

H.P. = High Pressure

Intercooler :

Compressed air is *cooled* between cylinders.

Reciprocating Compressor – Multistage



Overall Pr. Range : $P_1 – P_3$

Single – stage cycle : 8-1-5-6

Without Intercooling :

L.P. : 8-1-4-7

H.P. : 7-4-5-6

With Intercooling :

L.P. : 8-1-4-7

H.P. : 7-2-3-6

Perfect Intercooling : After initial compression in L.P. cylinder, air is cooled in the

Intercooler to its original temperature, before entering H.P. cylinder

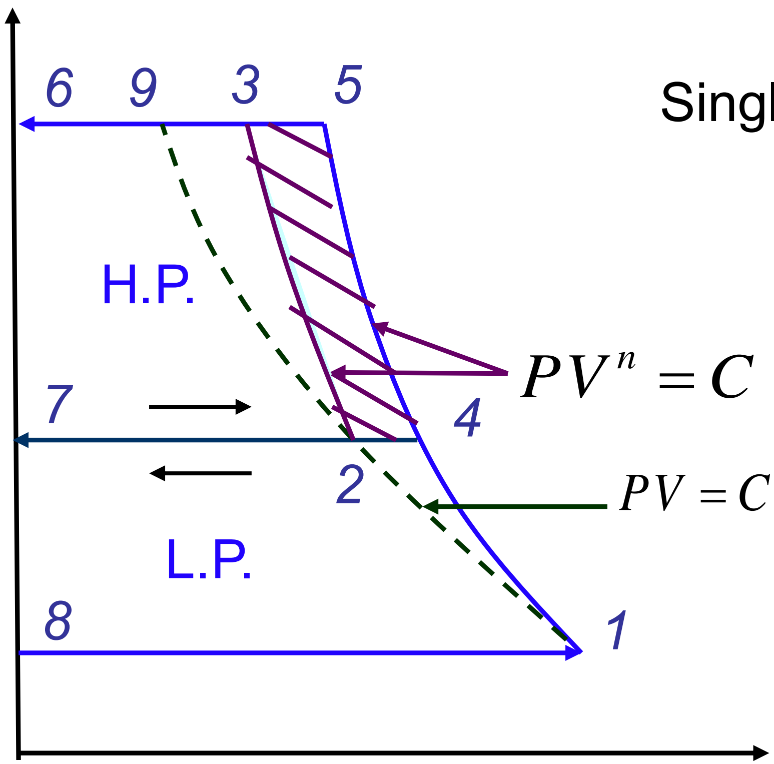
i.e. $T_2 = T_1$ OR

Points 1 and 2 are on SAME Isothermal line.

Reciprocating Compressor – Multistage

Ideal Conditions for Multi – Stage Compressors :

A. Single – Stage Compressor :



Single – stage cycle : 8-1-5-6

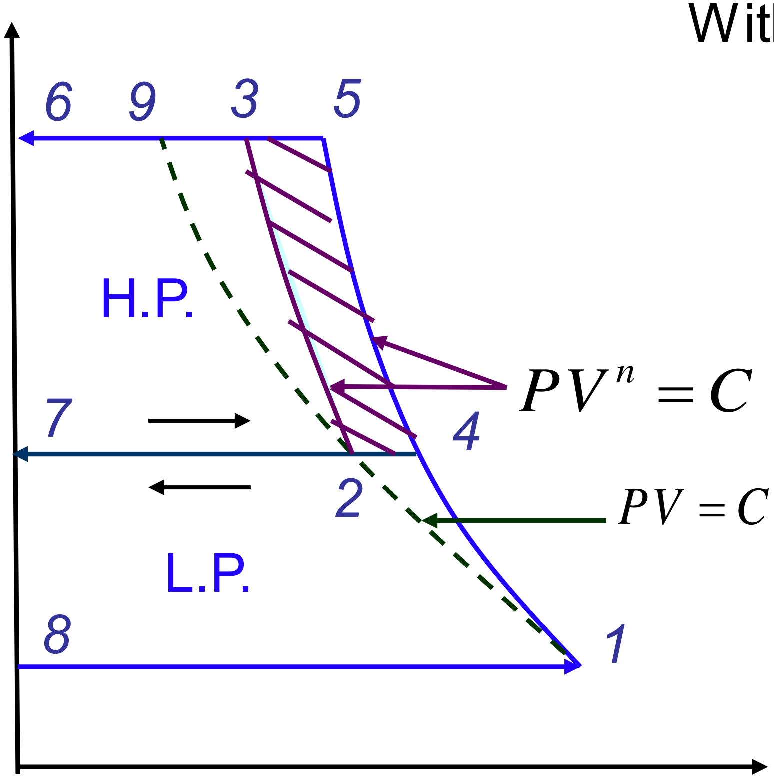
$$W = \frac{n}{n-1} P_1 V_1 \left[1 - \left(\frac{P_5}{P_1} \right)^{\frac{n}{n-1}} \right]$$

Delivery Temperature,

$$T_5 = T_1 \left(\frac{P_5}{P_1} \right)^{\frac{n-1}{n}}$$

Reciprocating Compressor – Multistage

B. <u>Two – Stage Compressor</u> *(Without Intercooling)* :



Without Intercooling :

L.P. : 8-1-4-7

H.P. : 7-4-5-6

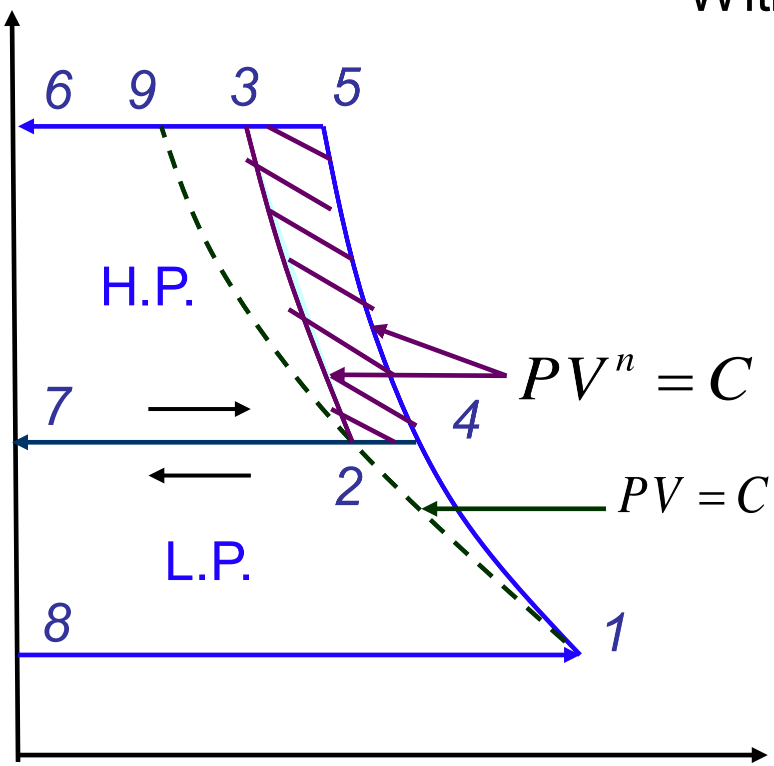
$$W = \frac{n}{n-1} P_1 V_1 \left[1 - \left(\frac{P_4}{P_1} \right)^{\frac{n-1}{n}} \right] + \frac{n}{n-1} P_4 V_4 \left[1 - \left(\frac{P_5}{P_4} \right)^{\frac{n-1}{n}} \right]$$

Without Intercooling $\Longrightarrow$ This is SAME as that of Work done in Single – S

Delivery Temperature also remains SAME.

Reciprocating Compressor – Multistage

C. <u>Two – Stage Compressor</u> *(With Perfect Intercooling)* :



With Intercooling :

L.P. : 8-1-4-7-8

H.P. : 7-2-3-6-7

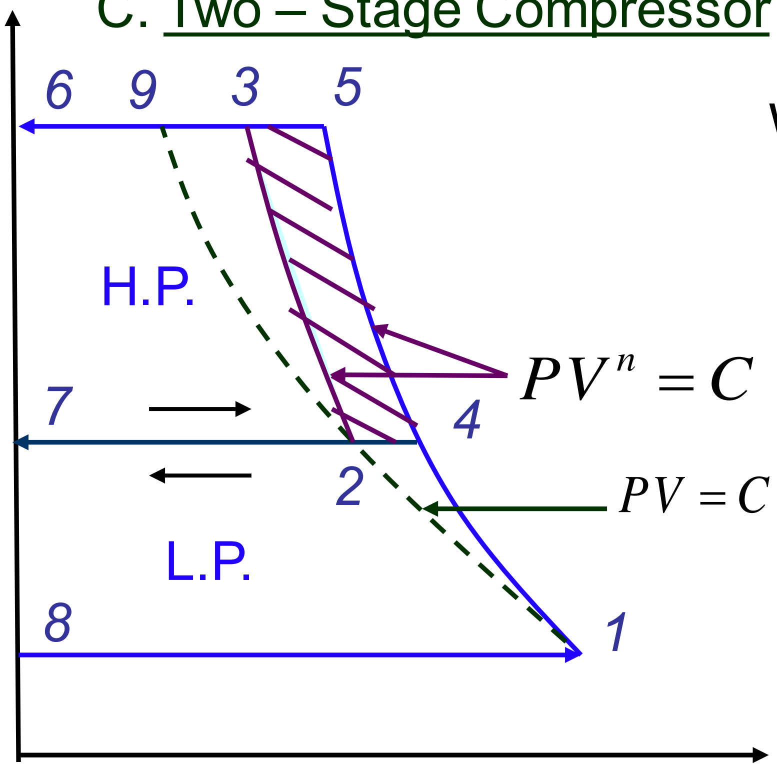
$$W = \frac{n}{n-1} P_1 V_1 \left[1 - \left(\frac{P_4}{P_1} \right)^{\frac{n-1}{n}} \right] + \frac{n}{n-1} P_2 V_2 \left[1 - \left(\frac{P_3}{P_2} \right)^{\frac{n-1}{n}} \right]$$

Delivery Temperature,

$$T_3 = T_2 \left(\frac{P_3}{P_2} \right)^{\frac{n-1}{n}} = T_1 \left(\frac{P_3}{P_2} \right)^{\frac{n-1}{n}}, \quad as\ T_2 = T_1$$

Reciprocating Compressor – Multistage

C. Two – Stage Compressor *(With Perfect Intercooling)* :



With Intercooling :

L.P. : 8-1-4-7-8

H.P. : 7-2-3-6-7

Now, $T_2 = T_1$

$P_2V_2 = P_1V_1$

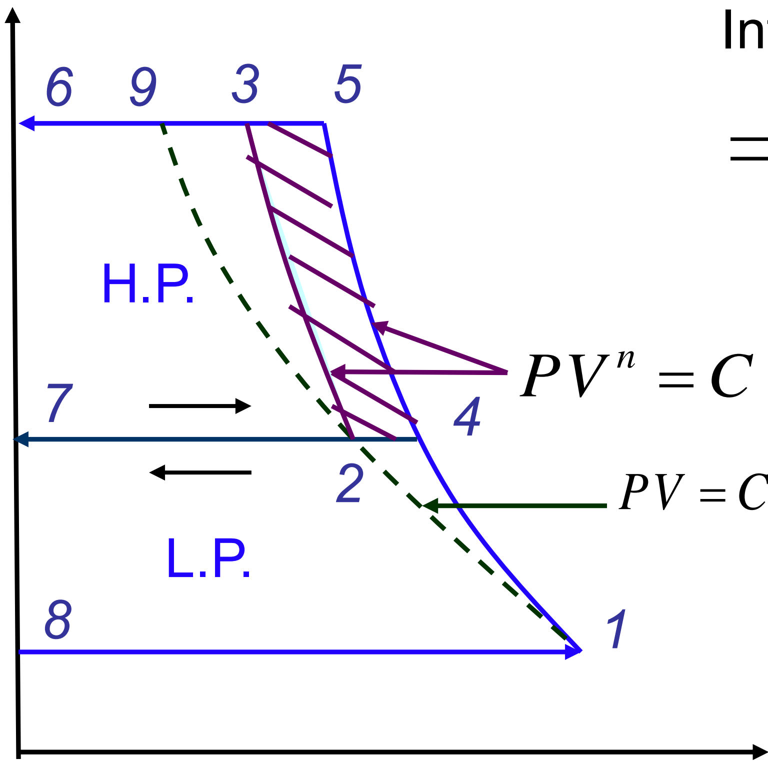
Also $P_4 = P_2$

$$W = \frac{n}{n-1} P_1 V_1 \left[2 - \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - \left(\frac{P_3}{P_2} \right)^{\frac{n-1}{n}} \right]$$

Shaded Area 2-4-5-3-2 : Work Saving due to Intercoo

Reciprocating Compressor – Multistage

Condition for Min. Work :



Intermediate Pr. $P_2 \rightarrow P_1$: Area 2-4-5-3-2 → 0

Intermediate Pr. $P_2 \rightarrow P_3$: Area 2-4-5-3-2 → 0

$\Longrightarrow$ There is an *Optimum* P_2 for which Area 2-4-5-3-2 is maximum,

i.e. Work is minimum…!!

$$W = \frac{n}{n-1} P_1 V_1 \left[2 - \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - \left(\frac{P_3}{P_2} \right)^{\frac{n-1}{n}} \right]$$

For min. Work,

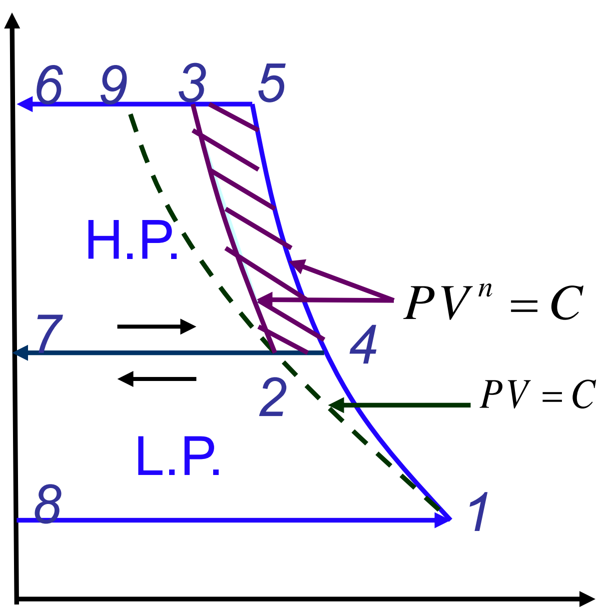
$$\frac{dW}{dP_2} = \frac{d\left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} + \left(\frac{P_3}{P_2} \right)^{\frac{n-1}{n}} \right]}{dP_2} = 0$$

Reciprocating Compressor – Multistage

Condition for Min. Work :

$$\frac{dW}{dP_2} = \frac{d\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} + \left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}}\right]}{dP_2} = 0$$

$$\frac{1}{(P_1)^{\frac{n-1}{n}}} \bullet \left(\frac{n-1}{n}\right)(P_2)^{\left(\frac{n-1}{n}\right)-1} + (P_3)^{\left(\frac{n-1}{n}\right)} \bullet \left[-\left(\frac{n-1}{n}\right)(P_2)^{\left(-\frac{n-1}{n}\right)-1}\right] = 0$$



$$\frac{(P_2)^{-1/n}}{(P_2)^{\left(\frac{-2n+1}{n}\right)}} = (P_1\,P_3)^{\left(\frac{n-1}{n}\right)}$$

$$(P_2)^2 = (P_1\,P_3)$$

$$\boxed{P_2 = \sqrt{P_1\,P_3} \quad OR \quad \frac{P_2}{P_1} = \frac{P_3}{P_2}}$$

Reciprocating Compressor – Multistage

P_2 obtained with this condition (Pr. Ratio per stage is equal) is the Ideal Intermediate Pr. Which, with Perfect Intercooling, gives Minimum Work, W_{min}. $\Rightarrow$ Equal Work per cylinder…!!

$$W = \frac{2n}{n-1} P_1 V_1 \left[1 - \left(\frac{(P_1 P_3)^{1/2}}{P_1} \right)^{\frac{n-1}{n}} \right]$$

$$W = \frac{2n}{n-1} P_1 V_1 \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right]$$

$$\boxed{W = \frac{2n}{n-1} P_1 V_1 \left[1 - \left(\frac{P_3}{P_1} \right)^{\frac{n-1}{2n}} \right]}$$

Reciprocating Compressor – Efficiency

Isothermal work done / cycle = Area of *P – V* Diagram

$$= P_1 V_1 \log_e(P_2/P_1)$$

Isothermal Power

$$= \frac{P_1 V_1 \log_e(P_2/P_1)\, N}{60 \times 1000} \text{ kW}$$

Indicated Power : Power obtained from the actual indicator card taken during a

$$\text{Compressor Efficiency} = \frac{\text{Isothermal Power}}{\text{Indicated Power}}$$

$$\text{Isothermal Efficiency} = \frac{\text{Isothermal Power}}{\text{Shaft Power}}$$

NOTE : Shaft Power = Brake Power required to drive the Compressor.

Reciprocating Compressor – Efficiency

Adiabatic Efficiency : Ratio of Power required to drive the Compressor;

compared

with the area of the hypothetical Indicator Diagram;

assuming

Adiabatic Compression.

$$\eta_{adiabatic} = \frac{\frac{\gamma}{\gamma - 1} P_1 V_1 \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} \right]}{Brake\ Power\ required\ to\ drive\ the\ Compressor}$$

Mechanical Efficiency : Ratio of mechanical output to mechanical input.

$$\text{Mechanical Efficiency, } \eta_{mech} = \frac{\text{Indicated Power}}{\text{Shaft Power}}$$

Reciprocating Compressor – Efficiency

How to Increase Isothermal Efficiency ?

A. Spray Injection : Assimilation of water into the compressor cylinder towards the compression stroke.

Object is to cool the air for next operation.

Demerits : 1. Requires *special gear* for injection.

2. Injected water interferes with the *cylinder lubri*

3. Damage to *cylinder walls* and *valves*.

4. Water must be separated before delivery of air.

B. Water Jacketing : Circulating water around the cylinder to help for cooling the air during compression.

Reciprocating Compressor – Efficiency

<u>How to Increase Isothermal Efficiency</u> ?

C. Inter – Cooling : For high speed and high Pr. Ratio compressors.

Compressed air from earlier stage is cooled to its original temperature before passing it to the next stage.

D. External Fins : For small capacity compressors, fins on external surfaces are useful.

E. Cylinder Proportions : Short stroke and large bore provides much greater surface

for cooling.

Cylinder head surface is far more effective than barrel surface.

Reciprocating Compressor – Efficiency

Clearance Volume : Consists of *two* spaces.

1. Space between cylinder end & the piston to allow for wear.
2. Space for reception of valves.

High – class H.P. compressors : Clearance Vol. = 3 % of Swept Vol.

: Lead (Pb) fuse wire used to measure the gap between cylinder end and piston.

Low – grade L.P. compressors : Clearance Vol. = 6 % of Swept Vol.

: Flattened ball of putty used to measure the gap between cylinder end and piston.

Effect of Clearance Vol. :

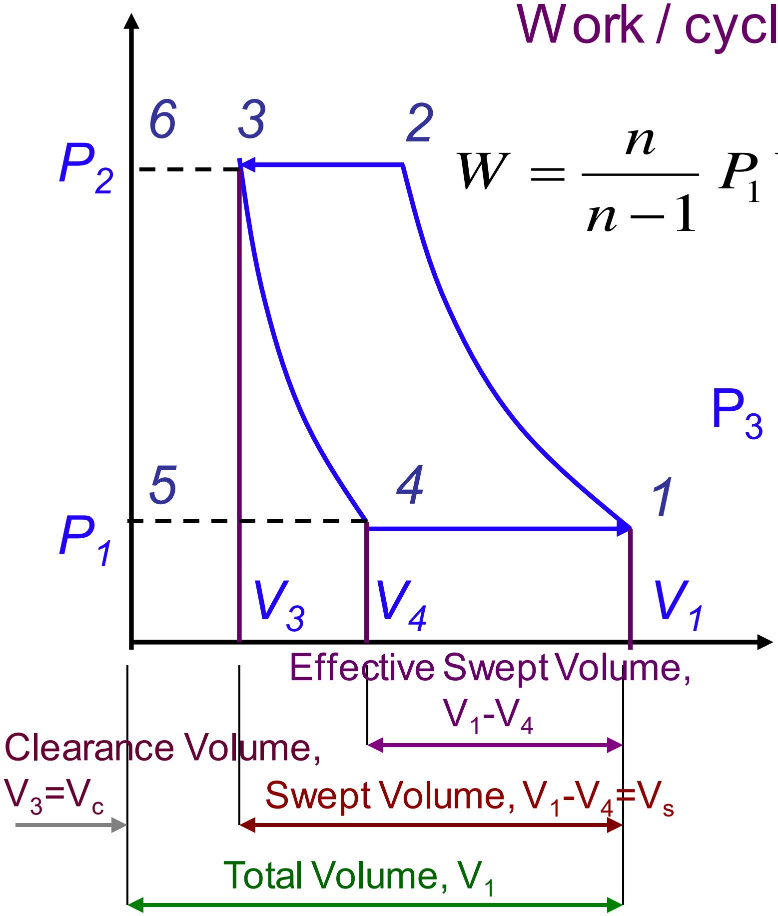
Vol. taken in per stroke < Swept Vol. ⇒ ↑ Size of compressor

↑ Power to drive compressor

Reciprocating Compressor – Work Done

Assumption : Compression and Expansion follow same Law.

Work / cycle = Area 1-2-3-4-1



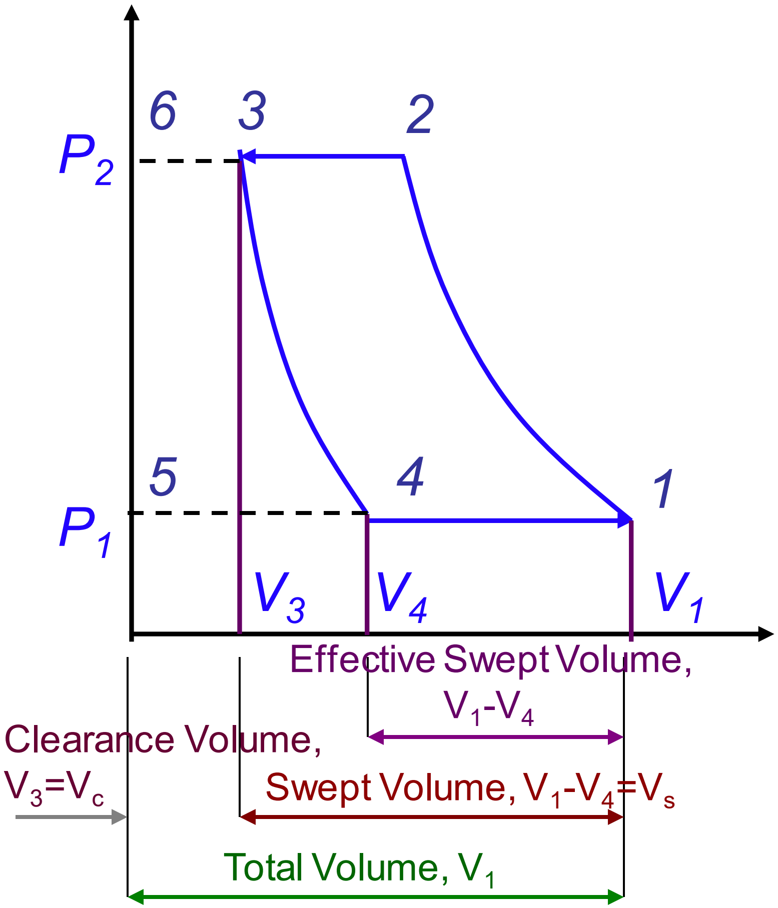
$$W = \frac{n}{n-1} P_1 V_1 \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right] - \frac{n}{n-1} P_4 V_4 \left[1 - \left(\frac{P_3}{P_4} \right)^{\frac{n-1}{n}} \right]$$

$P_3 = P_2$ and $P_4 = P_1$

$$W = \frac{n}{n-1} P_1 (V_1 - V_4) \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right]$$

$$= \frac{n}{n-1} P_1 V_a \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right]$$

Reciprocating Compressor – Work Done



$$W = \frac{n}{n-1} m_1 R T_1 \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right]$$

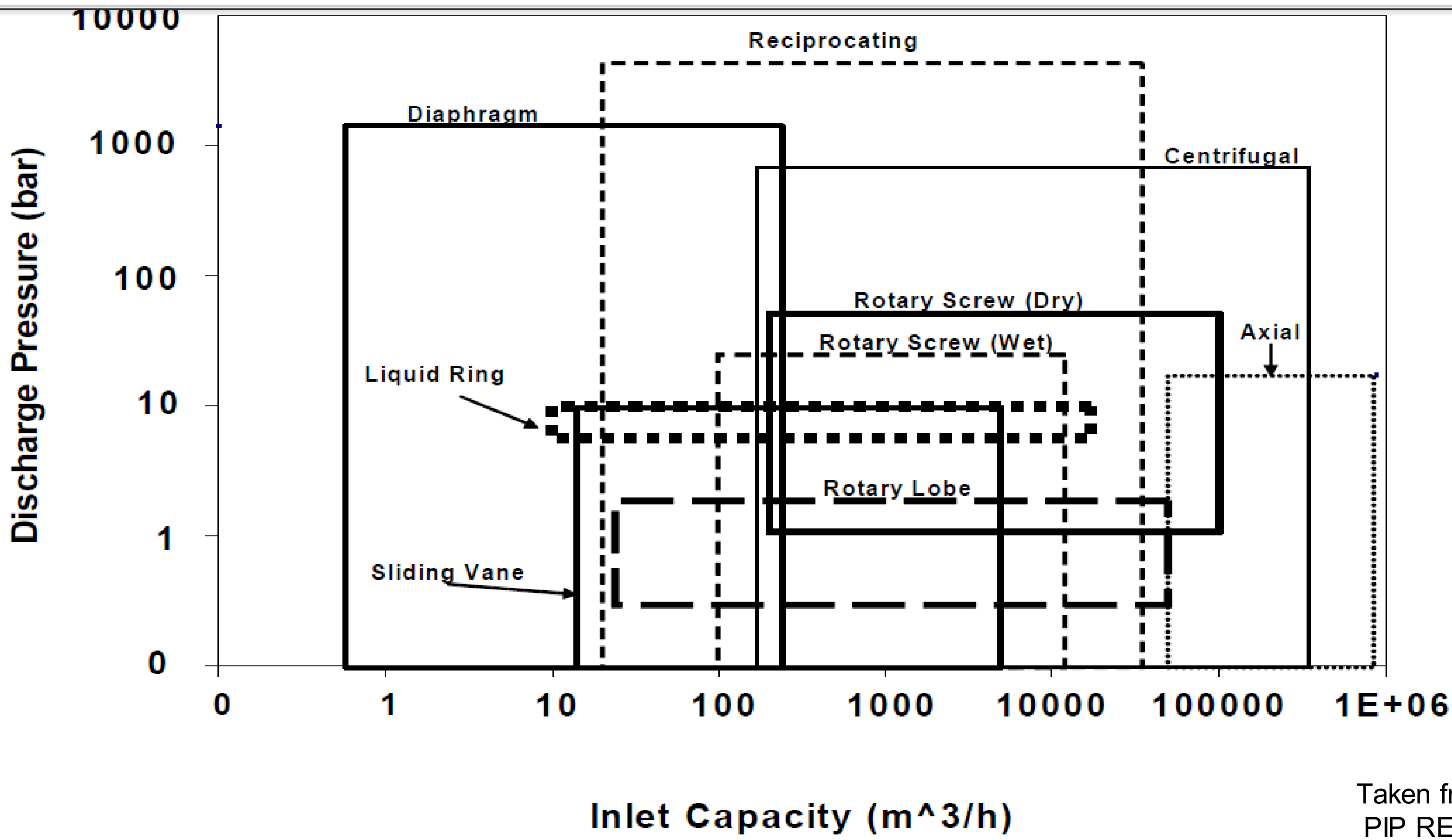
m_1 is the actual mass of air delivered.

Work done / kg of air delivered :

$$W = \frac{n}{n-1} R T_1 \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right]$$

How to select a particular type of compressor ?

Graph showing operating regions of various compressors



Taken from
PIP REEC001
Compressor Selection
Guidelines

Advantages and Disadvantages of Dynamic compressors

	Advantages	Disadvantages
Dynamic Compressors		
Centrifugal	•Wide operating range •High reliability •Low Maintenance	•Instability at reduced flow •Sensitive to gas composition change
Axial	•High Capacity for given size •High efficiency •Heavy duty •Low maintenance	•Low Compression ratios •Limited turndown

Advantages and Disadvantages of Positive displacement compressors

	Advantages	Disadvantages
Positive displacement compressor		
Reciprocating	•Wide pressure ratios •High efficiency	•Heavy foundation required •Flow pulsation •High maintenance
Diaphragm	•Very high pressure •Low flow •No moving seal	•Limited capacity range •Periodic replacement of diaphragm
Screw	•Wide application •High efficiency •High pressure ratio	•Expensive •Unsuitable for corrosive or dirty gases

References

Thermodynamics: DePablo, Juan, Schieber, Jay: Amazon.sg: **Book**		•https://www.amazon.sg/Thermodynamics-Juan-DePablo/dp/0071254188
Wikibooks		https://en.wikibooks.org/wiki/Engineering_Thermodynamics

Thank You

Govt. Polytechnic, Nanakpur (Panchkula)
e-Notes

Name : Er. **Shalander Singh (Mor),** Lect. (Mech. Engg.)

Semester : 3rd

Subject : **Workshop Technology-I**

DETAILED CONTENTS

- WELDING
- FOUNDRY TECHNIQUES
- METAL FORMING PROCESSES
- PLASTIC PROCESSING

LEARNING OUTCOMES

After undergoing the subject, students will be able to:

- Fabricate welding joints using gas welding, arc welding, TIG and MIG welding techniques.
- Select suitable (most appropriate) process, electrodes, various parameters of process for a given job.
- Explain respective principle of operations of modern welding processes.
- Inspect various welding joints, castings forgings.
- Prepare pattern for given job.
- Select material and type of patterns, cores.
- Prepare sand moulds manually and on manually and on machine.
- Select type of moulding sand, adhesives, compact, strength and parameters of sand for given job.
- Cast a mould.
- Identify a suitable furnace, alloying elements.
- Carry out deburring of castings.
- Test the properties of moulding sand (permeability, strength, refractoriness, adhesiveness, cohesiveness).
- Explain the principle of forging, rolling, extrusion and drawing process.

Unit I-Welding

1.1-WELDING PROCESSES

1. Resistance Welding
2. Oxyfuel Gas Welding
3. Other Fusion Welding Processes
4. Solid State Welding
5. Weld Quality
6. Weldability
7. Design Considerations in Welding
8. Arc Welding

Two Categories of Welding Processes

- Fusion welding - coalescence is accomplished by melting the two parts to be joined, in some cases adding filler metal to the joint
  - Examples: arc welding, resistance spot welding, oxyfuel gas welding
- Solid state welding - heat and/or pressure are used to achieve coalescence, but no melting of base metals occurs and no filler metal is added
  - Examples: forge welding, diffusion welding, friction welding

Arc Welding (AW)

A fusion welding process in which coalescence of the metals is achieved by the heat from an electric arc between an electrode and the work

- Electric energy from the arc produces temperatures ~ 10,000 F (5500 C), hot enough to melt any metal
- Most AW processes add filler metal to increase volume and strength of weld joint

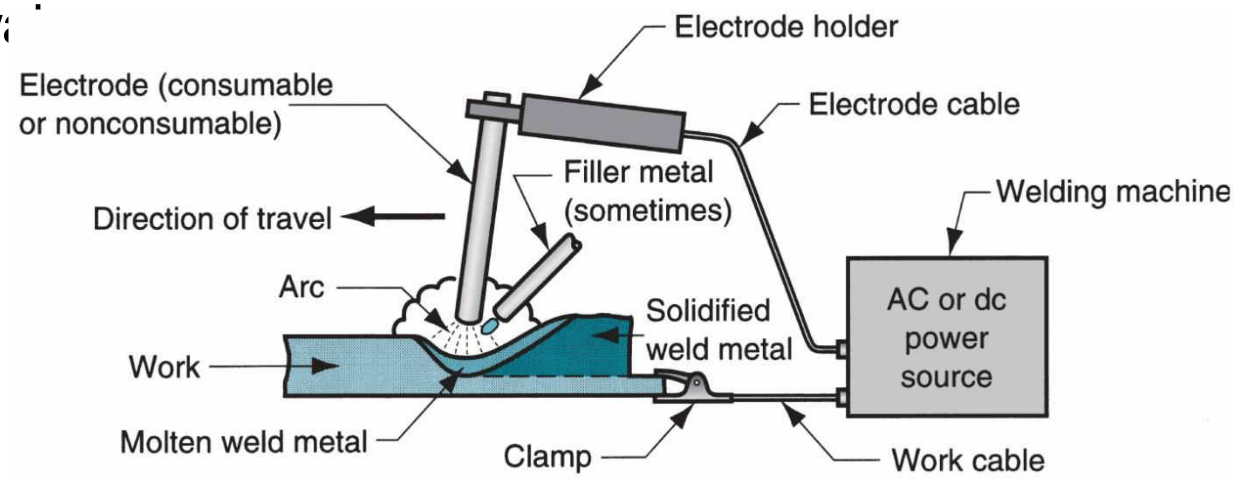
What is an Electric Arc?

An electric arc is a discharge of electric current across a gap in a circuit

- It is sustained by an ionized column of gas (*plasma*) through which the current flows
- To initiate the arc in AW, electrode is brought into contact with work and then quickly separated from it by a short distance

Arc Welding

- A pool of molten metal is formed near electrode tip, and as electrode is moved along joint, molten weld pool solidifies in its wa



Manual Arc Welding and Arc Time

- Problems with manual welding:
  - Weld joint quality
  - Productivity
- Arc Time = (time arc is on) divided by (hours worked)
  - Also called "arc-on time"
  - Manual welding arc time = 20%
  - Machine welding arc time ~ 50%

Two Basic Types of AW Electrodes

- Consumable – consumed during welding process
  - Source of filler metal in arc welding
- Nonconsumable – not consumed during welding process
  - Filler metal must be added separately if it is added

Consumable Electrodes

- Forms of consumable electrodes
  - Welding rods (a.k.a. sticks) are 9 to 18 inches and 3/8 inch or less in diameter and must be changed frequently
  - Weld wire can be continuously fed from spools with long lengths of wire, avoiding frequent interruptions
- In both rod and wire forms, electrode is consumed by the arc and added to weld joint as filler metal

Nonconsumable Electrodes

- Made of tungsten which resists melting
- Gradually depleted during welding (vaporization is principal mechanism)
- Any filler metal must be supplied by a separate wire fed into weld pool

Arc Shielding

- At high temperatures in AW, metals are chemically reactive to oxygen, nitrogen, and hydrogen in air
  - Mechanical properties of joint can be degraded by these reactions
  - To protect operation, arc must be shielded from surrounding air in AW processes
- Arc shielding is accomplished by:
  - Shielding gases, e.g., argon, helium, CO_2
  - Flux

Flux

A substance that prevents formation of oxides and other contaminants in welding, or dissolves them and facilitates removal

- Provides protective atmosphere for welding
- Stabilizes arc
- Reduces spattering

Various Flux Application Methods

- Pouring granular flux onto welding operation
- Stick electrode coated with flux material that melts during welding to cover operation
- Tubular electrodes in which flux is contained in the core and released as electrode is consumed

Power Source in Arc Welding

- Direct current (DC) vs. Alternating current (AC)
  - AC machines less expensive to purchase and operate, but generally restricted to ferrous metals
  - DC equipment can be used on all metals and is generally noted for better arc control

Consumable Electrode AW Processes

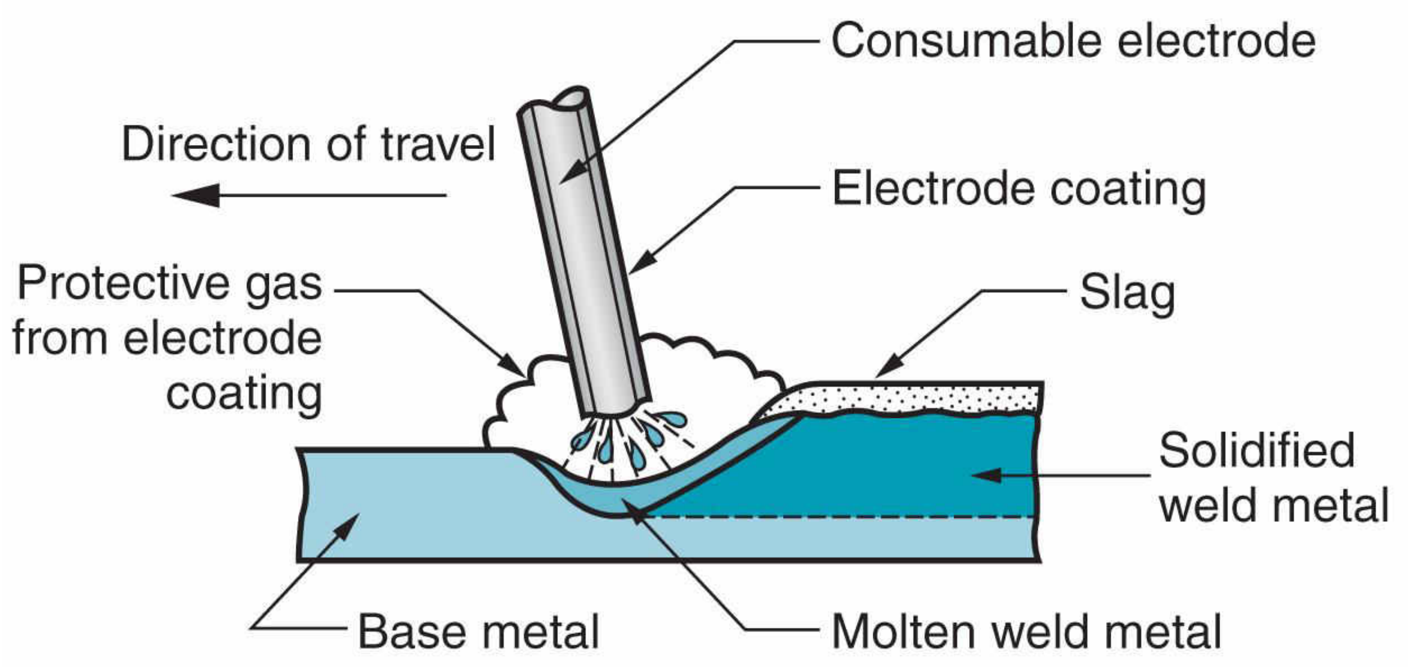
- Shielded Metal Arc Welding
- Gas Metal Arc Welding
- Flux-Cored Arc Welding
- Electrogas Welding
- Submerged Arc Welding

Shielded Metal Arc Welding (SMAW)

Uses a consumable electrode consisting of a filler metal rod coated with chemicals that provide flux and shielding

- Sometimes called "stick welding"
- Power supply, connecting cables, and electrode holder available for a few thousand dollars

Shielded Metal Arc Welding (SMAW)

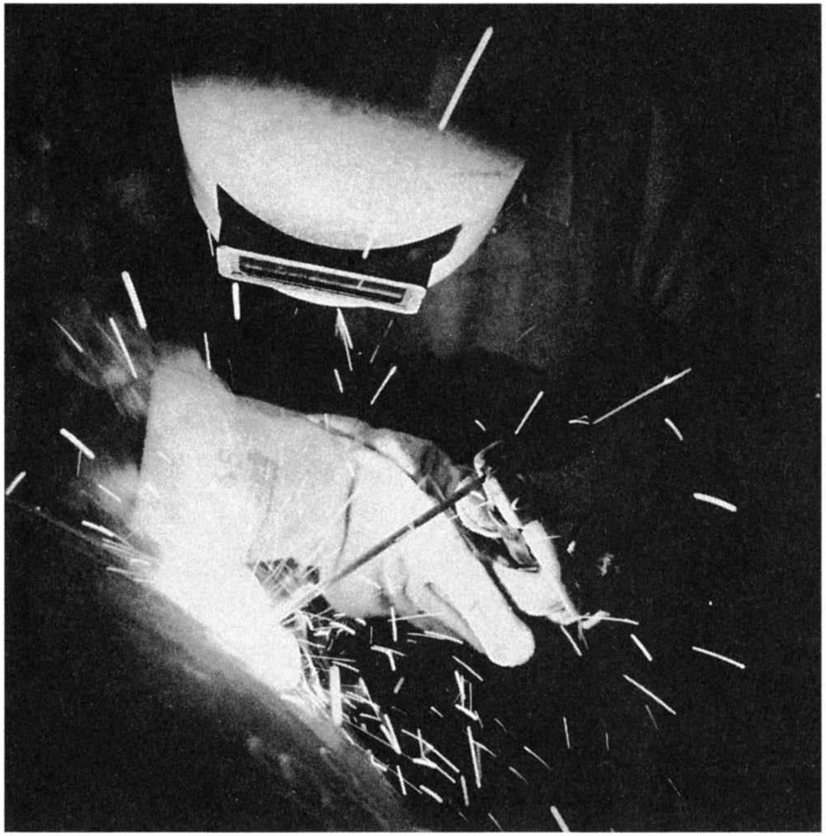


Welding Stick in SMAW

- Composition of filler metal usually close to base metal
- Coating: powdered cellulose mixed with oxides and carbonates, and held together by a silicate binder
- Welding stick is clamped in electrode holder connected to power source
- Disadvantages of stick welding:
  - Sticks must be periodically changed
  - High current levels may melt coating prematurely

Shielded Metal Arc Welding

- Shielded metal arc welding (stick welding) performed by a human welder (photo courtesy of Hobart Brothers Co.)



1.4 SMAW Applications

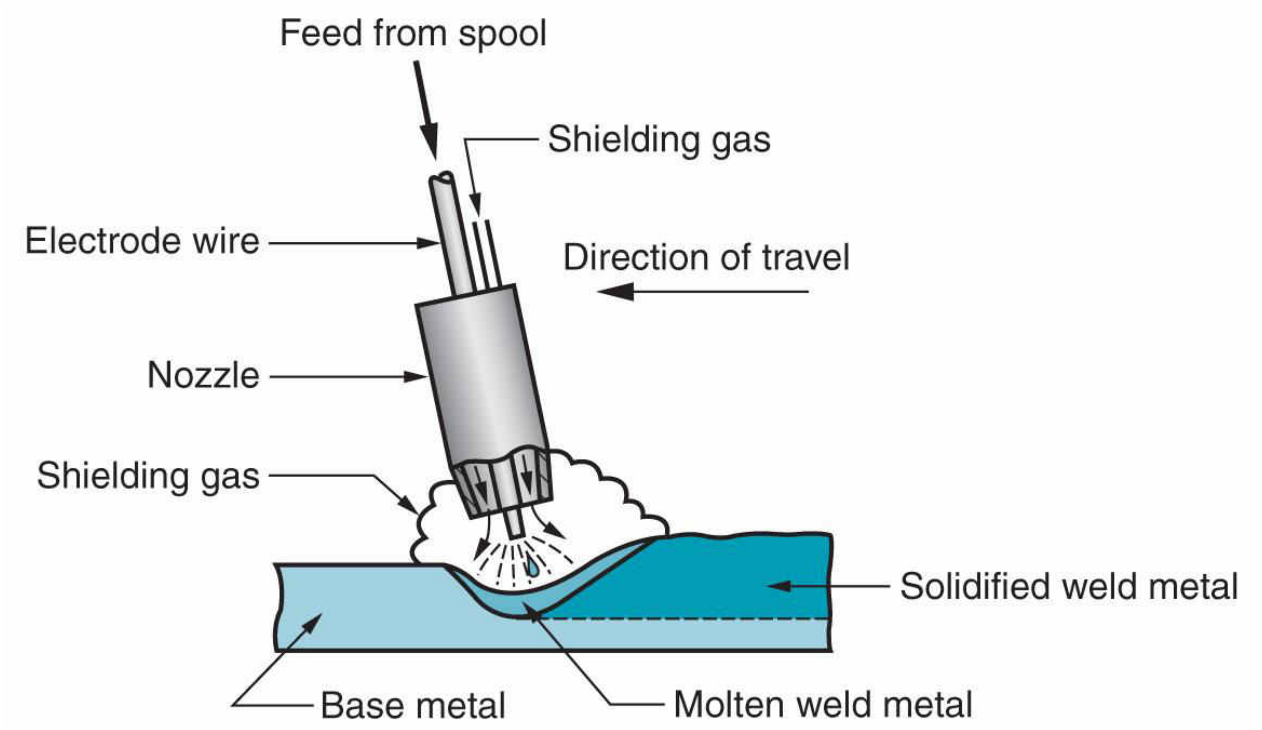
- Used for steels, stainless steels, cast irons, and certain nonferrous alloys
- Not used or rarely used for aluminum and its alloys, copper alloys, and titanium

Gas Metal Arc Welding (GMAW)

Uses a consumable bare metal wire as electrode with shielding by flooding arc with a gas

- Wire is fed continuously and automatically from a spool through the welding gun
- Shielding gases include argon and helium for aluminum welding, and CO_2 for steel welding
- Bare electrode wire plus shielding gases eliminate slag on weld bead
  - No need for manual grinding and cleaning of slag

Gas Metal Arc Welding



GMAW Advantages over SMAW

- Better arc time because of continuous wire electrode
  - Sticks must be periodically changed in SMAW
- Better use of electrode filler metal than SMAW
  - End of stick cannot be used in SMAW
- Higher deposition rates
- Eliminates problem of slag removal
- Can be readily automated

Flux-Cored Arc Welding (FCAW)

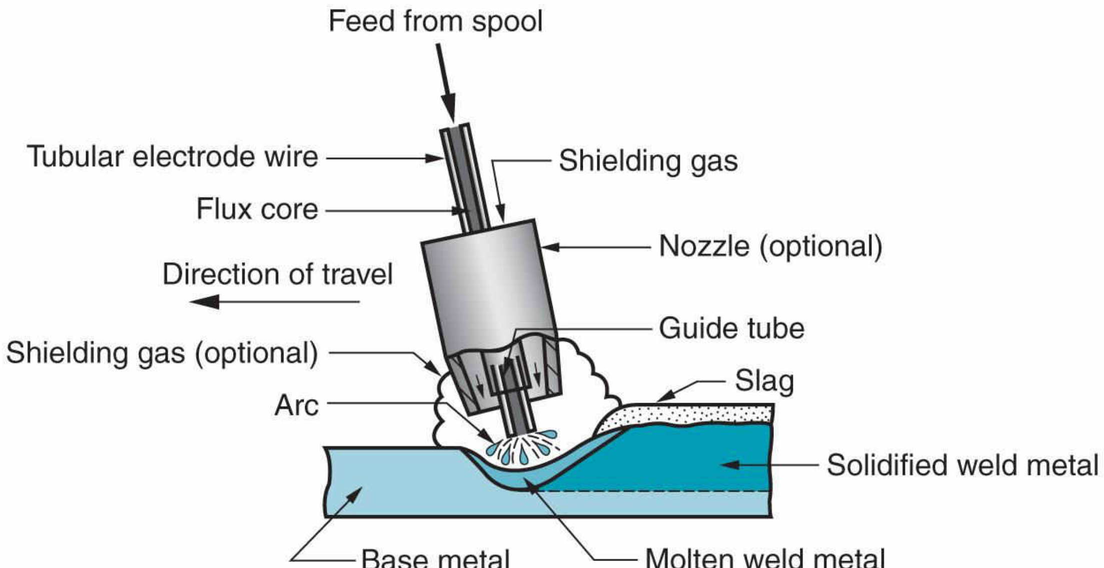
Adaptation of shielded metal arc welding, to overcome limitations of stick electrodes - two versions

- Self-shielded FCAW - core includes compounds that produce shielding gases
- Gas-shielded FCAW - uses externally applied shielding gases

- Electrode is a continuous consumable tubing (in coils) containing flux and other ingredients (e.g., alloying elements) in its core

Flux-Cored Arc Welding

Presence or absence of externally supplied shielding gas distinguishes: (1) self-shielded - core provides ingredients for shielding, (2) gas-shielded - uses external shielding gases



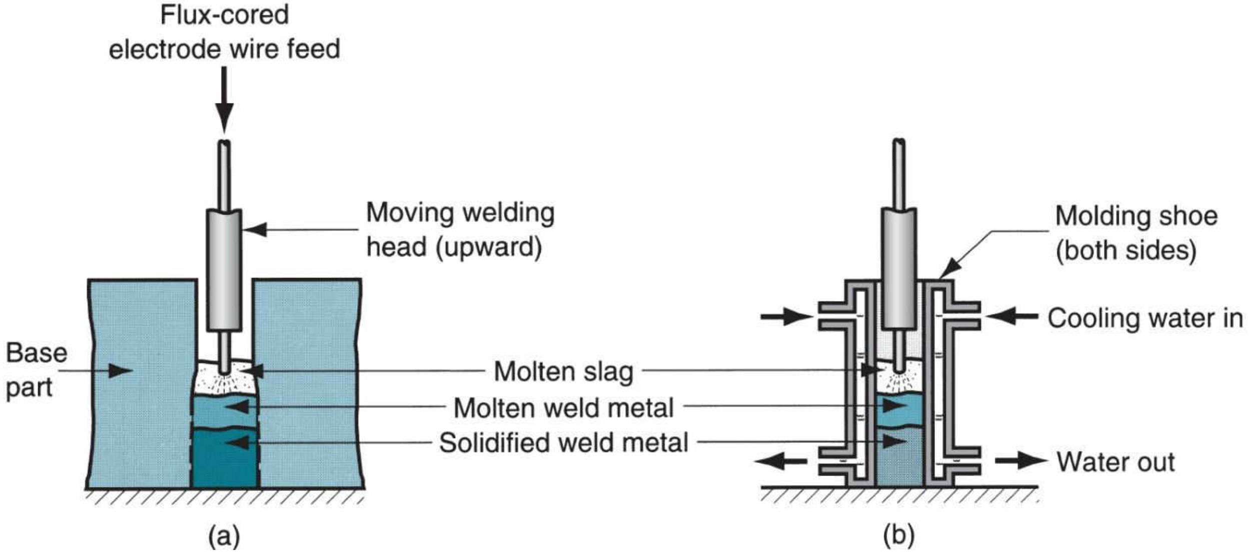
Electrogas Welding (EGW)

Uses a continuous consumable electrode, flux-cored wire or bare wire with externally supplied shielding gases, and molding shoes to contain molten metal

- When flux-cored electrode wire is used and no external gases are supplied, then special case of self-shielded FCAW
- When a bare electrode wire used with shielding gases from external source, then special case of GMAW

Electrogas Welding

- Electrogas welding using flux-cored electrode wire: (a) front view with molding shoe removed for clarity, and (b) side view showing molding shoes on both sides

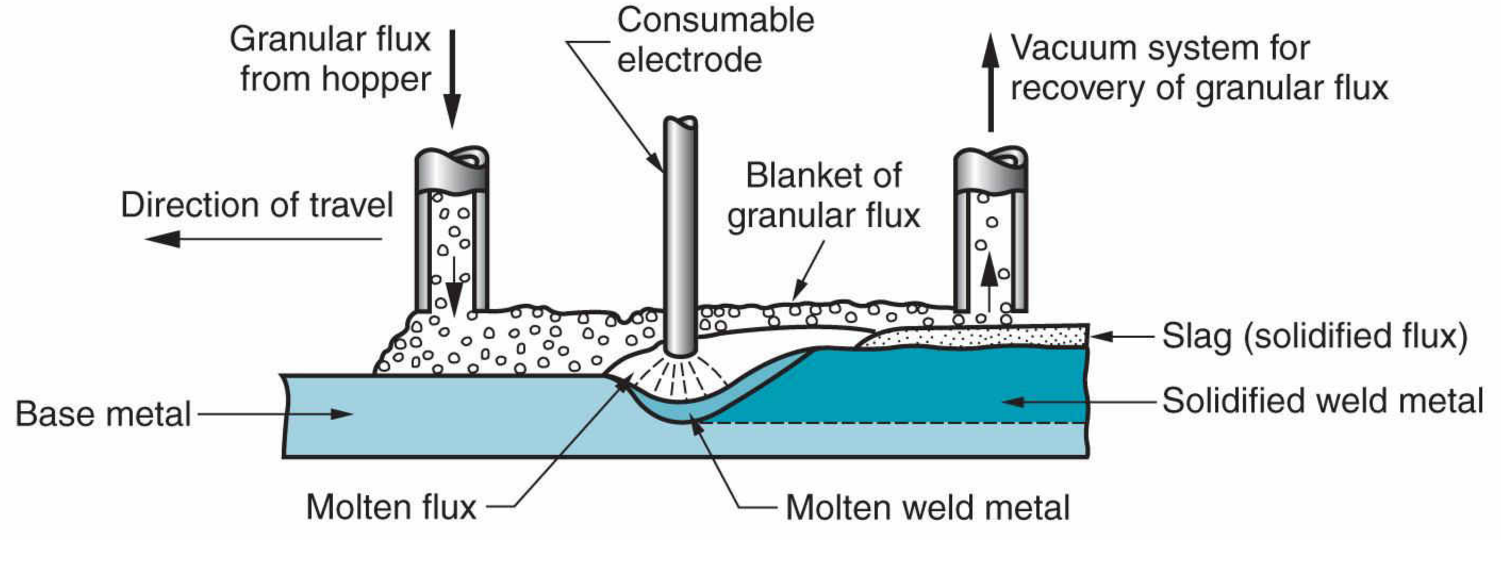


Submerged Arc Welding (SAW)

Uses a continuous, consumable bare wire electrode, with arc shielding by a cover of granular flux

- Electrode wire is fed automatically from a coil
- Flux introduced into joint slightly ahead of arc by gravity from a hopper
  - Completely submerges operation, preventing sparks, spatter, and radiation

Submerged Arc Welding



SAW Applications and Products

- Steel fabrication of structural shapes (e.g., I-beams)
- Seams for large diameter pipes, tanks, and pressure vessels
- Welded components for heavy machinery
- Most steels (except hi C steel)
- Not good for nonferrous metals

Nonconsumable Electrode Processes

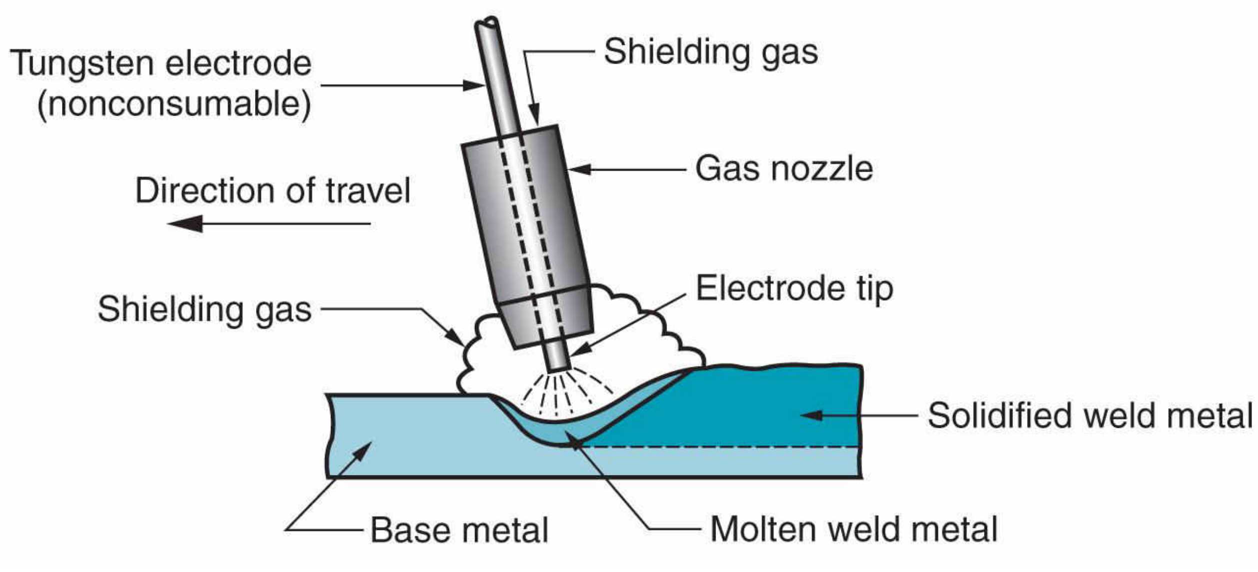
- Gas Tungsten Arc Welding
- Plasma Arc Welding
- Carbon Arc Welding
- Stud Welding

1.5 Gas Tungsten Arc Welding (GTAW)

Uses a nonconsumable tungsten electrode and an inert gas for arc shielding

- Melting point of tungsten = 3410°C (6170°F)
- A.k.a. Tungsten Inert Gas (TIG) welding
  - In Europe, called "WIG welding"
- Used with or without a filler metal
  - When filler metal used, it is added to weld pool from separate rod or wire
- Applications: aluminum and stainless steel mostly

Gas Tungsten Arc Welding



Advantages and Disadvantages of GTAW

Advantages:

- High quality welds for suitable applications
- No spatter because no filler metal through arc
- Little or no post-weld cleaning because no flux

Disadvantages:

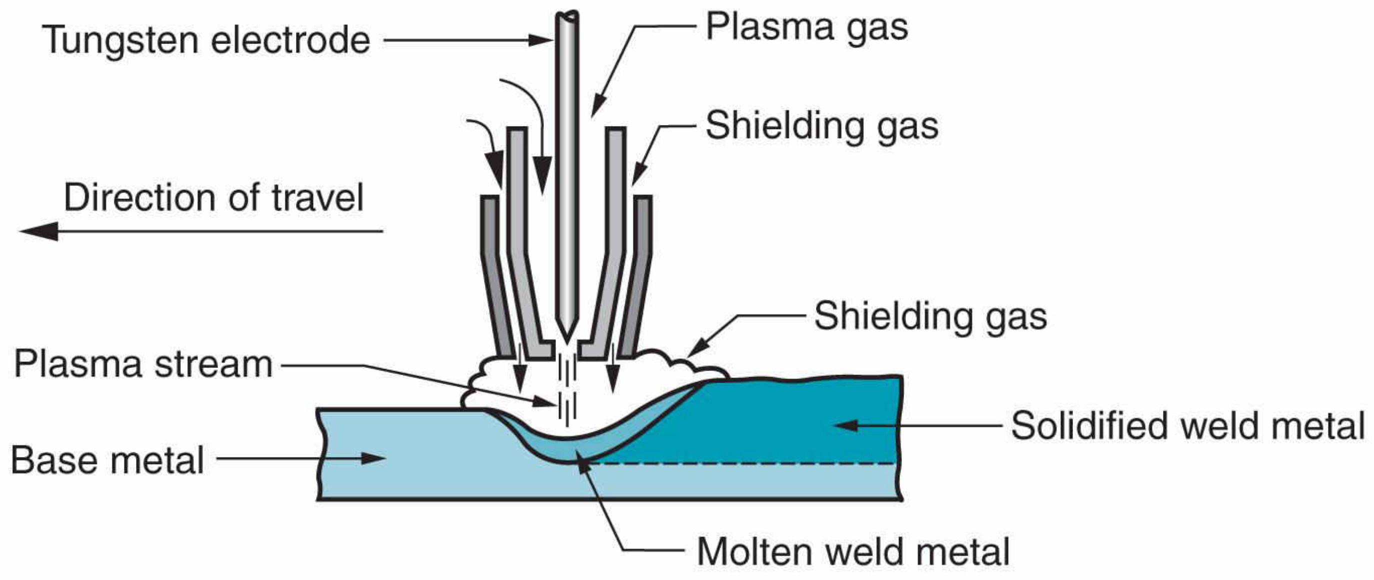
- Generally slower and more costly than consumable electrode AW processes

Plasma Arc Welding (PAW)

Special form of GTAW in which a constricted plasma arc is directed at weld area

- Tungsten electrode is contained in a nozzle that focuses a high velocity stream of inert gas (argon) into arc region to form a high velocity, intensely hot plasma arc stream
- Temperatures in PAW reach 28,000°C (50,000°F), due to constriction of arc, producing a plasma jet of small diameter and very high energy density

Plasma Arc Welding



Advantages and Disadvantages of PAW

Advantages:

- Good arc stability and excellent weld quality
- Better penetration control than other AW processes
- High travel speeds
- Can be used to weld almost any metals

Disadvantages:

- High equipment cost
- Larger torch size than other AW processes
  - Tends to restrict access in some joints

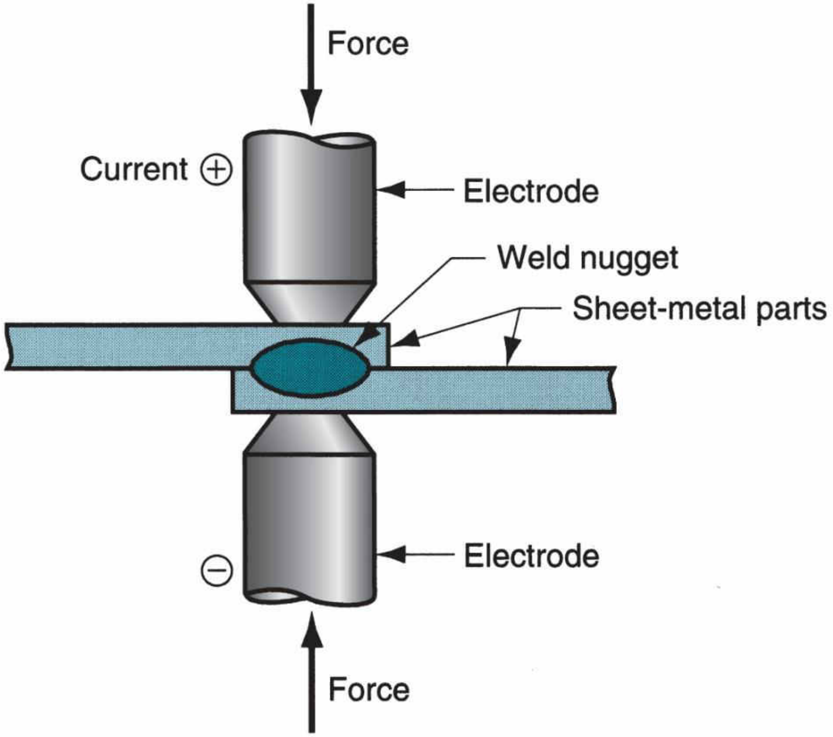
Resistance Welding (RW)

A group of fusion welding processes that use a combination of heat and pressure to accomplish coalescence

- Heat generated by electrical resistance to current flow at junction to be welded
- Principal RW process is resistance spot welding (RSW)

Resistance Welding

- Resistance welding, showing components in spot welding, the main process in the RW group



Components in Resistance Spot Welding

- Parts to be welded (usually sheet metal)
- Two opposing electrodes
- Means of applying pressure to squeeze parts between electrodes
- Power supply from which a controlled current can be applied for a specified time duration

Advantages and Drawbacks of Resistance Welding

Advantages:

- No filler metal required
- High production rates possible
- Lends itself to mechanization and automation
- Lower operator skill level than for arc welding
- Good repeatability and reliability

Disadvantages:

- High initial equipment cost
- Limited to lap joints for most RW processes

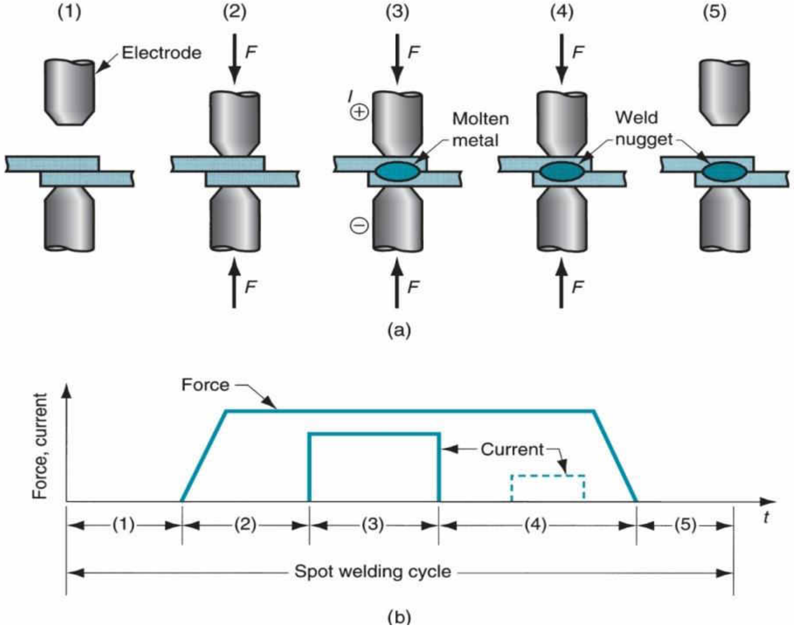
Resistance Spot Welding (RSW)

Resistance welding process in which fusion of faying surfaces of a lap joint is achieved at one location by opposing electrodes

- Used to join sheet metal parts
- Widely used in mass production of automobiles, metal furniture, appliances, and other sheet metal products
  - Typical car body has ~ 10,000 spot welds
  - Annual production of automobiles in the world is measured in tens of millions of units

Spot Welding Cycle

- (a) Spot welding cycle
- (b) Plot of force and current
- Cycle: (1) parts inserted between electrodes, (2) electrodes close, (3) current on, (4) current off, (5) electrodes opened

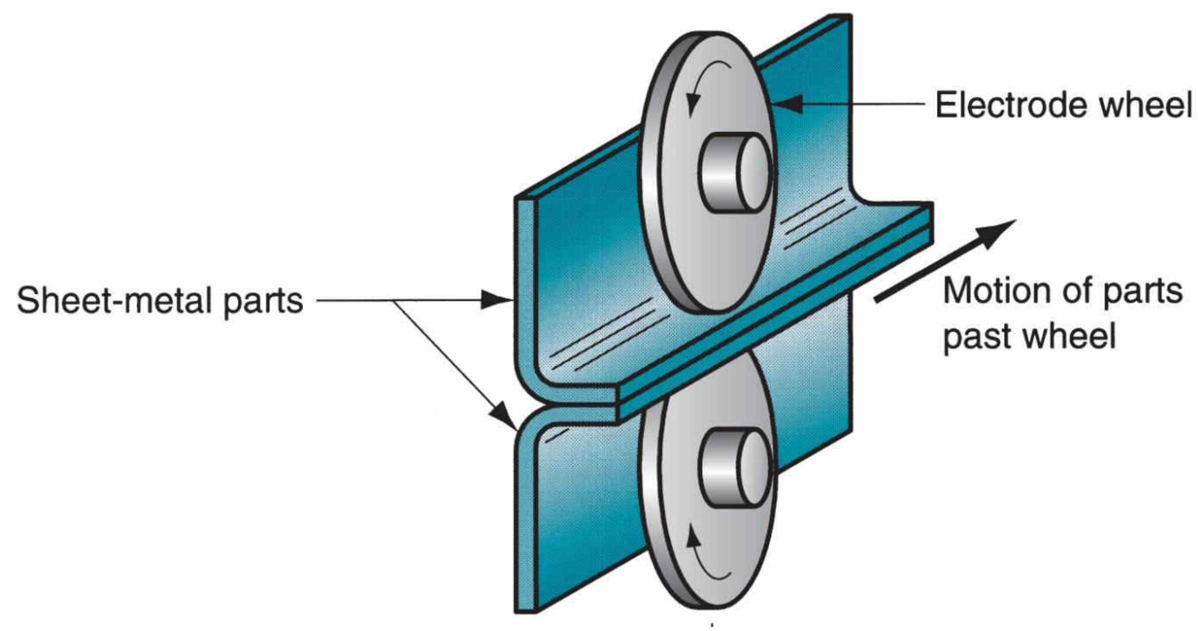


Resistance Seam Welding (RSEW)

Uses rotating wheel electrodes to produce a series of overlapping spot welds along lap joint

- Can produce air-tight joints
- Applications:
  - Gasoline tanks
  - Automobile mufflers
  - Various sheet metal containers

Resistance Seam Welding



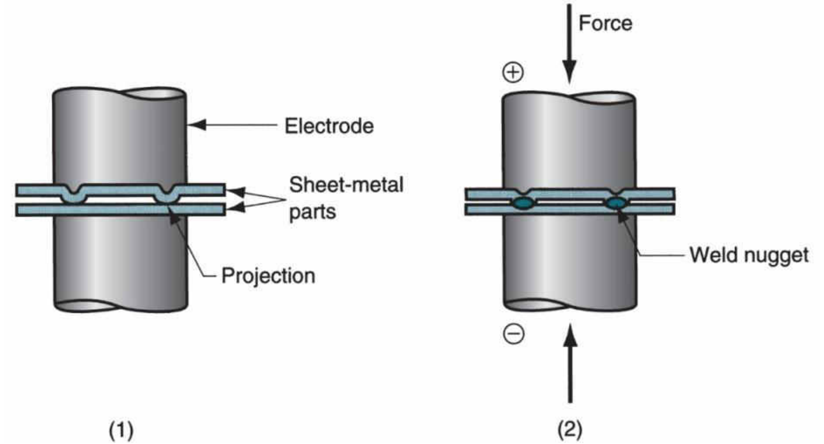
Resistance Projection Welding (RPW)

A resistance welding process in which coalescence occurs at one or more small contact points on the parts

- Contact points determined by design of parts to be joined
  - May consist of projections, embossments, or localized intersections of parts

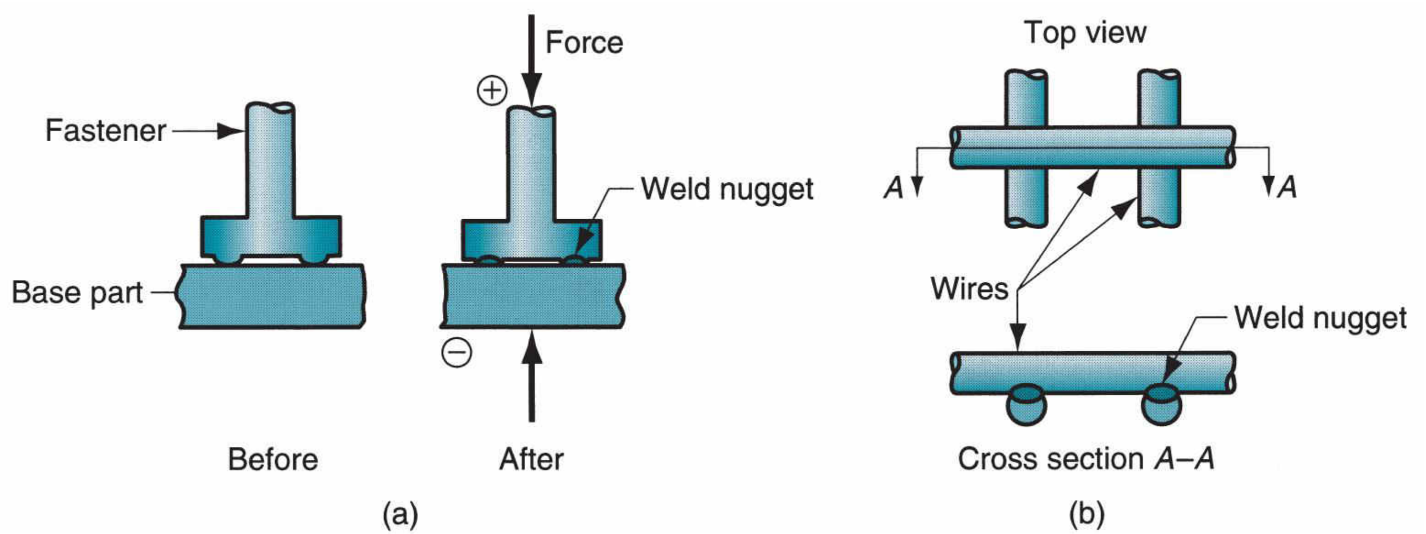
Resistance Projection Welding

- (1) Start of operation, contact between parts is at projections; (2) when current is applied, weld nuggets similar to spot welding are formed at the projections



Other Resistance Projection Welding Operations

- (a) Welding of fastener on Sheetmetal and (b) cross-wire welding



Oxyfuel Gas Welding (OFW)

Group of fusion welding operations that burn various fuels mixed with oxygen

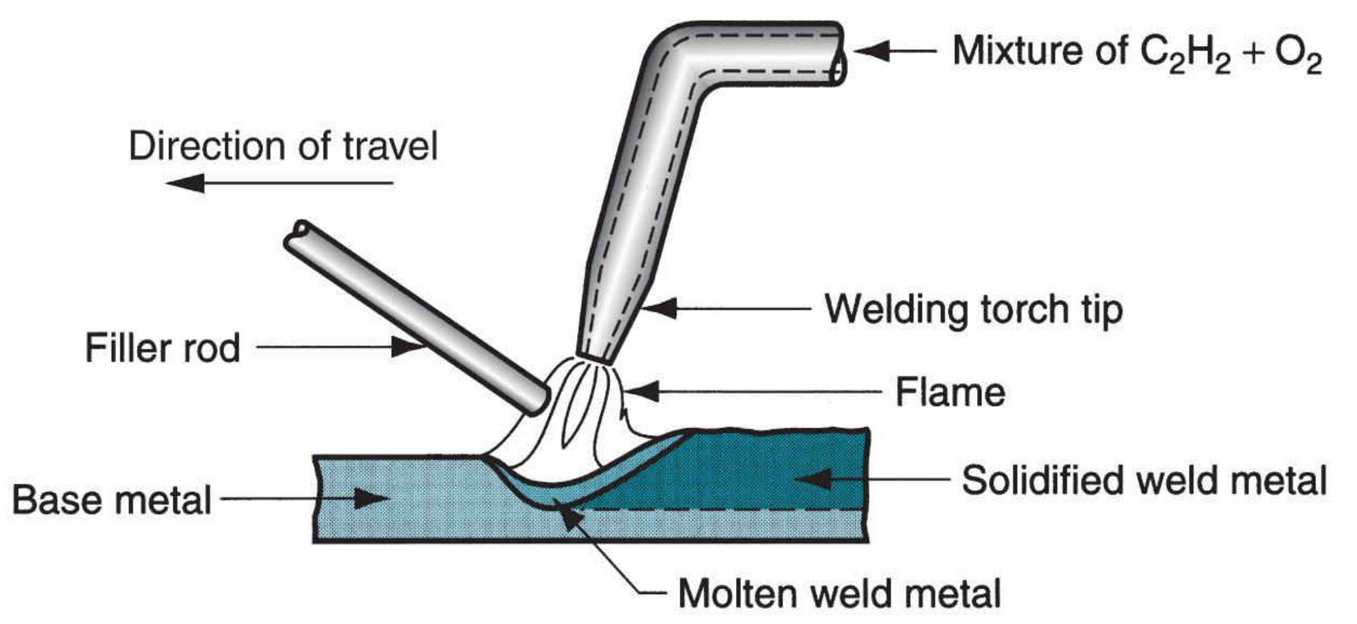
- OFW employs several types of gases, which is the primary distinction among the members of this group
- Oxyfuel gas is also used in flame cutting torches to cut and separate metal plates and other parts
- Most important OFW process is oxyacetylene welding

Oxyacetylene Welding (OAW)

Fusion welding performed by a high temperature flame from combustion of acetylene and oxygen

- Flame is directed by a welding torch
- Filler metal is sometimes added
  - Composition must be similar to base metal
  - Filler rod often coated with *flux* to clean surfaces and prevent oxidation

Oxyacetylene Welding



Acetylene (C_2H_2)

- Most popular fuel among OFW group because it is capable of higher temperatures than any other
  - Up to 3480°C (6300°F)
- Two stage reaction of acetylene and oxygen:
  - First stage reaction (inner cone of flame)

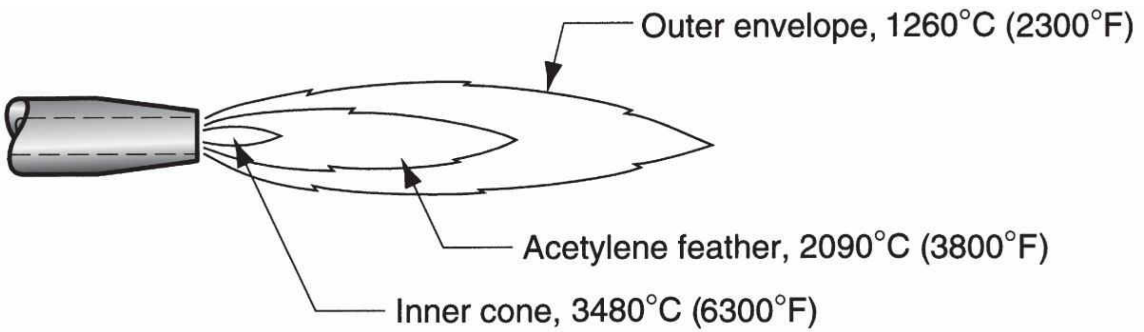
    $$C_2H_2 + O_2 \rightarrow 2CO + H_2 + heat$$

  - Second stage reaction (outer envelope)

    $$2CO + H_2 + 1.5O_2 \rightarrow 2CO_2 + H_2O + heat$$

Oxyacetylene Torch

- Maximum temperature reached at tip of inner cone, while outer envelope spreads out and shields work surface from atmosphere
- Shown below is neutral flame of oxyacetylene torch indicating temperatures achieved



Safety Issue in OAW

- Together, acetylene and oxygen are highly flammable
- C_2H_2 is colorless and odorless
  - It is therefore processed to have characteristic garlic odor

OAW Safety Issue

- C_2H_2 is physically unstable at pressures much above 15 lb./in^2 (about 1 atm)
  - Storage cylinders are packed with porous filler material saturated with acetone (CH_3COCH_3)
  - Acetone dissolves about 25 times its own volume of acetylene
- Different screw threads are standard on C_2H_2 and O_2 cylinders and hoses to avoid accidental connection of wrong gases

Alternative Gases for OFW

- Methylacetylene-Propadiene (MAPP)
- Hydrogen
- Propylene
- Propane
- Natural Gas

Other Fusion Welding Processes

FW processes that cannot be classified as arc, resistance, or oxyfuel welding

- Use unique technologies to develop heat for melting
- Applications are typically unique
- Processes include:
  - Electron beam welding
  - Laser beam welding
  - Electroslag welding
  - Thermit welding

Electron Beam Welding (EBW)

Fusion welding process in which heat for welding is provided by a highly-focused, high-intensity stream of electrons striking work surface

- Electron beam gun operates at:
  - High voltage (e.g., 10 to 150 kV typical) to accelerate electrons
  - Beam currents are low (measured in milliamps)
- Power in EBW not exceptional, but power density is

EBW Vacuum Chamber

- When first developed, EBW had to be carried out in a vacuum chamber to minimize disruption of electron beam by air molecules
  - Serious inconvenience in production
    - Pumpdown time can take as long as an hour

Three Vacuum Levels in EBW

1. High-vacuum welding – welding in same vacuum chamber as beam generation to produce highest quality weld
2. Medium-vacuum welding – welding in separate chamber but partial vacuum reduces pump-down time
3. Non-vacuum welding – welding done at or near atmospheric pressure, with work positioned close to electron beam generator - requires vacuum divider to separate work from beam generator

EBW Advantages and Disadvantages of EBW

Advantages:

- High-quality welds, deep and narrow profiles
- Limited heat affected zone, low thermal distortion
- No flux or shielding gases needed

Disadvantages:

- High equipment cost
- Precise joint preparation & alignment required
- Vacuum chamber required
- Safety concern: EBW generates x-rays

Laser Beam Welding (LBW)

Fusion welding process in which coalescence is achieved by energy of a highly concentrated, coherent light beam focused on joint

- LBW normally performed with shielding gases to prevent oxidation
- Filler metal not usually added
- High power density in small area
  - So LBW often used for small parts

Comparison: LBW vs. EBW

- No vacuum chamber required for LBW
- No x-rays emitted in LBW
- Laser beams can be focused and directed by optical lenses and mirrors
- LBW not capable of the deep welds and high depth-to-width ratios of EBW
  - Maximum LBW depth = ~ 19 mm (3/4 in), whereas EBW depths = 50 mm (2 in)

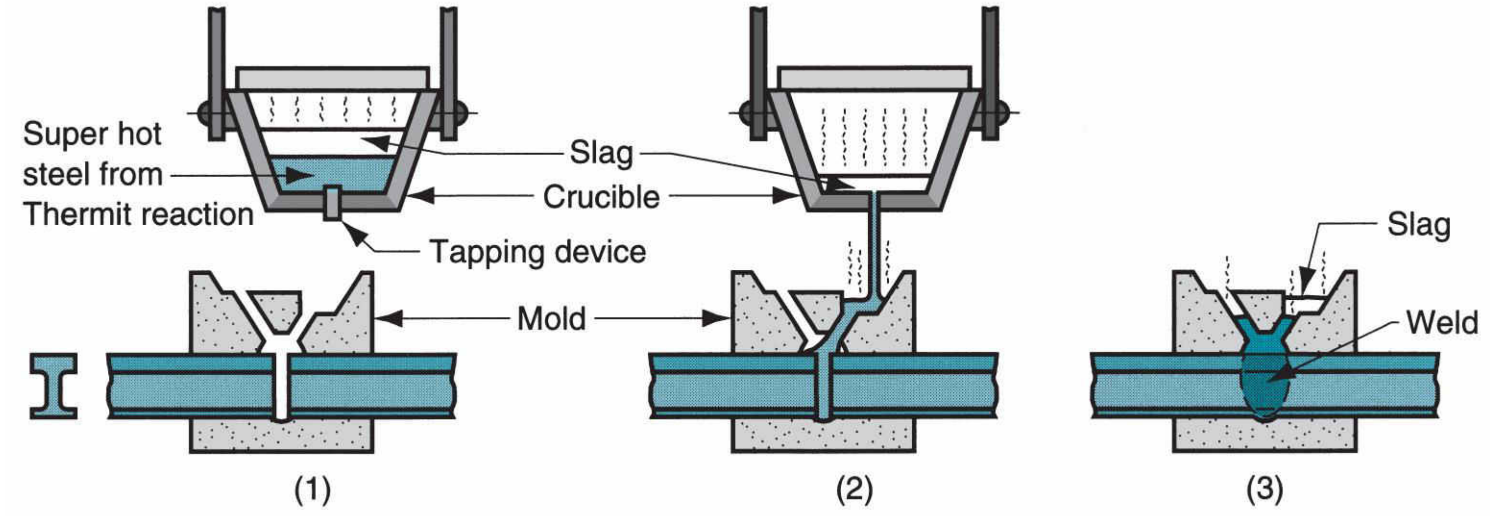
Thermit Welding (TW)

FW process in which heat for coalescence is produced by superheated molten metal from the chemical reaction of thermite

- *Thermite* = mixture of Al and Fe_3O_4 fine powders that produce an exothermic reaction when ignited
- Also used for incendiary bombs
- Filler metal obtained from liquid metal
- Process used for joining, but has more in common with casting than welding

Thermit Welding

- (1) Thermit ignited; (2) crucible tapped, superheated metal flows into mold; (3) metal solidifies to produce weld joint



TW Applications

- Joining of railroad rails
- Repair of cracks in large steel castings and forgings
- Weld surface is often smooth enough that no finishing is required

Solid State Welding (SSW)

- Coalescence of part surfaces is achieved by:
  - Pressure alone, or
  - Heat and pressure
    - If both heat and pressure are used, heat is not enough to melt work surfaces
  - For some SSW processes, time is also a factor
- No filler metal is added
- Each SSW process has its own way of creating a bond at the faying surfaces

Success Factors in SSW

- Essential factors for a successful solid state weld are that the two faying surfaces must be:
  - Very clean
  - In very close physical contact with each other to permit atomic bonding

SSW Advantages over FW Processes

- If no melting, then no heat affected zone, so metal around joint retains original properties
- Many SSW processes produce welded joints that bond the entire contact interface between two parts rather than at distinct spots or seams
- Some SSW processes can be used to bond dissimilar metals, without concerns about relative melting points, thermal expansions, and other problems that arise in FW

Solid State Welding Processes

- Forge welding
- Cold welding
- Roll welding
- Hot pressure welding
- Diffusion welding
- Explosion welding
- Friction welding
- Ultrasonic welding

Forge Welding

Welding process in which components to be joined are heated to hot working temperature range and then forged together by hammering or similar means

- Historic significance in development of manufacturing technology
  - Process dates from about 1000 B.C., when blacksmiths learned to weld two pieces of metal
- Of minor commercial importance today except for its variants

Cold Welding (CW)

SSW process done by applying high pressure between clean contacting surfaces at room temperature

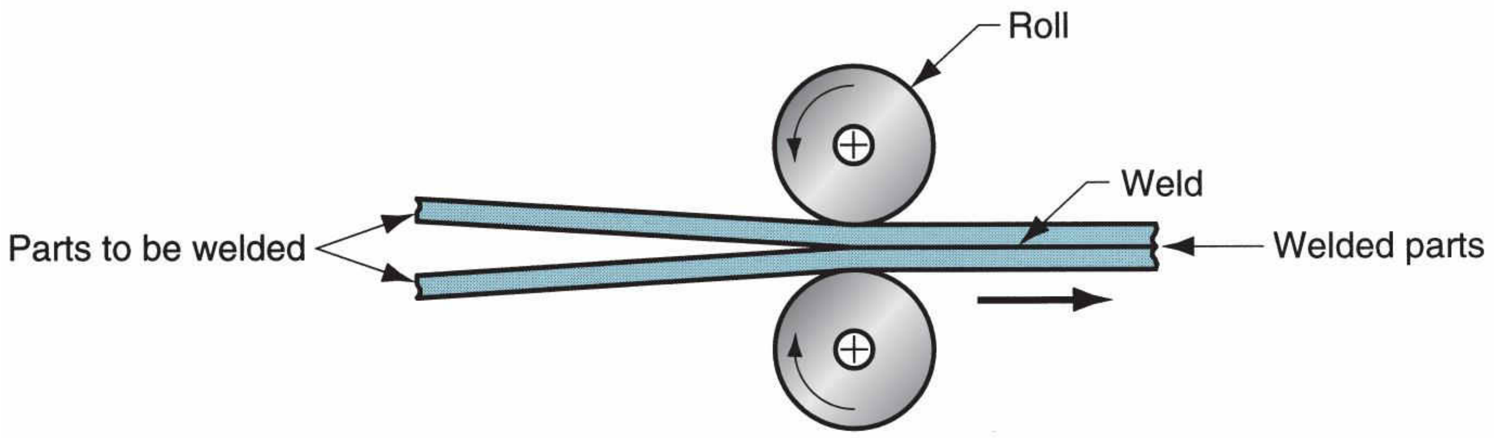
- Cleaning usually done by degreasing and wire brushing immediately before joining
- No heat is applied, but deformation raises work temperature
- At least one of the metals, preferably both, must be very ductile
  - Soft aluminum and copper suited to CW
- Applications: making electrical connections

Roll Welding (ROW)

SSW process in which pressure sufficient to cause coalescence is applied by means of rolls, either with or without external heat

- Variation of either forge welding or cold welding, depending on whether heating of workparts is done prior to process
  - If no external heat, called *cold roll welding*
  - If heat is supplied, *hot roll welding*

Roll Welding



Roll Welding Applications

- Cladding stainless steel to mild or low alloy steel for corrosion resistance
- Bimetallic strips for measuring temperature
- "Sandwich" coins for U.S mint

Diffusion Welding (DFW)

SSW process uses heat and pressure, usually in a controlled atmosphere, with sufficient time for diffusion and coalescence to occur

- Temperatures $\leq 0.5\ T_m$
- Plastic deformation at surfaces is minimal
- Primary coalescence mechanism is solid state diffusion
- Limitation: time required for diffusion can range from seconds to hours

DFW Applications

- Joining of high-strength and refractory metals in aerospace and nuclear industries
- Can be used to join either similar and dissimilar metals
  - For joining dissimilar metals, a filler layer of different metal is often sandwiched between base metals to promote diffusion

Explosion Welding (EXW)

SSW process in which rapid coalescence of two metallic surfaces is caused by the energy of a detonated explosive

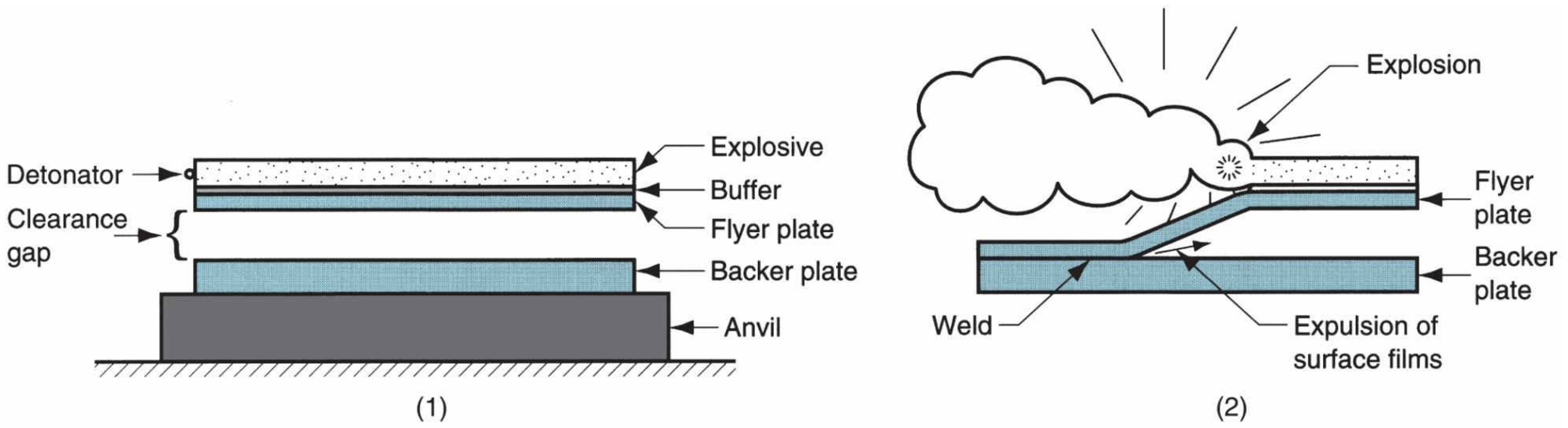
- No filler metal used
- No external heat applied
- No diffusion occurs - time is too short
- Bonding is metallurgical, combined with mechanical interlocking that results from a rippled or wavy interface between the metals

Explosive Welding

Commonly used to bond two dissimilar metals, in particular to clad one metal on top of a base metal over large areas

Explosive Welding

- Commonly used to bond two dissimilar metals, e.g., to clad one metal on top of a base metal over large areas
- (1) Setup in parallel configuration, and (2) during detonation of the explosive charge



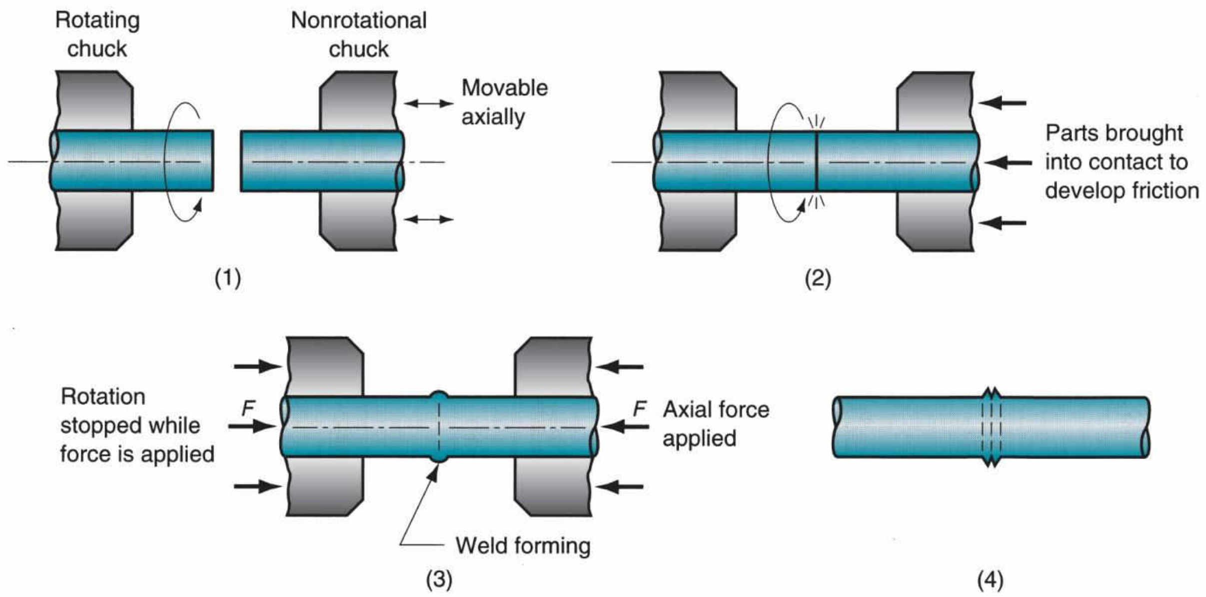
Friction Welding (FRW)

SSW process in which coalescence is achieved by frictional heat combined with pressure

- When properly carried out, no melting occurs at faying surfaces
- No filler metal, flux, or shielding gases normally used
- Process yields a narrow HAZ
- Can be used to join dissimilar metals
- Widely used commercial process, amenable to automation and mass production

Friction Welding

- (1) Rotating part, no contact; (2) parts brought into contact to generate friction heat; (3) rotation stopped and axial pressure applied; and (4) weld created



Applications and Limitations of Friction Welding

Applications:

- Shafts and tubular parts
- Industries: automotive, aircraft, farm equipment, petroleum and natural gas

Limitations:

- At least one of the parts must be rotational
- Flash must usually be removed (extra operation)
- Upsetting reduces the part lengths (which must be taken into consideration in product design)

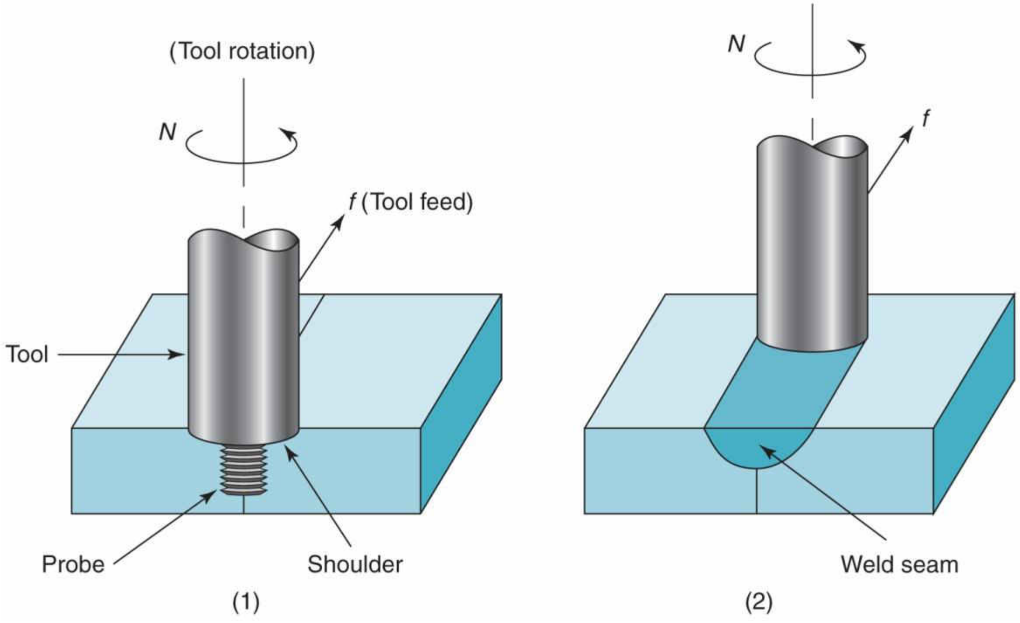
Friction Stir Welding (FSW)

SSW process in which a rotating tool is fed along a joint line between two workpieces, generating friction heat and mechanically stirring the metal to form the weld seam

- Distinguished from FRW because heat is generated by a separate wear-resistant tool rather than the parts
- Applications: butt joints in large aluminum parts in aerospace, automotive, and shipbuilding

Friction Stir Welding

- (1) Rotating tool just before entering work, and (2) partially completed weld seam

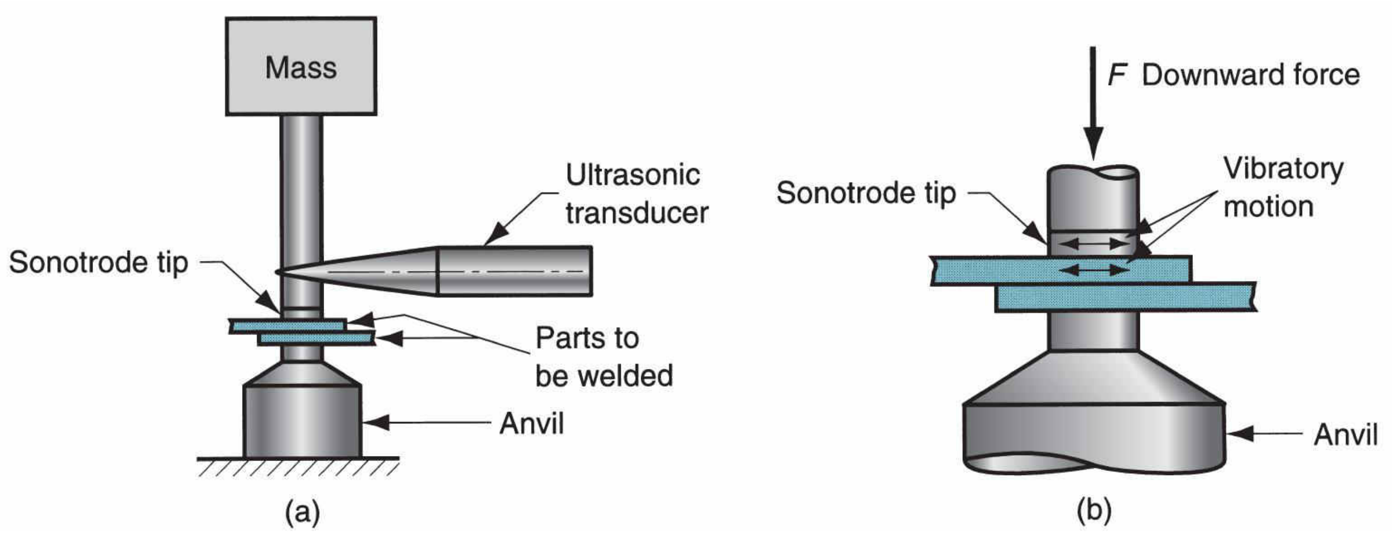


Advantages and Disadvantages of Friction Stir Welding

- Advantages
  - Good mechanical properties of weld joint
  - Avoids toxic fumes, warping, and shielding issues
  - Little distortion or shrinkage
  - Good weld appearance
- Disadvantages
  - An exit hole is produce when tool is withdrawn
  - Heavy duty clamping of parts is required

Ultrasonic Welding

- (a) General setup for a lap joint; and (b) close-up of weld area



USW Applications

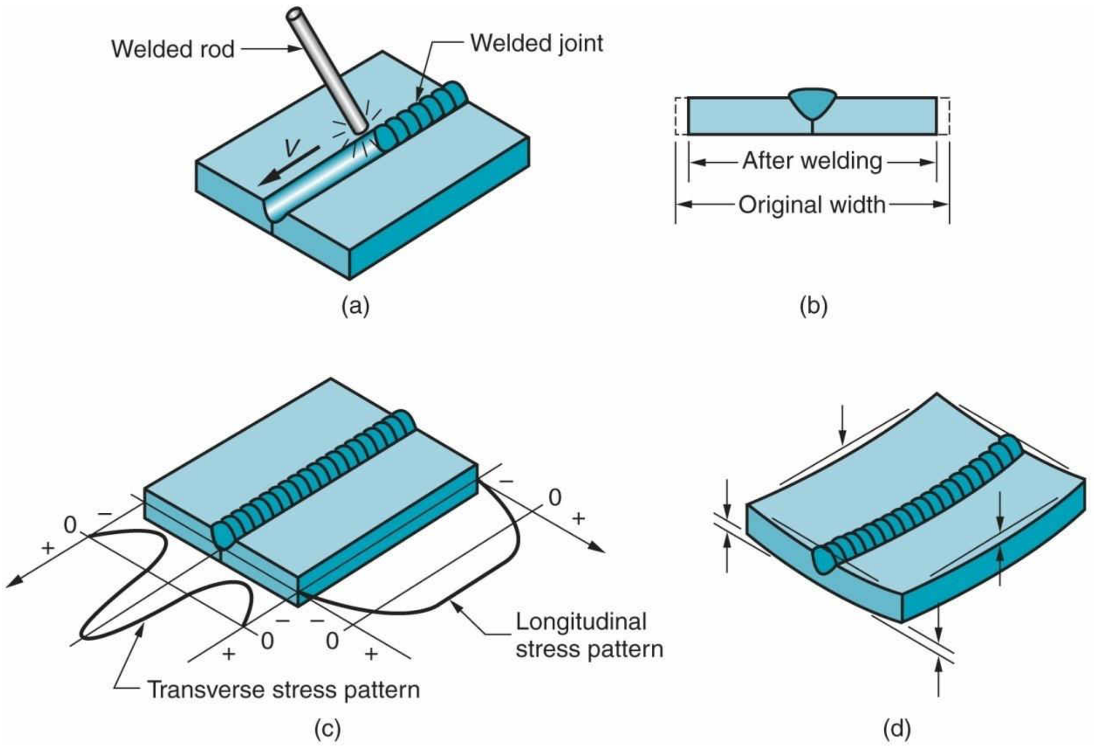
- Wire terminations and splicing in electrical and electronics industry
  - Eliminates need for soldering
- Assembly of aluminum sheet metal panels
- Welding of tubes to sheets in solar panels
- Assembly of small parts in automotive industry

Residual Stresses and Distortion

- Rapid heating and cooling in localized regions during FW result in thermal expansion and contraction that cause residual stresses
- These stresses, in turn, cause distortion and warpage
- Situation in welding is complicated because:
  - Heating is very localized
  - Melting of base metals in these regions
  - Location of heating and melting is in motion (at least in AW)

Residual Stresses and Distortion

- (a) Butt welding two plates
- (b) Shrinkage
- (c) Residual stress patterns
- (d) Likely warping of weldment



Techniques to Minimize Warpage

- Welding fixtures to physically restrain parts
- Heat sinks to rapidly remove heat
- Tack welding at multiple points along joint to create a rigid structure prior to seam welding
- Selection of welding conditions (speed, amount of filler metal used, etc.) to reduce warpage
- Preheating base parts
- Stress relief heat treatment of welded assembly
- Proper design of weldment

Welding Defects

- Cracks
- Cavities
- Solid inclusions
- Imperfect shape or unacceptable contour
- Incomplete fusion
- Miscellaneous defects

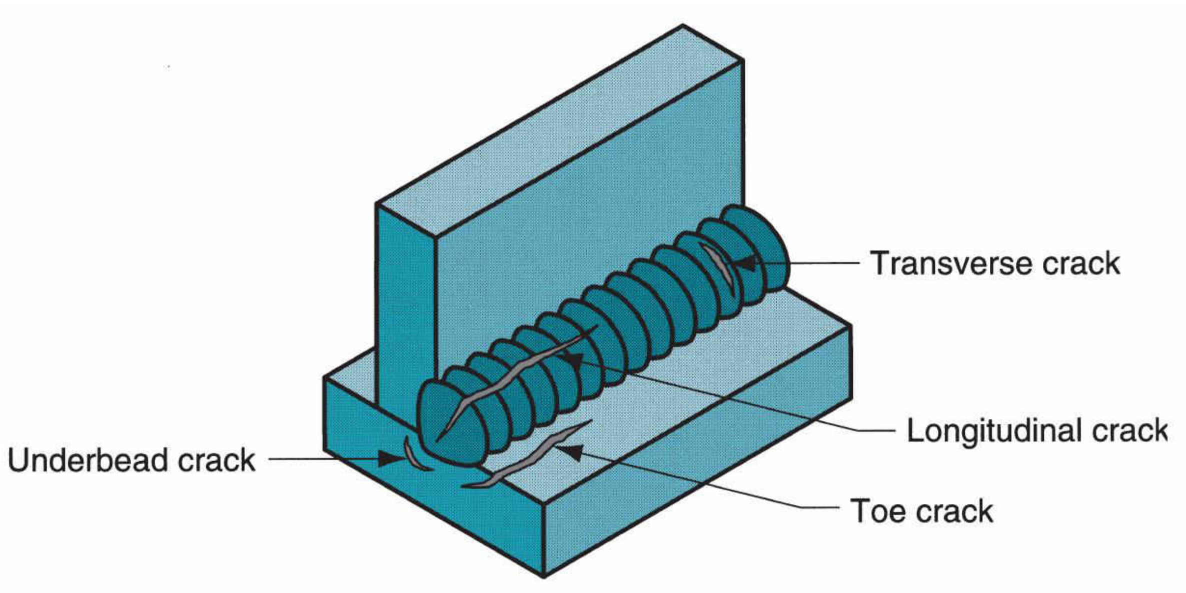
Welding Cracks

Fracture-type interruptions either in weld or in base metal adjacent to weld

- Serious defect because it is a discontinuity in the metal that significantly reduces strength
- Caused by embrittlement or low ductility of weld and/or base metal combined with high restraint during contraction
- In general, this defect must be repaired

Welding Cracks

- Various forms of welding cracks



Cavities

Two defect types, similar to defects found in castings:

1. Porosity - small voids in weld metal formed by gases entrapped during solidification
   - Caused by inclusion of atmospheric gases, sulfur in weld metal, or surface contaminants
2. Shrinkage voids - cavities formed by shrinkage during solidification

Solid Inclusions

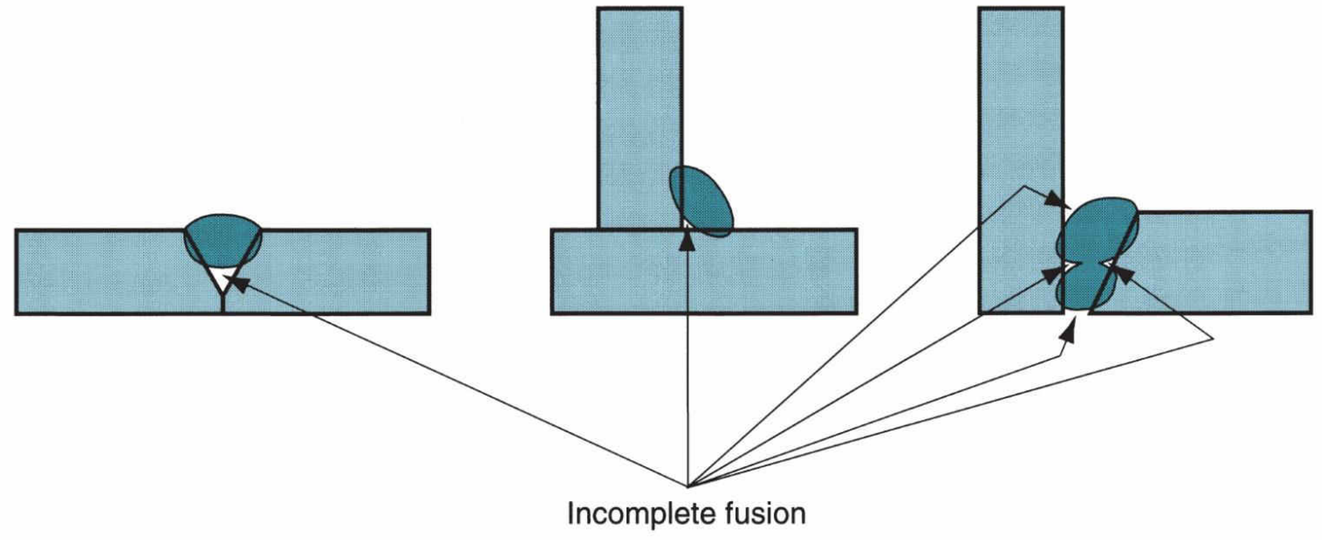
Nonmetallic material entrapped in weld metal

- Most common form is slag inclusions generated during AW processes that use flux
  - Instead of floating to top of weld pool, globules of slag become encased during solidification
- Other forms: metallic oxides that form during welding of certain metals such as aluminum, which normally has a surface coating of Al_2O_3

Incomplete Fusion

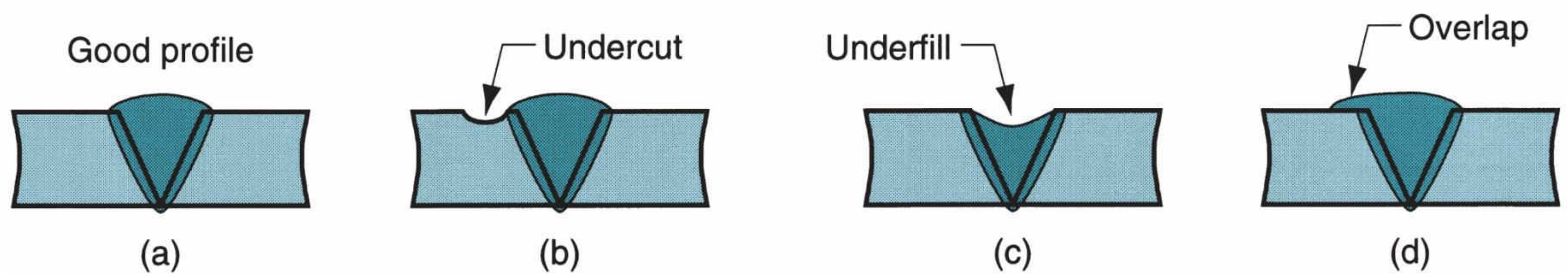
A weld bead in which fusion has not occurred throughout entire cross section of joint

- Several forms of incomplete fusion are shown below



Weld Profile in AW

- (a) Desired profile for single V-groove weld joint, (b) undercut - portion of base metal melted away, (c) underfill - depression in weld below adjacent base metal surface, and (d) overlap - weld metal spills beyond joint onto part surface but no fusion occurs



Inspection and Testing Methods

- Visual inspection
- Nondestructive evaluation
- Destructive testing

Visual Inspection

- Most widely used welding inspection method
- Human inspector visually examines for:
  - Conformance to dimensions, wWarpage
  - Cracks, cavities, incomplete fusion, and other surface defects
- Limitations:
  - Only surface defects are detectable
  - Welding inspector must also decide if additional tests are warranted

Nondestructive Evaluation (NDE) Tests

- Ultrasonic testing - high frequency sound waves through specimen to detect cracks and inclusions
- Radiographic testing - x-rays or gamma radiation provide photograph of internal flaws
- Dye-penetrant and fluorescent-penetrant tests - to detect small cracks and cavities at part surface
- Magnetic particle testing – iron filings sprinkled on surface reveal subsurface defects by distorting magnetic field in part

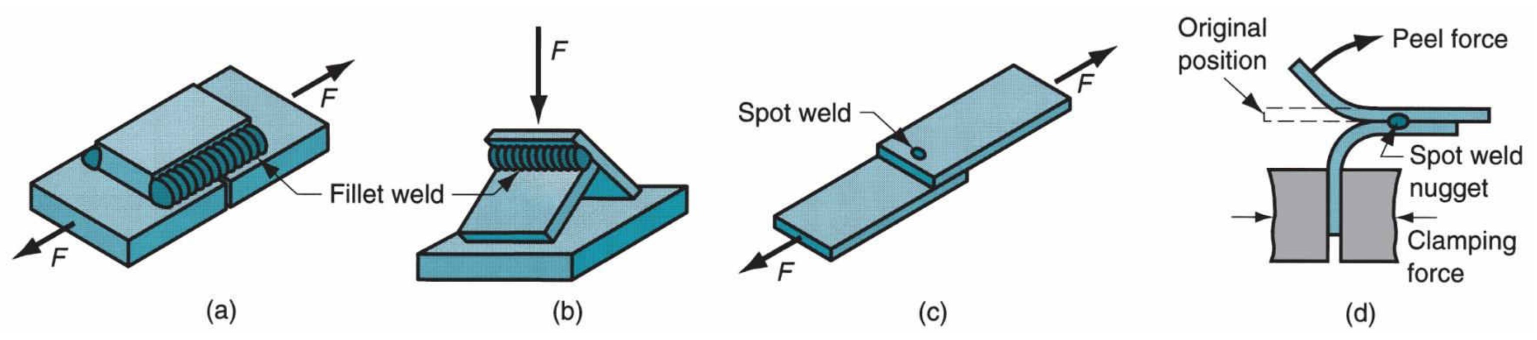
Destructive Testing

Tests in which weld is destroyed either during testing or to prepare test specimen

- Mechanical tests - purpose is similar to conventional testing methods such as tensile tests, shear tests, etc
- Metallurgical tests - preparation of metallurgical specimens (e.g., photomicrographs) of weldment to examine metallic structure, defects, extent and condition of heat affected zone, and similar phenomena

Mechanical Tests in Welding

- (a) Tension-shear test, (b) fillet break test, (c) tension-shear of spot weld, and (d) peel test for spot weld



Weldability

Capacity of a metal or combination of metals to be welded into a suitable structure, and for the resulting weld joint(s) to possess the required metallurgical properties to perform satisfactorily in intended service

- Good weldability characterized by:
  - Ease with which welding is accomplished
  - Absence of weld defects
  - Strength, ductility, and toughness in welded joint

Weldability Factors – Welding Process

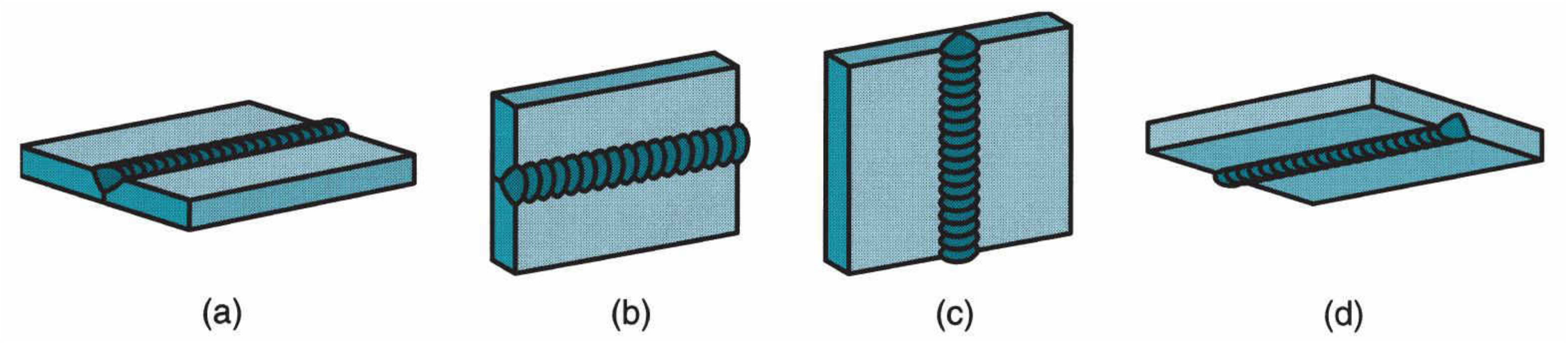
- Some metals or metal combinations can be readily welded by one process but are difficult to weld by others
  - Example: stainless steel readily welded by most AW and RW processes, but difficult to weld by OFW

Weldability Factors – Base Metal

- Some metals melt too easily; e.g., aluminum
- Metals with high thermal conductivity transfer heat away from weld, which causes problems; e.g., copper
- High thermal expansion and contraction in metal causes distortion problems
- Dissimilar metals pose problems in welding when their physical and/or mechanical properties are substantially different

Arc Welding Positions

- Welding positions defined here for groove welds: (a) flat, (b) horizontal, (c) vertical, and (d) overhead



Design Guidelines - RSW

- Low-carbon sheet steel up to 0.125 (3.2 mm) is ideal metal for RSW
- How additional strength and stiffness can be obtained in large flat sheet metal components
  - Spot welding reinforcing parts into them
  - Forming flanges and embossments
- Spot welded assembly must provide access for electrodes to reach welding area
- Sufficient overlap of sheet metal parts required for electrode tip to make proper contact

METAL CASTING PROCESSES

1. Sand Casting
2. Other Expendable Mold Casting Processes
3. Permanent Mold Casting Processes
4. Foundry Practice
5. Casting Quality
6. Metals for Casting
7. Product Design Considerations

- Sand casting weighing over 680 kg (1500 lb) for an air compressor frame (photo courtesy of Elkhart Foundry).

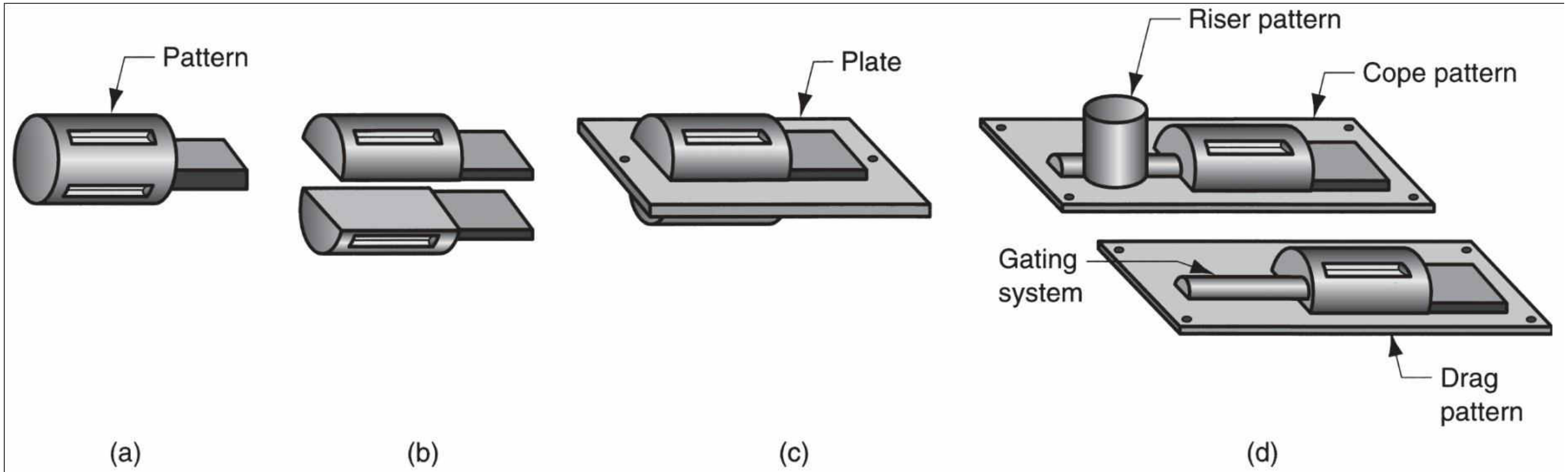
The Pattern

Full-sized model of part, slightly enlarged to account for shrinkage and machining allowances in the casting

- Pattern materials:
  - Wood - common material because it is easy to work, but it warps
  - Metal - more expensive to fabricate, but lasts longer
  - Plastic - compromise between wood and metal

Types of Patterns

- Types of patterns used in sand casting: (a) solid pattern, (b) split pattern, (c) match-plate pattern, (d) cope and drag pattern



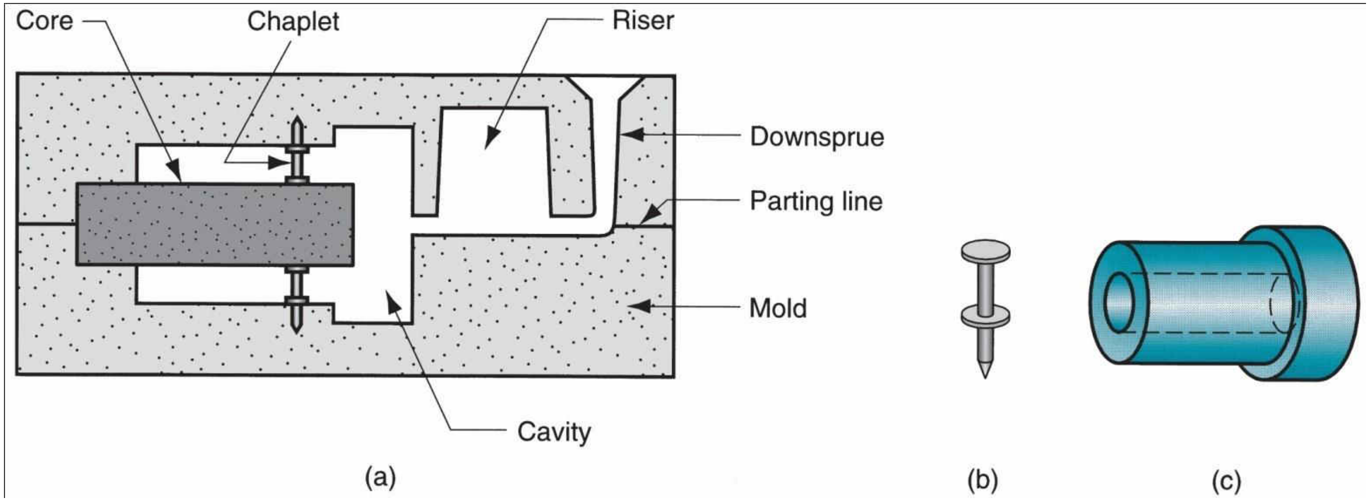
Core

Full-scale model of interior surfaces of part

- Inserted into mold cavity prior to pouring
- The molten metal flows and solidifies between the mold cavity and the core to form the casting's external and internal surfaces
- May require supports to hold it in position in the mold cavity during pouring, called *chaplets*

Core in Mold

- (a) Core held in place in the mold cavity by chaplets, (b) possible chaplet design, (c) casting



Foundry Sand

Silica (SiO_2) or silica mixed with other minerals

- Good refractory properties - for high temperatures
- Small grain size for better surface finish on cast part
- Large grain size is more permeable, allowing gases to escape during pouring
- Irregular grain shapes strengthen molds due to interlocking, compared to round grains
  - Disadvantage: interlocking reduces permeability

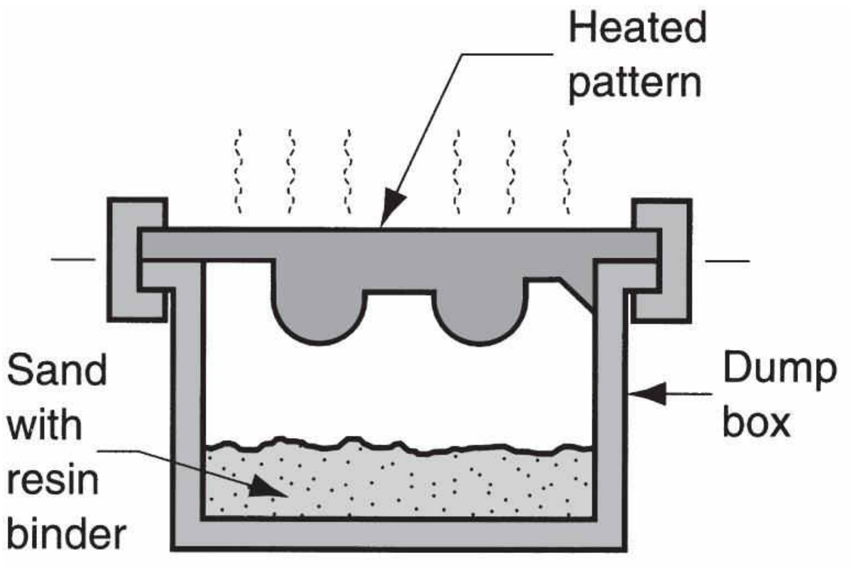
Other Expendable Mold Processes

- Shell Molding
- Vacuum Molding
- Expanded Polystyrene Process
- Investment Casting
- Plaster Mold and Ceramic Mold Casting

Shell Molding

Casting process in which the mold is a thin shell of sand held together by thermosetting resin binder

- Steps: (1) A metal pattern is heated and placed over a box containing sand mixed with thermosetting resin



(1)

Vacuum Molding

Uses sand mold held together by vacuum pressure rather than by a chemical binder

- The term "vacuum" refers to mold making rather than casting operation itself
- Developed in Japan around 1970

Investment Casting:
Advantages and Disadvantages

- Advantages:
  - Parts of great complexity and intricacy can be cast
  - Close dimensional control and good surface finish
  - Wax can usually be recovered for reuse
  - This is a net shape process
    - Additional machining is not normally required
- Disadvantages:
  - Many processing steps are required
  - Relatively expensive process

Permanent Mold Casting Processes

- Economic disadvantage of expendable mold casting:
  - A new mold is required for every casting
- In permanent mold casting, the mold is reused many times
- The processes include:
  - Basic permanent mold casting
  - Die casting
  - Centrifugal casting

The Basic Permanent Mold Process

Uses a metal mold constructed of two sections designed for easy, precise opening and closing

- Molds used for casting lower melting point alloys are commonly made of steel or cast iron
- Molds used for casting steel must be made of refractory material, due to the very high pouring temperatures

Die Casting

A permanent mold casting process in which molten metal is injected into mold cavity under high pressure

- Pressure is maintained during solidification, then mold is opened and part is removed
- Molds in this casting operation are called *dies*; hence the name die casting
- Use of high pressure to force metal into die cavity is what distinguishes this from other permanent mold processes

Die Casting Machines

- Designed to hold and accurately close two mold halves and keep them closed while liquid metal is forced into cavity
- Two main types:

1. Hot-chamber machine
2. Cold-chamber machine

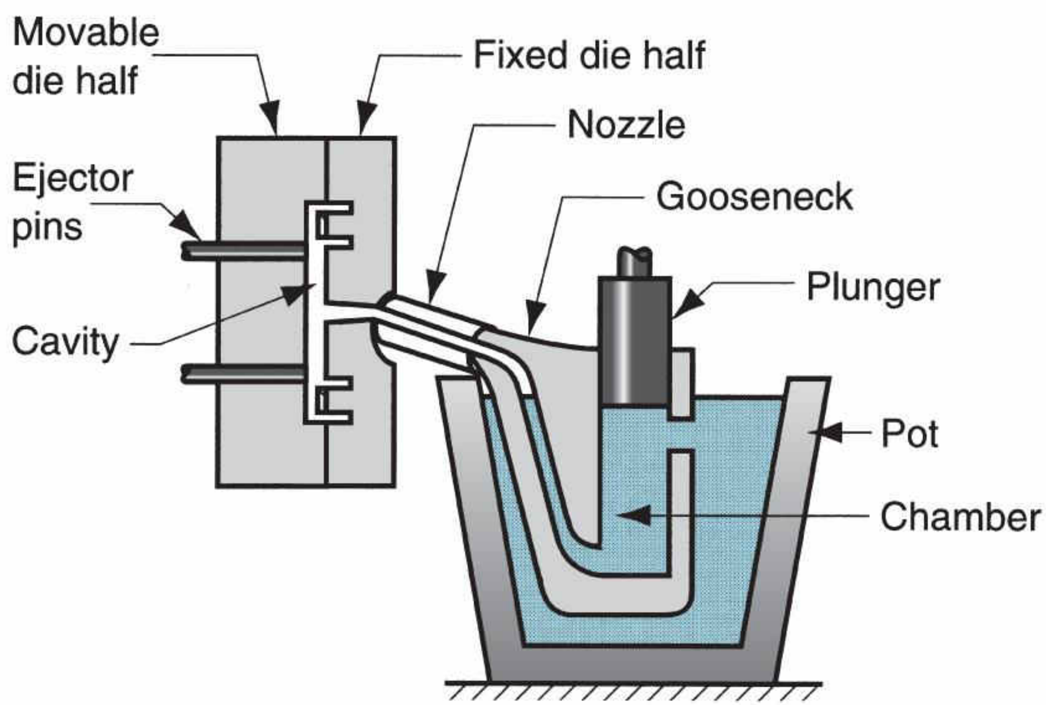
Hot-Chamber Die Casting

Metal is melted in a container, and a piston injects liquid metal under high pressure into the die

- High production rates
  - 500 parts per hour not uncommon
- Applications limited to low melting-point metals that do not chemically attack plunger and other mechanical components
- Casting metals: zinc, tin, lead, and magnesium

Hot-Chamber Die Casting

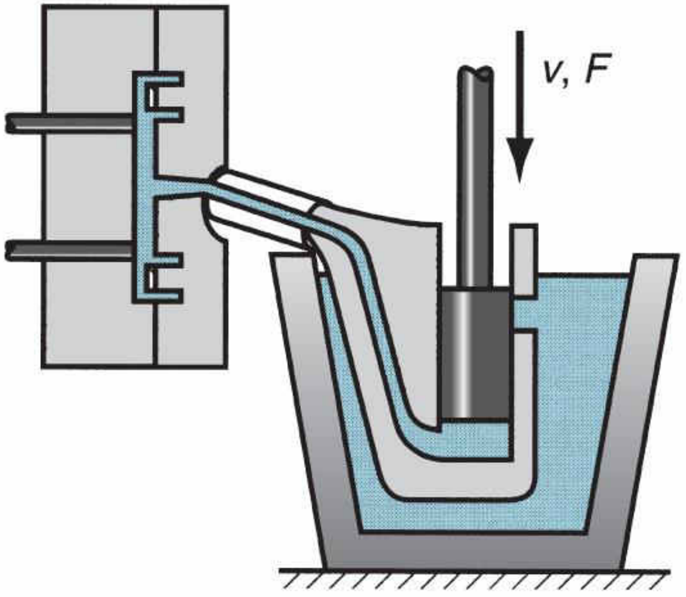
- Hot-chamber die casting cycle: (1) with die closed and plunger withdrawn, molten metal flows into the chamber



(1)

Hot-Chamber Die Casting

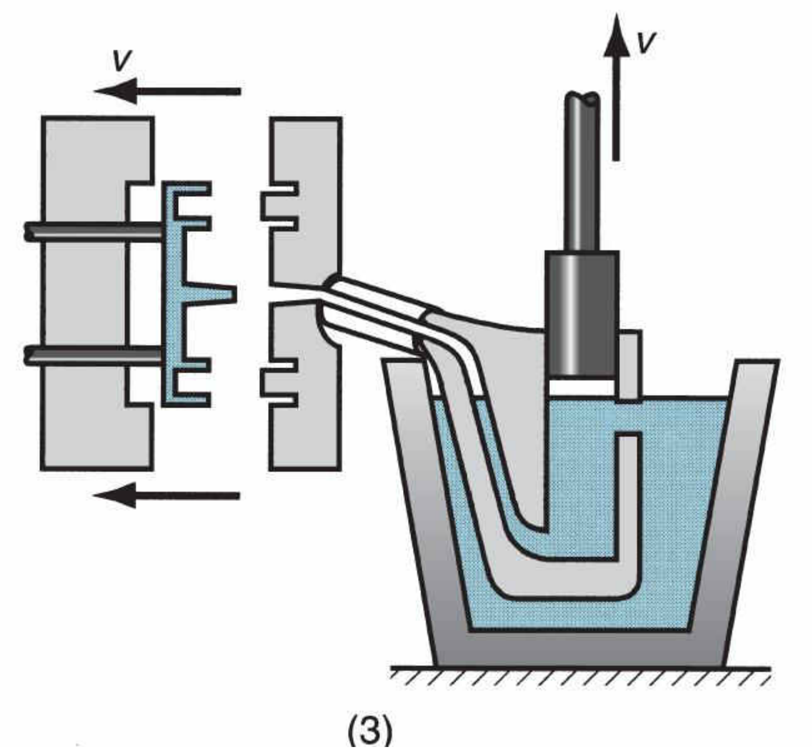
- (2) plunger forces metal in chamber to flow into die, maintaining pressure during cooling and solidification.



(2)

Hot-Chamber Die Casting

- (3) Plunger is withdrawn, die is opened, and casting is ejected



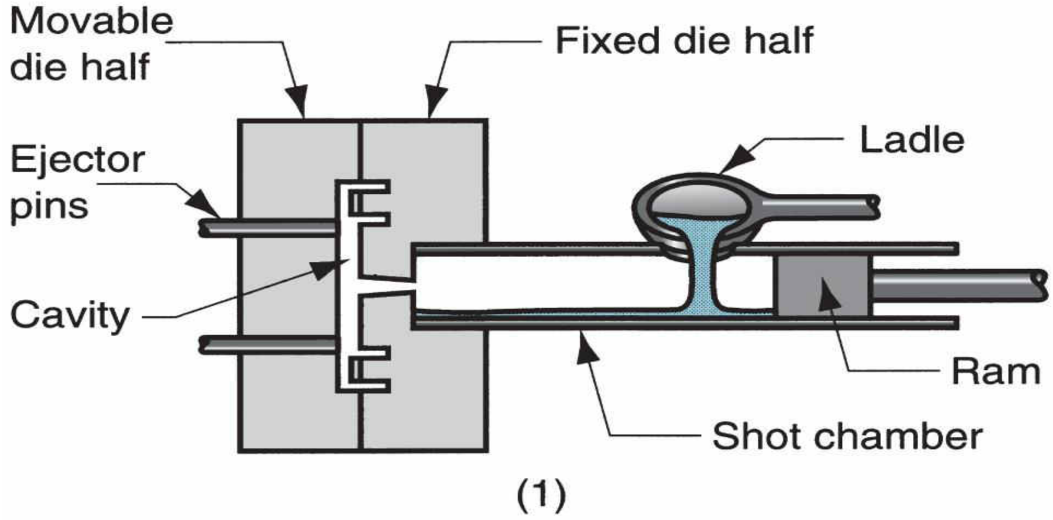
Cold-Chamber Die Casting Machine

Molten metal is poured into unheated chamber from external melting container, and a piston injects metal under high pressure into die cavity

- High production but not usually as fast as hot-chamber machines because of pouring step
- Casting metals: aluminum, brass, and magnesium alloys
- Advantages of hot-chamber process favor its use on low melting-point alloys (zinc, tin, lead)

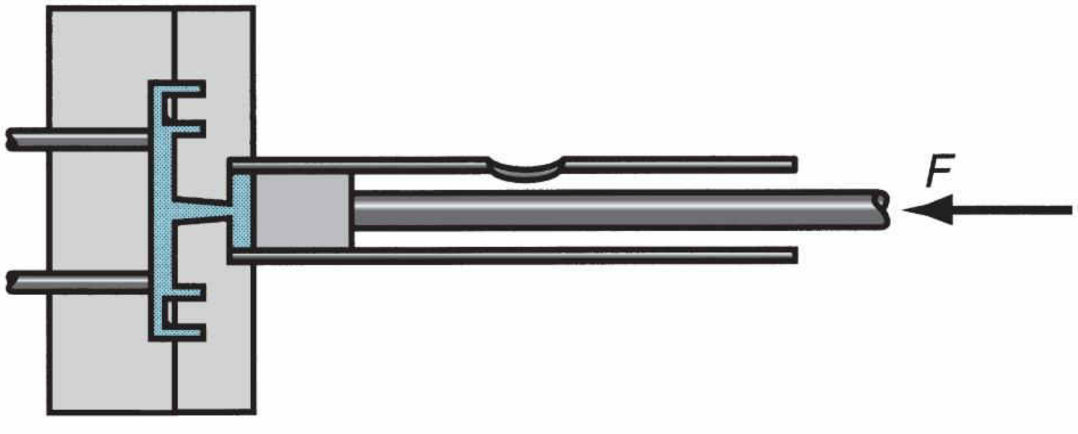
Cold-Chamber Die Casting Cycle

- (1) With die closed and ram withdrawn, molten metal is poured into the chamber



Cold-Chamber Die Casting Cycle

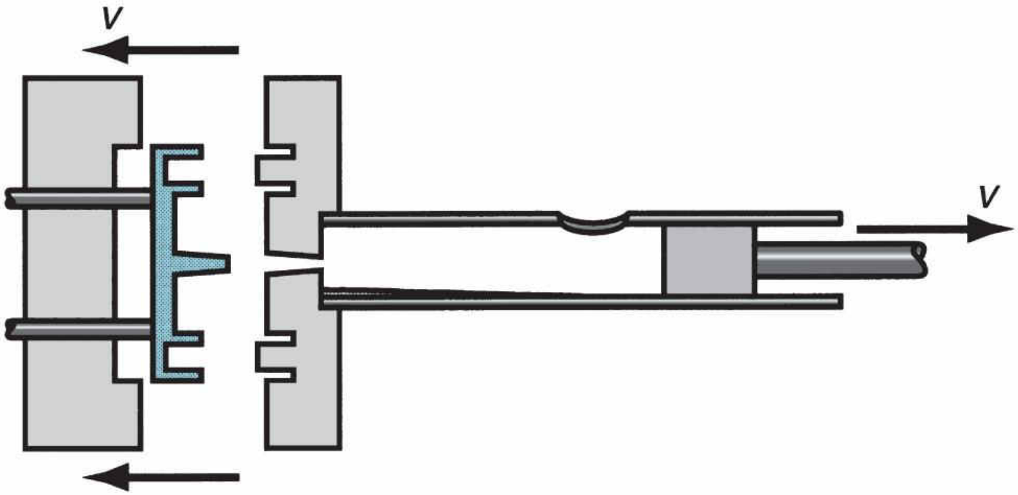
- (2) Ram forces metal to flow into die, maintaining pressure during cooling and solidification



(2)

Cold-Chamber Die Casting Cycle

- (3) Ram is withdrawn, die is opened, and part is ejected



(3)

Molds for Die Casting

- Usually made of tool steel, mold steel, or maraging steel
- Tungsten and molybdenum (good refractory qualities) used to die cast steel and cast iron
- Ejector pins required to remove part from die when it opens
- Lubricants must be sprayed onto cavity surfaces to prevent sticking

Die Casting: Advantages and Limitations

- Advantages:
  - Economical for large production quantities
  - Good accuracy and surface finish
  - Thin sections possible
  - Rapid cooling means small grain size and good strength in casting
- Disadvantages:
  - Generally limited to metals with low metal points
  - Part geometry must allow removal from die

Squeeze Casting

Combination of casting and forging in which a molten metal is poured into a preheated lower die, and the upper die is closed to create the mold cavity after solidification begins

- Differs from usual closed-mold casting processes in which die halves are closed before introduction of the molten metal
- Compared to conventional forging, pressures are less and finer surface details can be achieved

Semi-Solid Metal Casting

Family of net-shape and near net-shape processes performed on metal alloys at temperatures between liquidus and solidus

- Thus, the alloy is a mixture of solid and molten metals during casting (mushy state)
  - To flow properly, the mixture must consist of solid metal globules in a liquid
    - Achieved by stirring the mixture to prevent dendrite formation

Semi-Solid Metal Casting: Advantages

- Complex part geometries
- Thin part walls possible
- Close tolerances
- Zero or low porosity, resulting in high strength of the casting

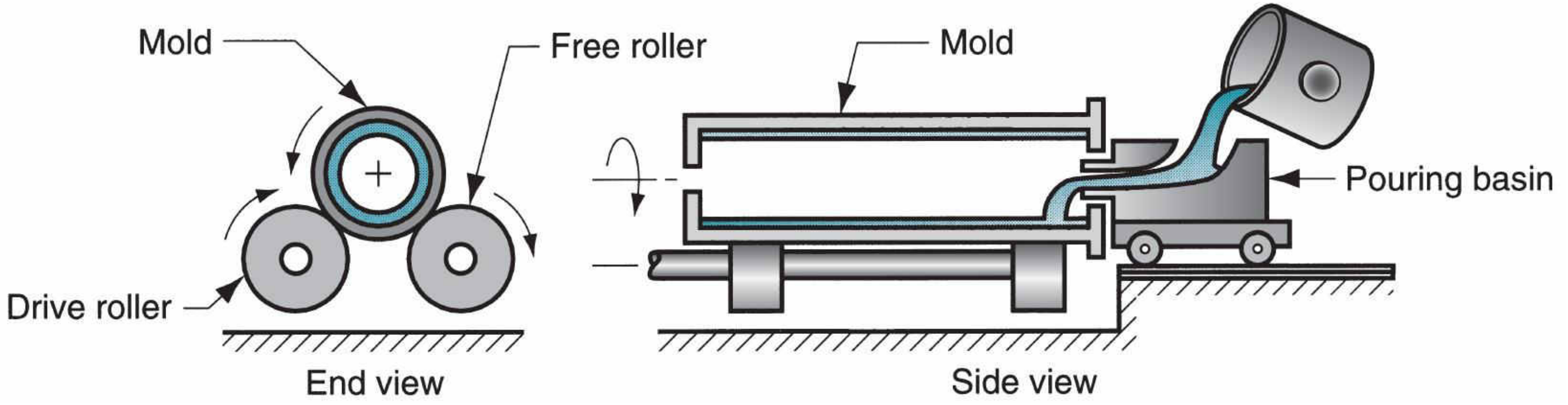
Centrifugal Casting

A family of casting processes in which the mold is rotated at high speed so centrifugal force distributes molten metal to outer regions of die cavity

- The group includes:
  - True centrifugal casting
  - Semicentrifugal casting
  - Centrifuge casting

True Centrifugal Casting

- Setup for true centrifugal casting

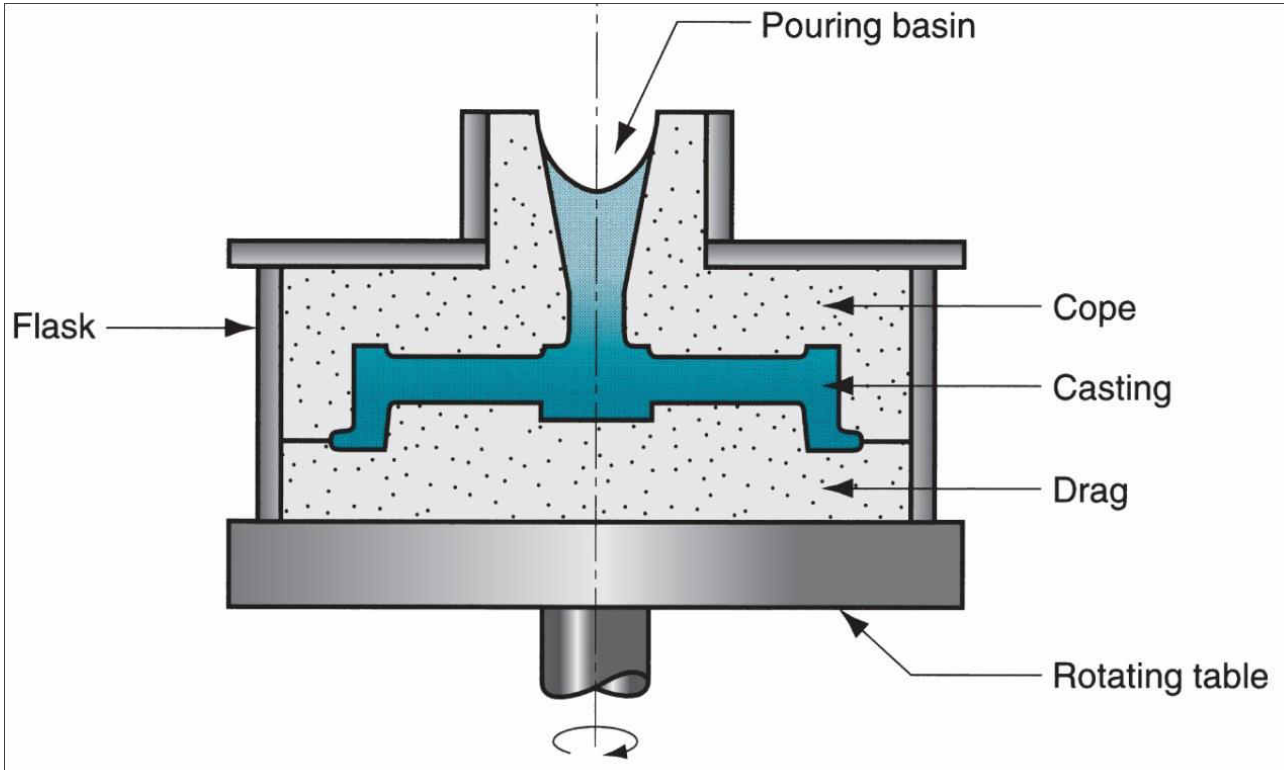


Semicentrifugal Casting

Centrifugal force is used to produce solid castings rather than tubular parts

- Molds use risers at center to supply feed metal
- Density of metal in final casting is greater in outer sections than at center of rotation
- Often used on parts in which center of casting is machined away, thus eliminating the portion where quality is lowest
  - Examples: wheels and pulleys

Semicentrifugal Casting

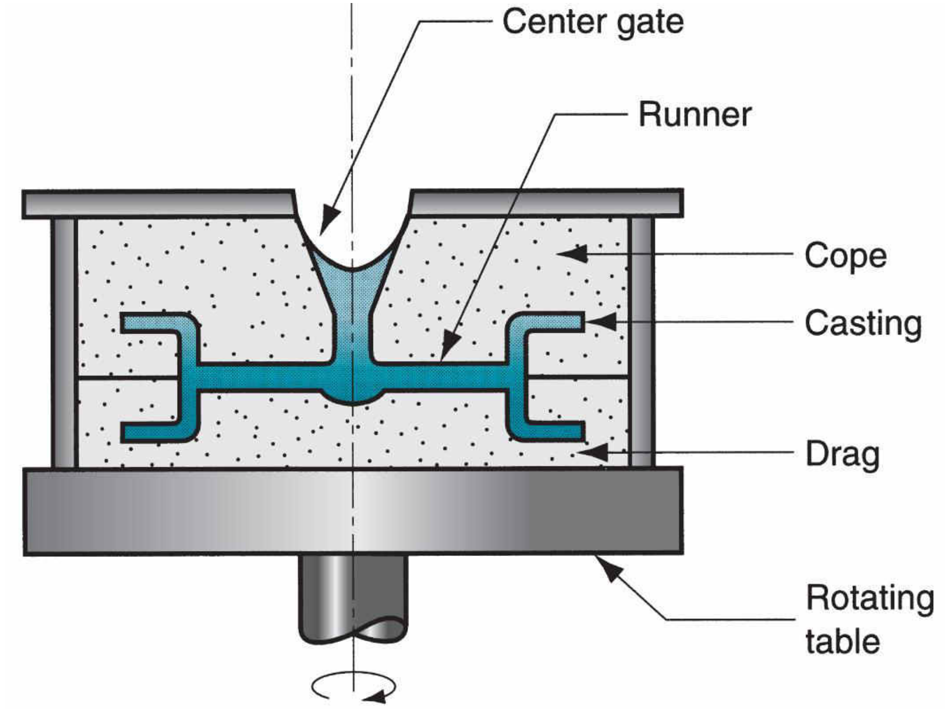


Centrifuge Casting

Mold is designed with part cavities located away from axis of rotation, so molten metal poured into mold is distributed to these cavities by centrifugal force

- Used for smaller parts
- Radial symmetry of part is not required as in other centrifugal casting methods

Centrifuge Casting



Furnaces for Casting Processes

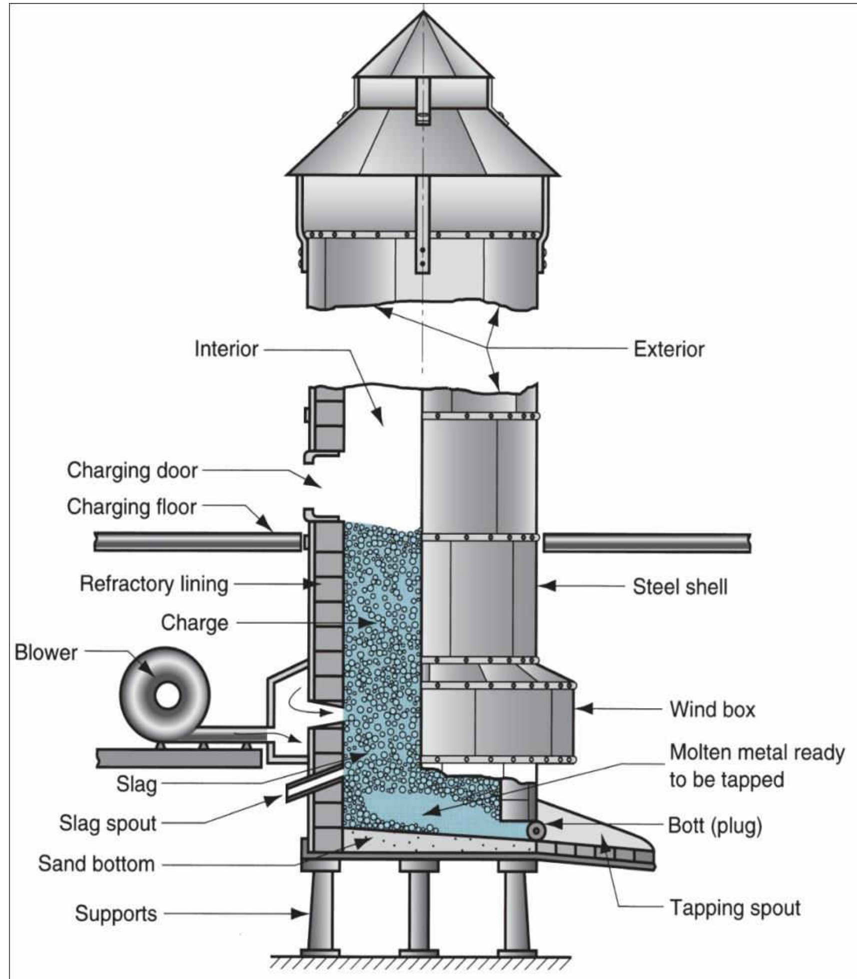
- Furnaces most commonly used in foundries:
  - Cupolas
  - Direct fuel-fired furnaces
  - Crucible furnaces
  - Electric-arc furnaces
  - Induction furnaces

Cupolas

Vertical cylindrical furnace equipped with tapping spout near base

- Used only for cast irons
  - Although other furnaces are also used, the largest tonnage of cast iron is melted in cupolas
- The "charge," consisting of iron, coke, flux, and any alloying elements, is loaded through a charging door located less than halfway up height of cupola

- Cupola for melting cast iron



Direct Fuel-Fired Furnaces

Small open-hearth in which charge is heated by natural gas fuel burners located on side of furnace

- Furnace roof assists heating action by reflecting flame down against charge
- At bottom of hearth is a tap hole to release molten metal
- Generally used for nonferrous metals such as copper-base alloys and aluminum

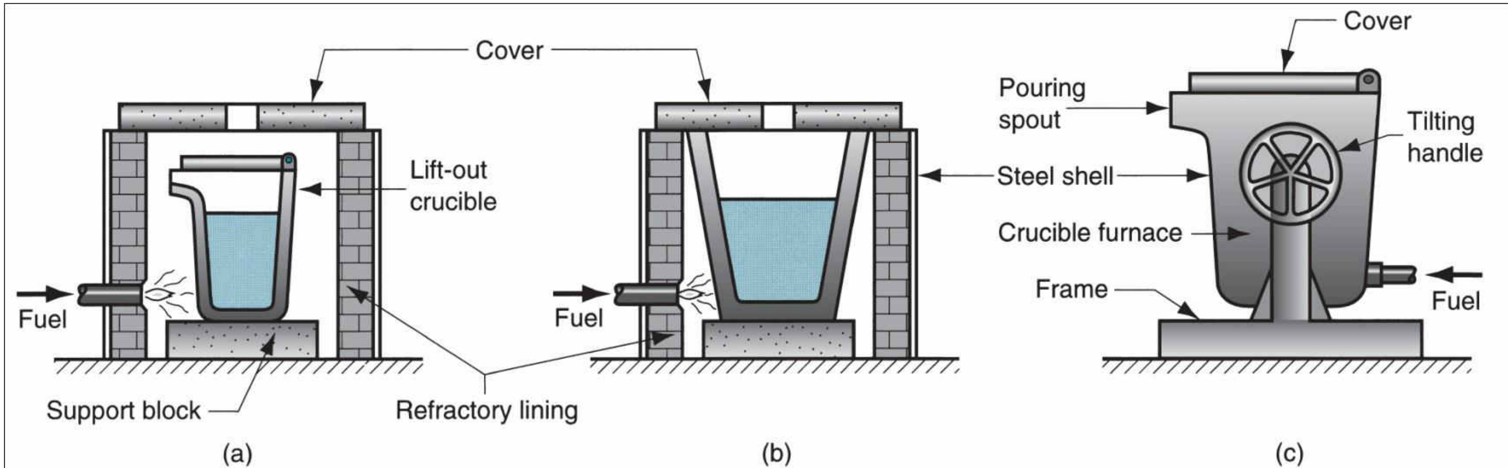
Crucible Furnaces

Metal is melted without direct contact with burning fuel mixture

- Sometimes called *indirect fuel-fired furnaces*
- Container (crucible) is made of refractory material or high-temperature steel alloy
- Used for nonferrous metals such as bronze, brass, and alloys of zinc and aluminum
- Three types used in foundries: (a) lift-out type, (b) stationary, (c) tilting

Three Types of Crucible Furnaces

- (a) Lift-out crucible, (b) stationary pot - molten metal must be ladled, and (c) tilting-pot furnace



Electric-Arc Furnaces

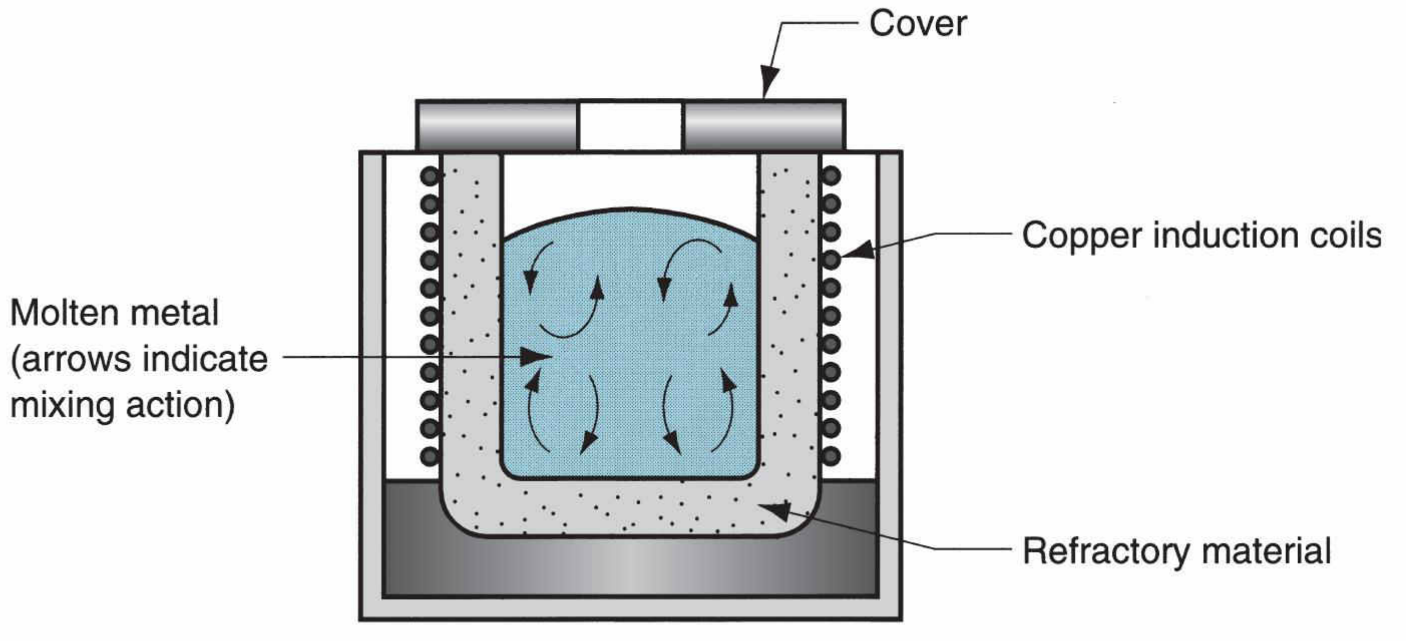
Charge is melted by heat generated from an electric arc

- High power consumption
  - But electric-arc furnaces can be designed for high melting capacity
- Used primarily for melting steel

Induction Furnaces

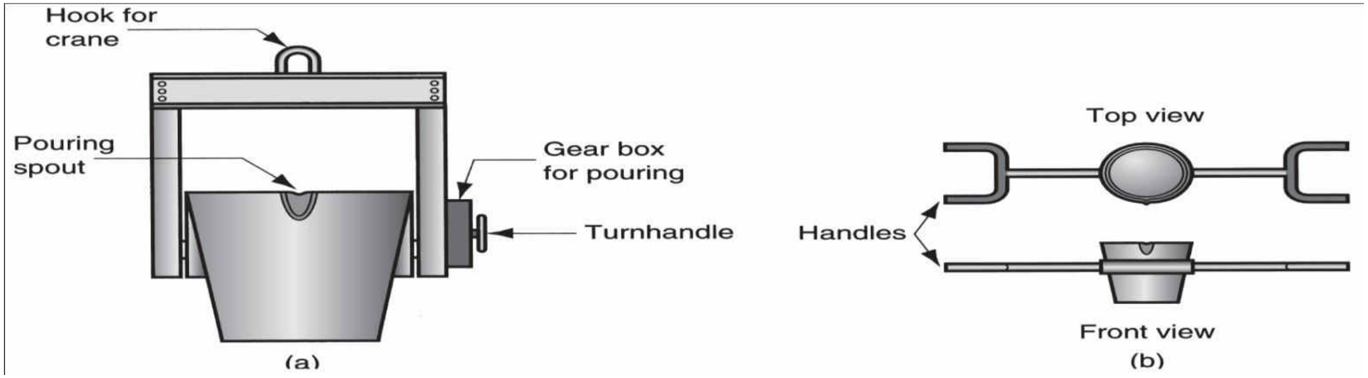
- Uses alternating current passing through a coil to develop magnetic field in metal
  - Induced current causes rapid heating and melting
  - Electromagnetic force field also causes mixing action
- Since metal does not contact heating elements, environment can be closely controlled to produce molten metals of high quality and purity
- Common alloys: steel, cast iron, and aluminum

Induction Furnace



Ladles

- Two common types of ladles to transfer molten metals to molds: (a) crane ladle, and (b) two-man ladle



Additional Steps After Solidification

- Trimming
- Removing the core
- Surface cleaning
- Inspection
- Repair, if required
- Heat treatment

Trimming

Removal of sprues, runners, risers, parting-line flash, fins, chaplets, and any other excess metal from the cast part

- For brittle casting alloys and when cross sections are relatively small, appendages can be broken off
- Otherwise, hammering, shearing, hack-sawing, band-sawing, abrasive wheel cutting, or various torch cutting methods are used

Removing the Core

- If cores have been used, they must be removed
  - Most cores are bonded, and they often fall out of casting as the binder deteriorates
  - In some cases, they are removed by shaking the casting, either manually or mechanically
  - In rare cases, cores are removed by chemically dissolving bonding agent
  - Solid cores must be hammered or pressed out

Surface Cleaning and Inspection

Removal of sand from casting surface and otherwise enhancing appearance of surface

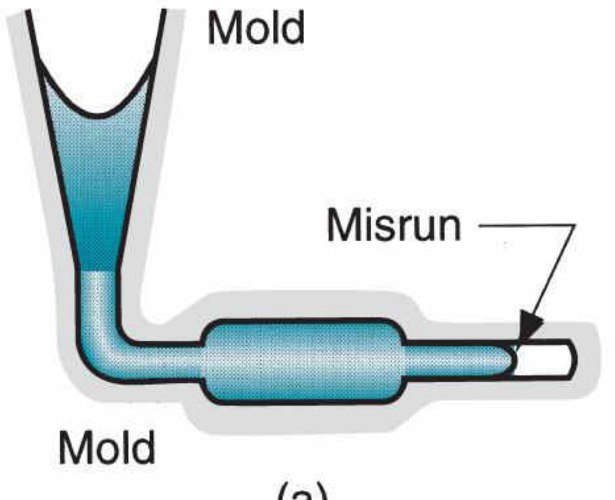
- Cleaning methods: tumbling, air-blasting with coarse sand grit or metal shot, wire brushing, buffing, and chemical pickling
- Surface cleaning is most important for sand casting
  - In many permanent mold processes, this step can be avoided
- Defects are possible in casting, and inspection is needed to detect their presence

Heat Treatment

- Castings are often heat treated to enhance properties
- Reasons for heat treating a casting:
  - For subsequent processing operations such as machining
  - To bring out the desired properties for the application of the part in service

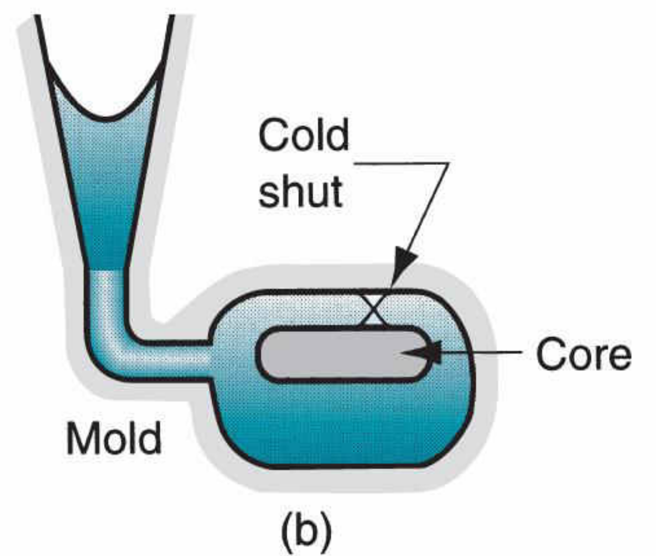
General Defects: Misrun

- A casting that has solidified before completely filling mold cavity



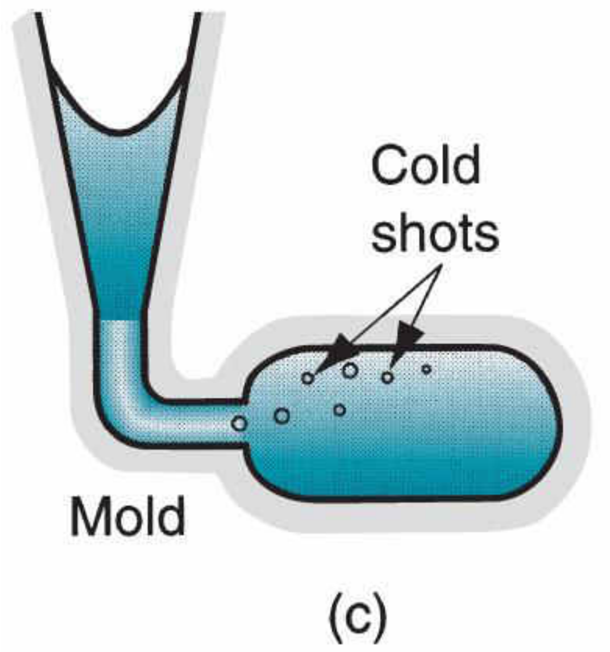
General Defects: Cold Shut

- Two portions of metal flow together but there is a lack of fusion due to premature freezing



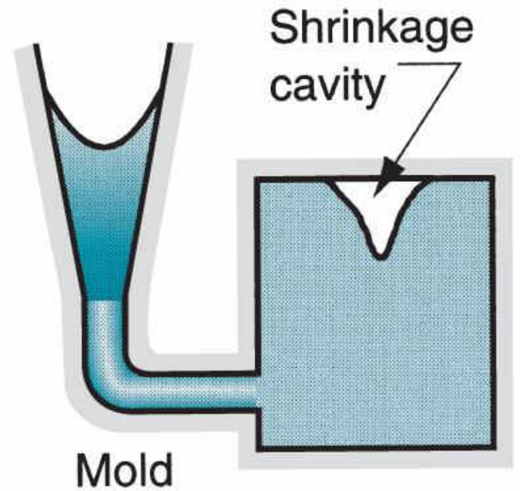
General Defects: Cold Shot

- Metal splatters during pouring and solid globules form and become entrapped in casting



General Defects: Shrinkage Cavity

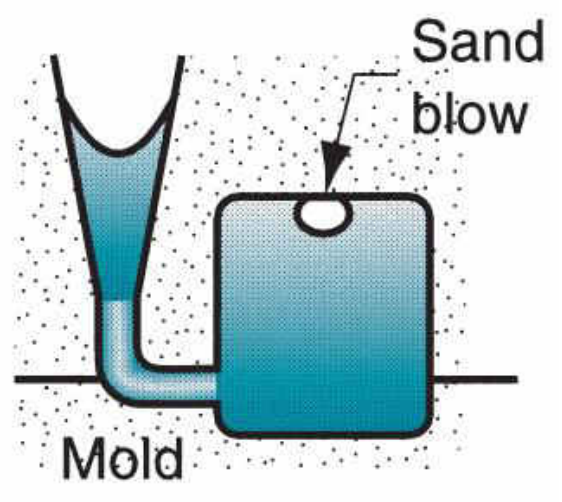
- Depression in surface or internal void caused by solidification shrinkage that restricts amount of molten metal available in last region to freeze



(d)

Sand Casting Defects: Sand Blow

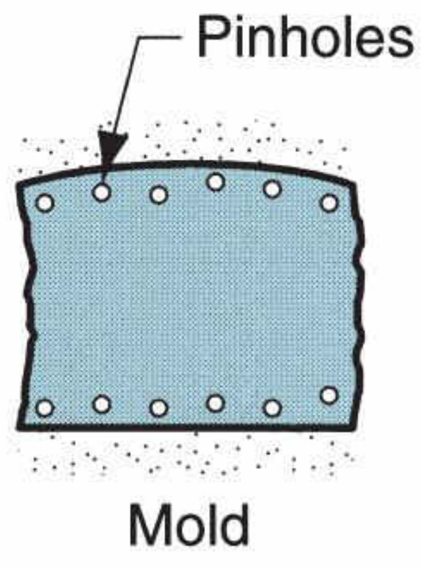
- Balloon-shaped gas cavity caused by release of mold gases during pouring



(a)

Sand Casting Defects: Pin Holes

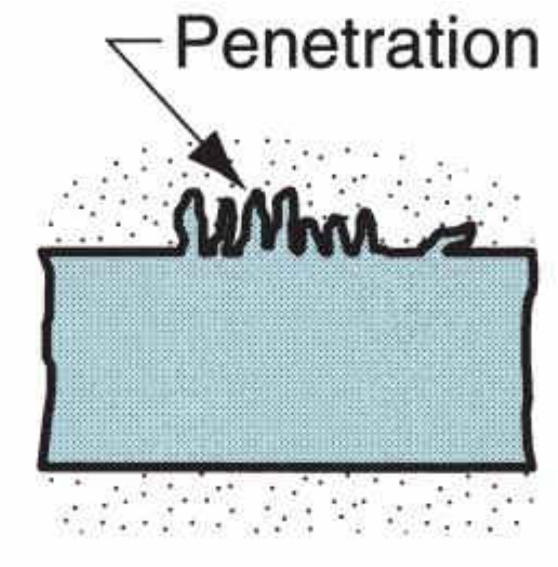
- Formation of many small gas cavities at or slightly below surface of casting



(b)

Sand Casting Defects: Penetration

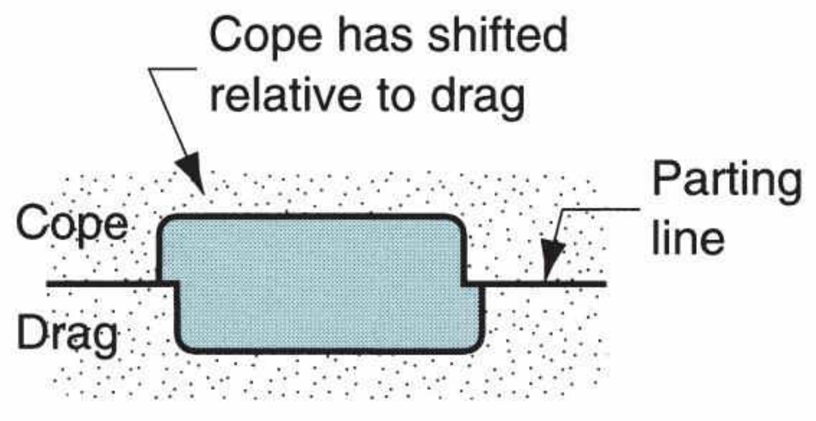
- When fluidity of liquid metal is high, it may penetrate into sand mold or core, causing casting surface to consist of a mixture of sand grains and metal



(e)

Sand Casting Defects: Mold Shift

- A step in the cast product at parting line caused by sidewise relative displacement of cope and drag



(f)

Metals for Casting

- Most commercial castings are made of alloys rather than pure metals
  - Alloys are generally easier to cast, and properties of product are better
- Casting alloys can be classified as:
  - Ferrous
  - Nonferrous

Ferrous Casting Alloys: Cast Iron

- Most important of all casting alloys
- Tonnage of cast iron castings is several times that of all other metals combined
- Several types: (1) gray cast iron, (2) nodular iron, (3) white cast iron, (4) malleable iron, and (5) alloy cast irons
- Typical pouring temperatures $\sim$ 1400°C (2500°F), depending on composition

Ferrous Casting Alloys: Steel

- The mechanical properties of steel make it an attractive engineering material
- The capability to create complex geometries makes casting an attractive shaping process
- Difficulties when casting steel:
  - Pouring temperature is high $\sim$ 1650°C (3000°F)
    - At such temperatures, steel readily oxidizes, so molten metal must be isolated from air
  - Molten steel has relatively poor fluidity

Nonferrous Casting Alloys: Copper Alloys

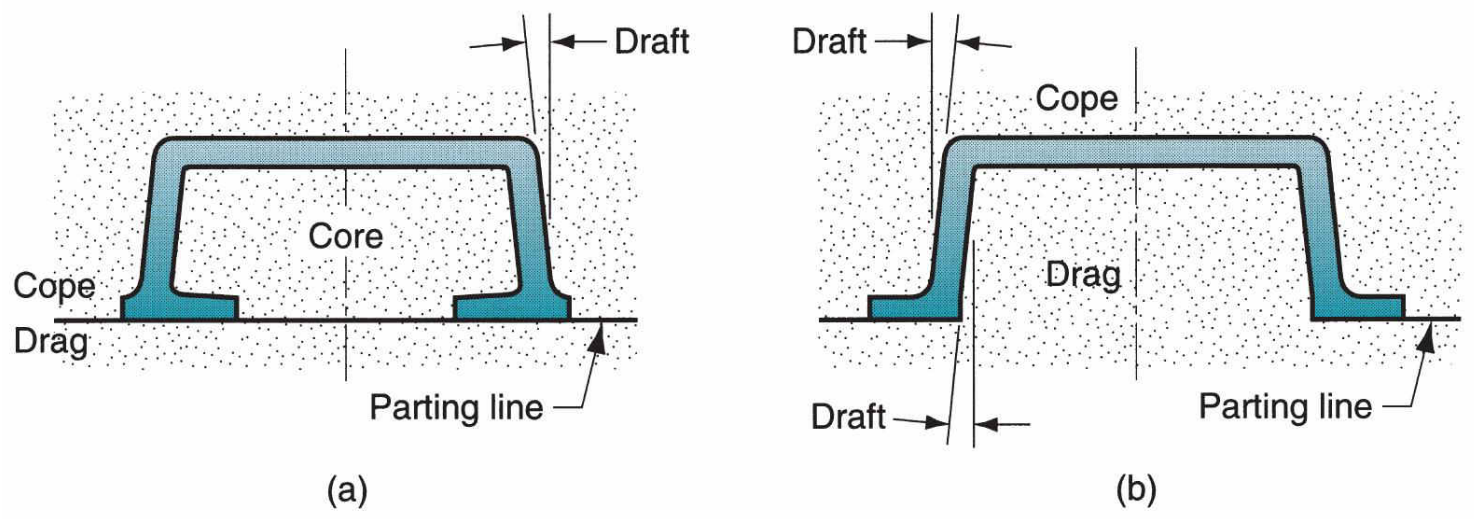
- Includes bronze, brass, and aluminum bronze
- Properties:
  - Corrosion resistance
  - Attractive appearance
  - Good bearing qualities
- Limitation: high cost of copper
- Applications: pipe fittings, marine propeller blades, pump components, ornamental jewelry

Nonferrous Casting Alloys: Zinc Alloys

- Very castable, commonly used in die casting
- Low pouring temperatures due to low melting temperature
  - Pure zinc T_m = 419°C (786°F)
- Good fluidity for ease of casting
- Properties:
  - Low creep strength, so castings cannot be subjected to prolonged high stresses

Draft

- Design change to eliminate need for using a core: (a) original design, and (b) redesign



Govt. Polytechnic, Nanakpur (Panchkula) e-Notes

- Name : Shalander Singh (Mor), Lect. (Mech. Engg.)
- Semester :3rd
- Subject : Workshop Technology-I

Metal Forming

Large group of manufacturing processes in which plastic deformation is used to change the shape of metal workpieces

- The tool, usually called a ***die***, applies stresses that exceed the yield strength of the metal
- The metal takes a shape determined by the geometry of the die

Stresses in Metal Forming

- Stresses to plastically deform the metal are usually compressive
  - Examples: rolling, forging, extrusion
- However, some forming processes
  - Stretch the metal (tensile stresses)
  - Others bend the metal (tensile and compressive)
  - Still others apply shear stresses (shear spinning)

Material Properties in Metal Forming

- Desirable material properties:
  - **Low yield strength**
  - **High ductility**
- These properties are affected by **temperature**:
  - Ductility increases and yield strength decreases when work temperature is raised
- Other factors:
  - Strain rate and friction

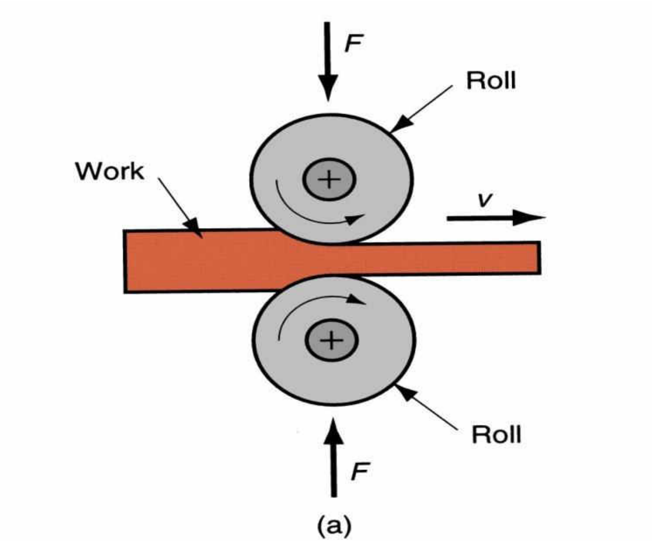
Basic Types of Deformation Processes

1. Bulk deformation
   - Rolling
   - Forging
   - Extrusion
   - Wire and bar drawing
2. Sheet metalworking
   - Bending
   - Deep drawing
   - Cutting

Bulk Deformation Processes

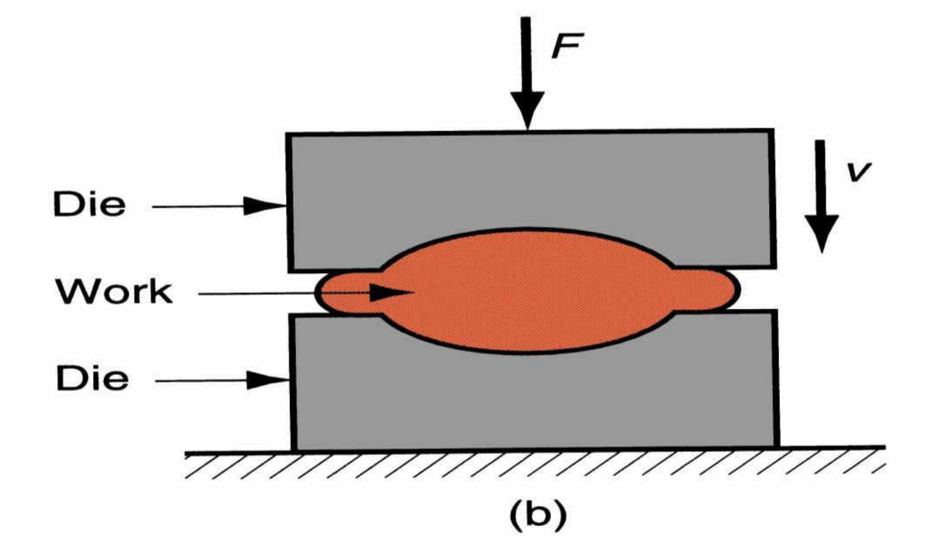
- Characterized by significant deformations and massive shape changes
- "Bulk" refers to workparts with relatively low surface area-to-volume ratios
- Starting work shapes include cylindrical billets and rectangular bars

Rolling



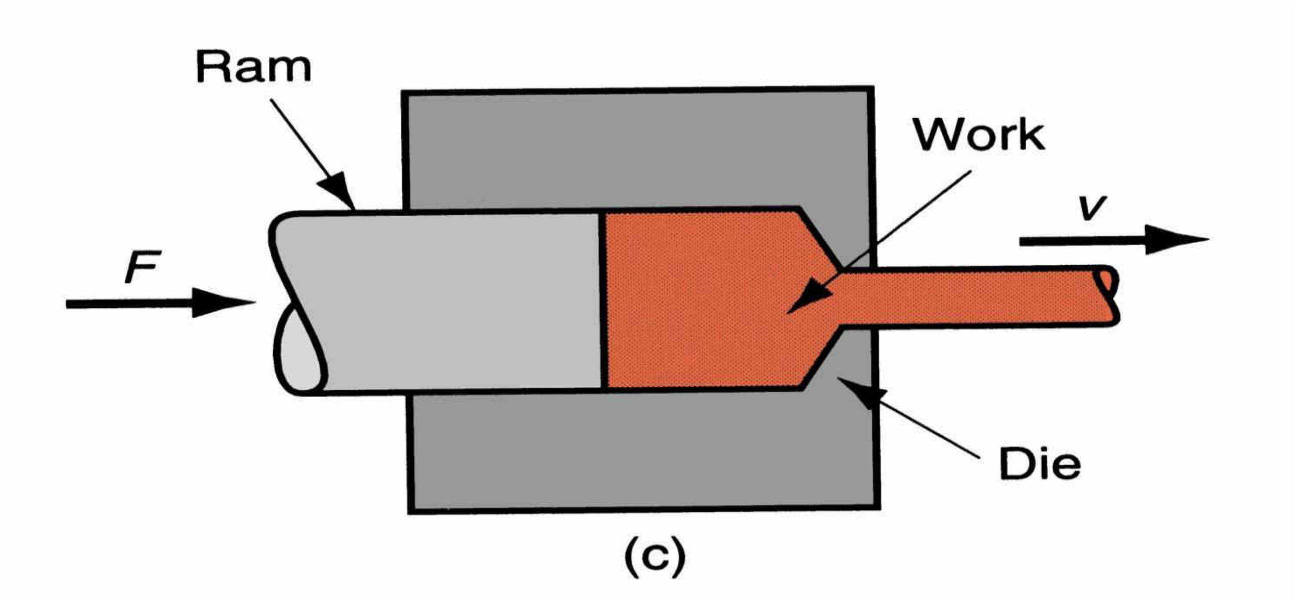
Basic bulk deformation processes: rolling

Forging



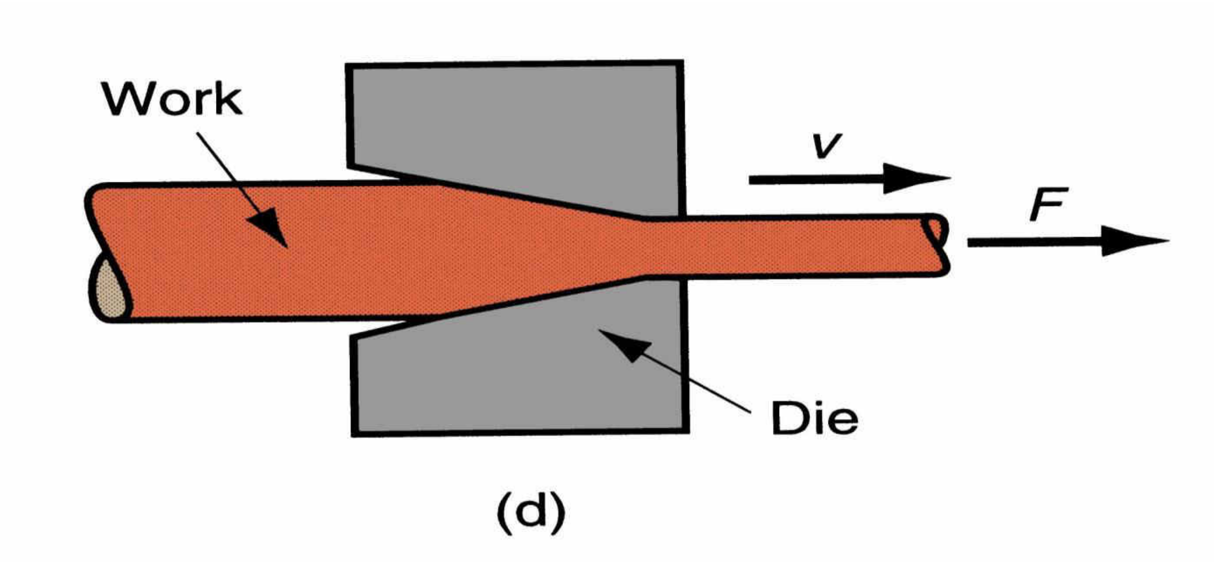
Basic bulk deformation processes: forging

Extrusion



Basic bulk deformation processes: (c) extrusion

Wire and Bar Drawing

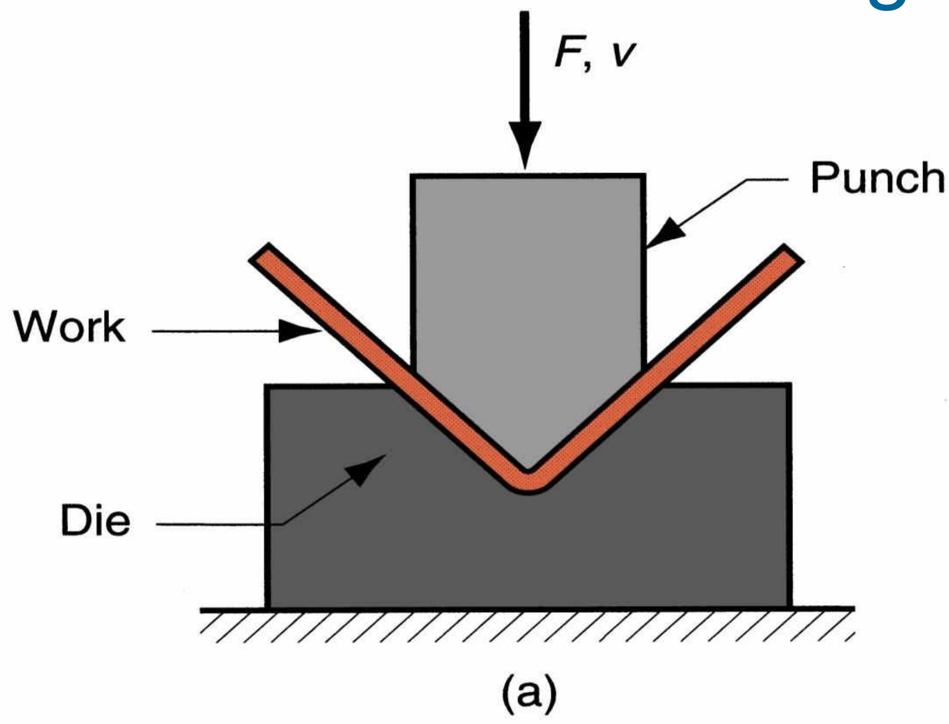


Basic bulk deformation processes: (d) drawing

Sheet Metalworking

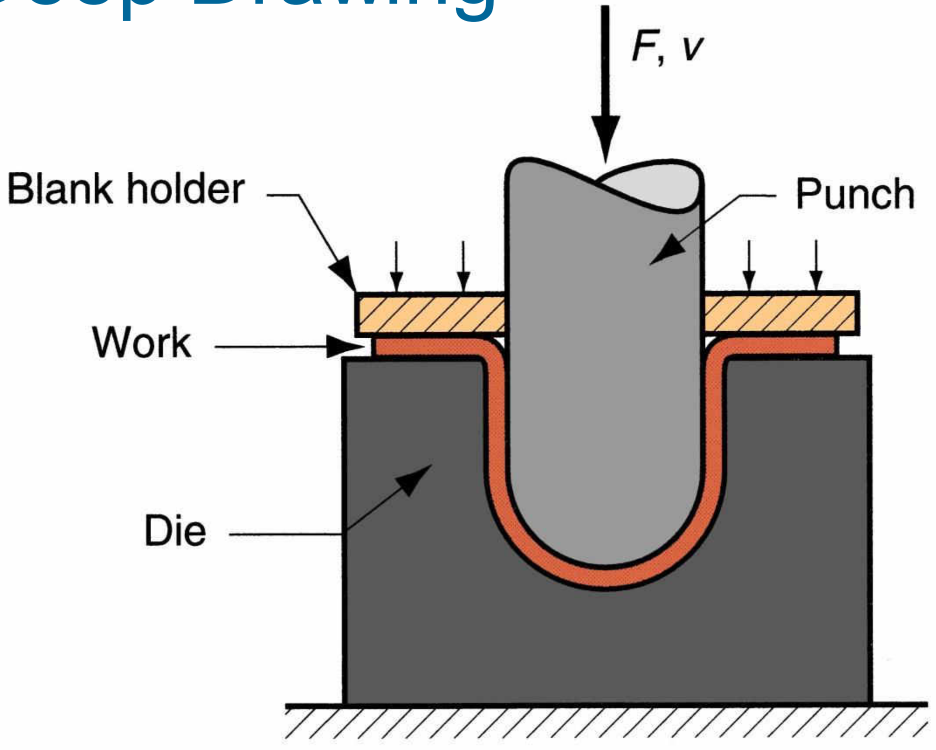
- Forming and related operations performed on metal sheets, strips, and coils
- High surface area-to-volume ratio of starting metal, which distinguishes these from bulk deformation
- Often called *pressworking* because presses perform these operations
  - Parts are called *stampings*
  - Usual tooling: *punch* and *die*

Sheet Metal Bending



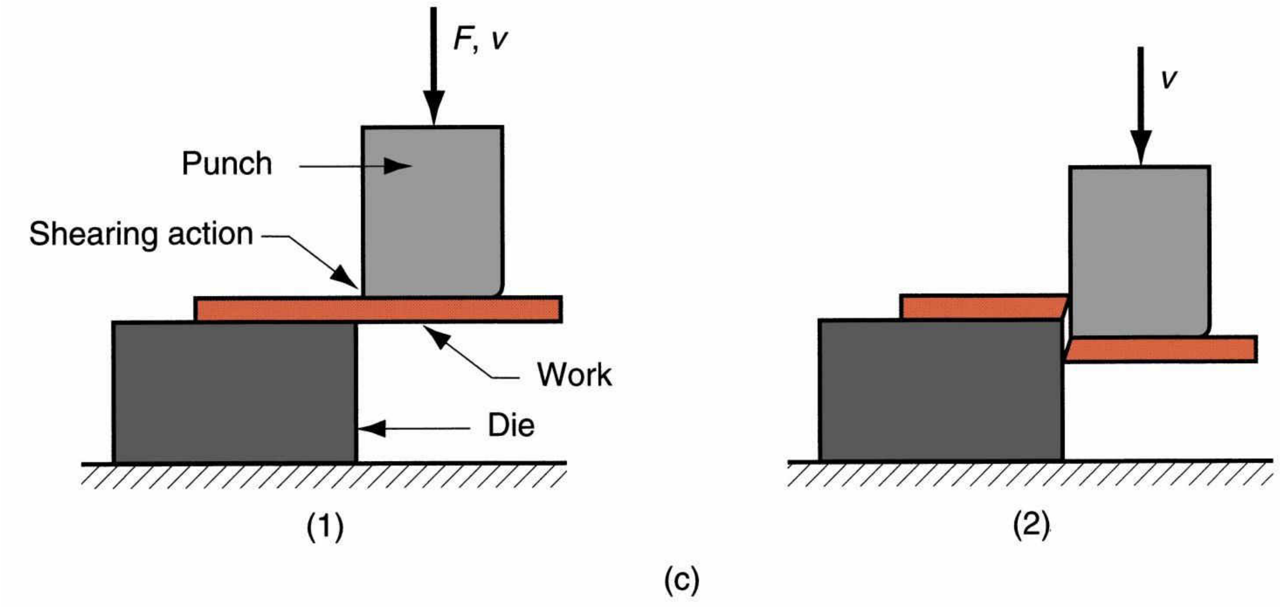
Basic sheet metalworking operations: bending

Deep Drawing



Basic sheet metalworking operations: drawing

Shearing of Sheet Metal



Basic sheet metalworking operations: shearing

Temperature in Metal Forming

- Any deformation operation can be accomplished with lower forces and power at elevated temperature
- Three temperature ranges in metal forming:
  - Cold working
  - Warm working
  - Hot working

Cold Working

- Performed at room temperature or slightly above
- Many cold forming processes are important mass production operations
- Minimum or no machining usually required

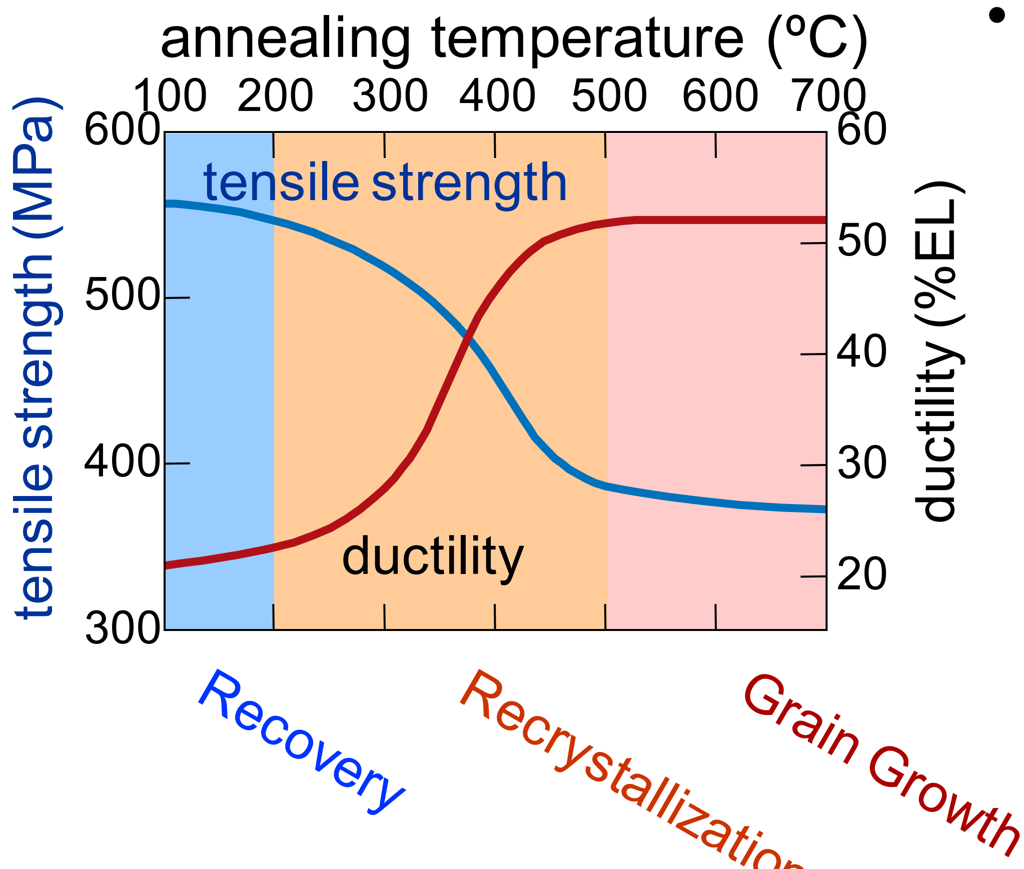
Advantages of Cold Forming

- Better accuracy, closer tolerances
- Better surface finish
- Strain hardening increases strength and hardness
- No heating of work required

Disadvantages of Cold Forming

- Higher forces and power required in the deformation operation
- Ductility and strain hardening limit the amount of forming that can be done
  - In some cases, metal must be annealed to allow further deformation
  - In other cases, metal is simply not ductile enough to be cold worked

Effect of Heat Treating After Cold Working



- Three Annealing stages:
  1. Recovery
  2. Recrystallization
  3. Grain Growth

Adapted from Fig. 8.22, *Callister & Rethwisch 4e.* (Fig. 8.22 is adapted from G. Sachs and K.R. van Horn, *Practical Metallurgy, Applied Metallurgy, and the Industrial Processing of Ferrous and Nonferrous Metals and Alloys*, American Society for Metals, 1940, p. 139.)

Recrystallization Temperature

T_R = recrystallization temperature = temperature at which recrystallization just reaches completion in 1 h.

$$0.3T_m < T_R < 0.6T_m$$

For a specific metal/alloy, T_R depends on:

- %CW -- T_R decreases with increasing %CW
- Purity of metal -- T_R decreases with increasing purity

Advantages of Warm Working

- Lower forces and power than in cold working
- More intricate work geometries possible
- Need for annealing may be reduced or eliminated
- Low spring back

Disadvantage:

1. Scaling of part surface

Hot Working

- Deformation at temperatures above the *recrystallization temperature*
- Recrystallization temperature = about one-half of melting point on absolute scale
  - In practice, hot working usually performed somewhat above $0.6T_m$
  - Metal continues to soften as temperature increases above $0.6T_m$, enhancing advantage of hot working above this level

Why Hot Working?

Capability for substantial plastic deformation of the metal - far more than possible with cold working or warm working

- Why?
  - Strength coefficient (***K***) is substantially less than at room temperature
  - Strain hardening exponent (*n*) is zero (theoretically)
  - Ductility is significantly increased

Advantages of Hot Working

- Work part shape can be significantly altered
- Lower forces and power required
- Metals that usually fracture in cold working can be hot formed
- Strength properties of product are generally isotropic
- No work hardening occurs during forming

Disadvantages of Hot Working

- Lower dimensional accuracy in case of bulk forming
- Higher total energy required (due to the thermal energy to heat the workpiece)
- Work surface oxidation (scale), poorer surface finish
- Shorter tool life

Govt. Polytechnic, Nanakpur (Panchkula) e-Notes

- Name : Shalander Singh (Mor), Lect. (Mech. Engg.)
- Semester :3rd
- Subject : Workshop Technology-I

INTRODUCTION TO PLASTICS PROCESSING

DEFINITION:

Raw Materials
(Powder/granules)

$\xrightarrow{\text{Processing}}$

Semi-finished
or
Finished Products

Plastics Processing – in a simple layman's language – can be defined as the process of converting the plastic raw materials into Semi-finished or finished products.

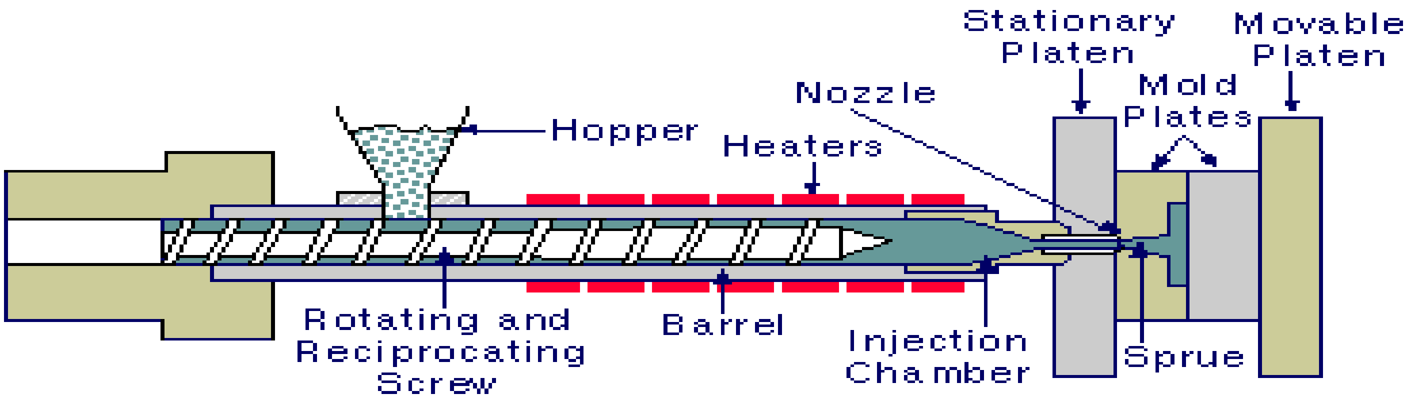
INJECTION MOULDING :

Very widely used. High automation of manufacturing is standard practice. Thermoplastic or thermoset is heated to plasticate in cylinder at controlled temperature, then forced under pressure through a nozzle into sprue, runners, gates, and cavities of mould. The resin undergoes solidification rapidly. The mould is opened, and the part ejected, Injection Moulding is growing in the making of glass-reinforced parts. High production runs, low labour costs, high reproducibility of complex details, and excellent surface finish are the merits.

Limitations:

High initial tool and die costs; not economically practical for small runs.

INJECTION MOULDING




Refrigerator cabinet in Lustran

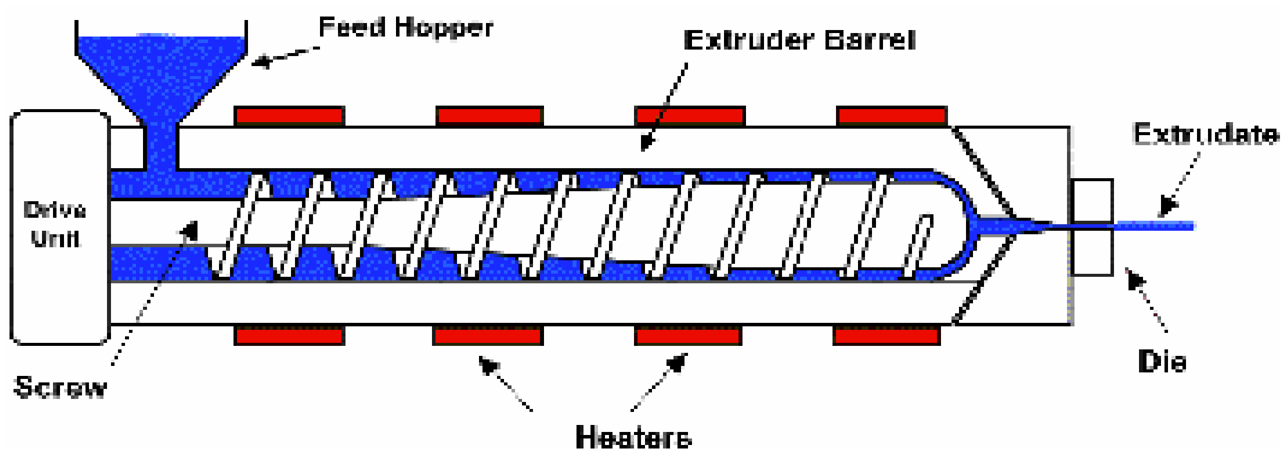
EXTRUSION :

Widely used for continuous production of film, sheet, tube, and other profiles; also used in conjunction with blow moulding. Thermoplastic moulding compound is fed from a hopper to a screw pump where it is heated to plasticate then pumped out through the shaping orifice (die) to achieve desired cross section. Production lines require input and takeoff equipment that can be complex. Low tool cost, numerous complex profile shapes possible, very rapid production rates, can apply coatings or jacketing to core materials (Such as wire).

Limitations:

Usually limited to sections of uniform cross section.

Extrusion



Feed Hopper
Extruder Barrel
Extrudate
Drive Unit
Die
Screw
Heaters

COMPRESSION MOULDING :

Thermoset compound, usually preformed, is positioned in a heated mould cavity; the mould is closed (heat and pressure are applied) and the material flows and fills the mould cavity. Heat completes polymerization and the part is ejected. The process is sometimes used for thermoplastics, e.g. Vinyl phonograph records. Little material waste is attainable; large, bulky parts can be moulded; process is adaptable to rapid automation.

Limitations:

Extremely intricate parts containing undercuts, side draws, small holes, delicate inserts, etc.; very close tolerances are difficult to produce. Time consuming process.

TRANSFER MOULDING

Widely used to produce Thermoset products with part complexity. Thermoset moulding compound is fed into transfer chamber where it is then heated to plasticate; it is then fed by a plunger through sprues, runners, and gates into a closed mould where it cures; mould is opened and part ejected. Good dimensional accuracy, rapid production rate, and very intricate parts can be produced.

Limitations:

High mould cost; high material loss in sprues and runners; size of parts is somewhat limited.

ROTATIONAL MOULDING

A predetermined amount of powdered thermoplastic material is poured into mould; mould is closed, heated, and rotated in the axis of two planes until contents have fused to the inner walls of mould; mould is then opened and part is removed. Low mould cost, large hollow parts in one piece can be produced, and moulded parts are essentially isotropic in nature.

Limitations:

Limited to hollow parts; production rates are usually slow.

ROTATIONAL MOULDING

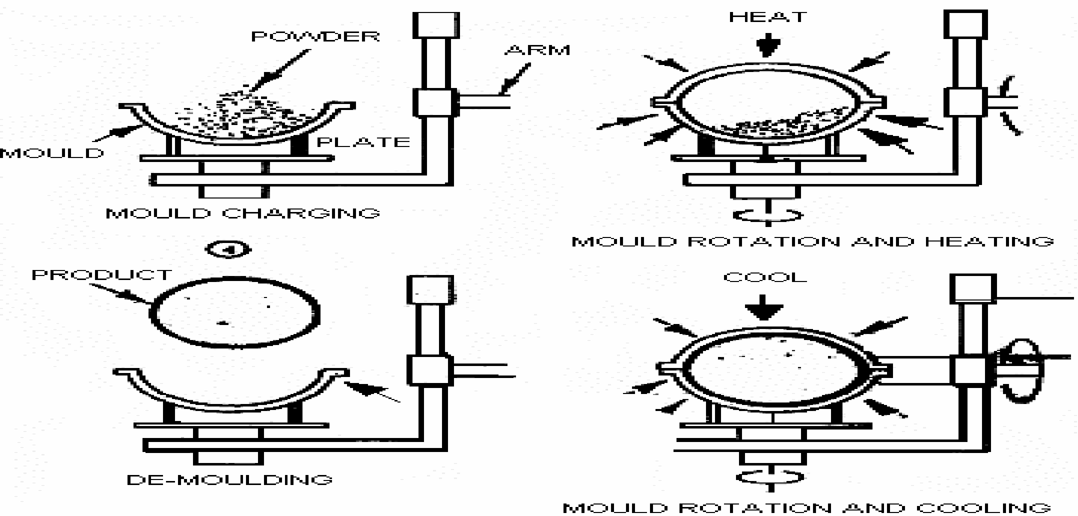


FIG-6.1 PRINCIPLE OF ROTATIONAL MOULDING

THERMOFORMING

Heat-softened thermoplastic sheet is positioned over male or female mould; air is evacuated between sheet and mould, forcing sheet to conform to contour of mould. Variations are vacuum snapback, plug assist, drape forming, etc. Tooling costs are generally low, large part production with thin sections possible, and often comes out economical for limited part production.

Limitations:

Limited to parts of simple configuration, high scrap, and limited number of materials from which to choose.

THERMOFORMING

CASTING

Liquid plastic which is generally thermoset except for acrylics is poured into a mould without pressure, cured, and taken from the mould. Cast thermoplastic films are produced via building up the material (either in solution or hot-melt form) against a highly polished supporting surface. Low mould cost, capability to form large parts with thick cross sections, good surface finish, and convenient for low-volume production.

Limitations:

Limited to relatively simple shapes. Most thermoplastics are not suitable for this method.

Except for cast films, method becomes uneconomical at high volume production rates.

CENTRIFUGAL CASTING:

Reinforcement is placed in mould and is rotated. Resin distributed through pipe; impregnates reinforcement through centrifugal action. Utilized for round objects, particularly pipe.

Limitations:

Limited to simple curvatures in single axis rotation. Low production rates.

COATING

Description :

Process methods vary. Both thermoplastics and thermosets widely used in coating of numerous materials. Roller coating similar to calendaring process. Spread coating employs blade in front of roller to position resin on material. Coatings also applied via brushings, spraying, and dipping.

MATCHED-DIE MOULDING :

A variation of the conventional compression moulding, this process employs two metal moulds possessing a close-fitting, telescoping area to seal in the plastic compound being moulded and to allow trim of the reinforcement. The mat or preform reinforcement is positioned in the mould and the mould is closed and heated under pressures of 150 – 400psi (1-3MPa). The mould is then opened and the part is removed after curing.

Prevalent high mould and equipment costs. Part often require expensive surface finishing.

SLUSH MOULDING :

Liquid thermoplastic material (Plastisol) is poured into a mould to capacity; mould is closed and heated for a predetermined time in order to achieve a specified buildup of partially fused material on mould walls; mould is opened and excess material is poured out; and semifused part is removed from mould and fully fused in oven. Low mould costs and economical for small production runs.

Limitations:

Limited to hollow parts; production rates are very slow; and limited choice of materials that can be processed.

- References:

https://www.pdfdrive.com/welding-books.html

The Welding Handbook - Wilhelmsen

https://www.wilhelmsen.com/globalassets/marine-products/welding/documents/wilhelmsen-ships-service---unitor-welding-handbook.pdf

Thank

You

Government Polytechnic Nanakpur

DEPARTMENT OF MECHANICAL ENGINEERING

E - NOTES

STRENGTH OF MATERIALS

HOD/O.I. (Mechanical) : Er. **SHALANDER MOR**

Subject Teacher: Er. Amit Kumar

Semester: 3rd Sem

By: Er. Amit Kumar
Designed, Mechanical Editing & Developmental Editing

LEARNING OUTCOMES

After undergoing this subject, the students will be able to:

- Interpret various concepts and terms related to strength of materials
- Calculate stresses in thin cylindrical shells.
- Calculate energy stored by materials subjected to axial loads.
- Calculate moment of inertia of different sections.
-
- Draw and calculate bending moment and shear force diagrams of beam under given loading
-
- Interpret the concept of bending and torsion and calculate stresses on different section of materials.
-
- Determine the diameter of a shaft under combined bending and torsion.
- Calculate critical axial loads on column under different end constraints.
- Determine the various parameters in closed coil helical and laminated springs
- Determine conformance of given materials sample to the prescribed Indian standards.

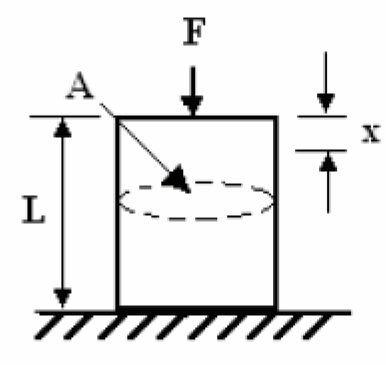
Stresses and Strains

- Topic cover
  - Stress and strain
    - Introduction to stress and strain, stress strain diagram
    - Elasticity and plasticity and Hooke’s law
    - Shear Stress and Shear strain
    - Load and stress limit
    - Axial force and deflection of body
  - Torsion
    - Introduction, round bar torsion, non-uniform torsion.
    - Relation between Young’s Modulus E, υ and G
    - Power transmission on round bar

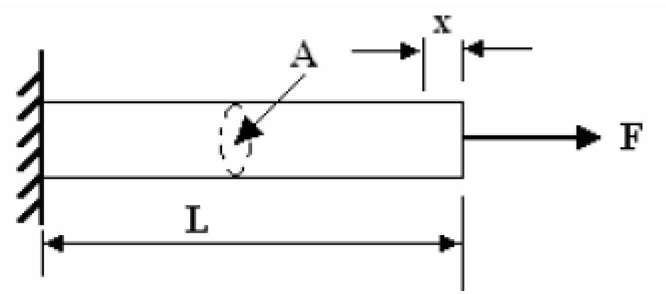
Stress and strain

DIRECT STRESS σ

- When a force is applied to an elastic body, the body deforms. The way in which the body deforms depends upon the type of force applied to it.



Compression force makes the body shorter.



A tensile force makes the body longer

Tensile and compressive forces are called DIRECT FORCES
Stress is the force per unit area upon which it acts.

$$Stress = \sigma = \frac{Force}{Area} = \frac{F}{A}$$

….. Unit is Pascal (Pa) or N/m^2

(Simbol – Sigma)

Note: Most of engineering fields used kPa, MPa, GPa.

DIRECT STRAIN , ε

In each case, a force F produces a deformation x. In engineering, we usually change this force into stress and the deformation into strain and we define these as follows:
Strain is the deformation per unit of the original length.

$$Strain = \varepsilon = \frac{x}{L}$$

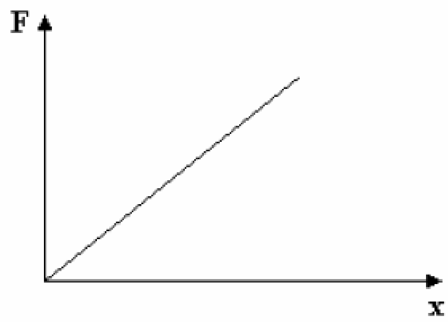
The symbol ε called EPSILON

Strain has no unit's since it is a ratio of length to length. Most engineering materials do not stretch very mush before they become damages, so strain values are very small figures. It is quite normal to change small numbers in to the exponent for 10^{-6}(micro strain).

MODULUS OF ELASTICITY (E)

- Elastic materials always spring back into shape when released. They also obey HOOKE's LAW.

- This is the law of spring which states that deformation is directly proportional to the force. F/x = stiffness = kN/m



- The stiffness is different for the different material and different sizes of the material. We may eliminate the size by using stress and strain instead of force and deformation:

- If F and x is refer to the direct stress and strain , then

$F = \sigma A$ $\quad$ $x = \varepsilon L$ $\quad$ hence $\dfrac{F}{x} = \dfrac{\sigma A}{\varepsilon L}$ and $\dfrac{FL}{Ax} = \dfrac{\sigma}{\varepsilon}$

•The stiffness is now in terms of stress and strain only and this constant is called the MODULUS of ELASTICITY (E)

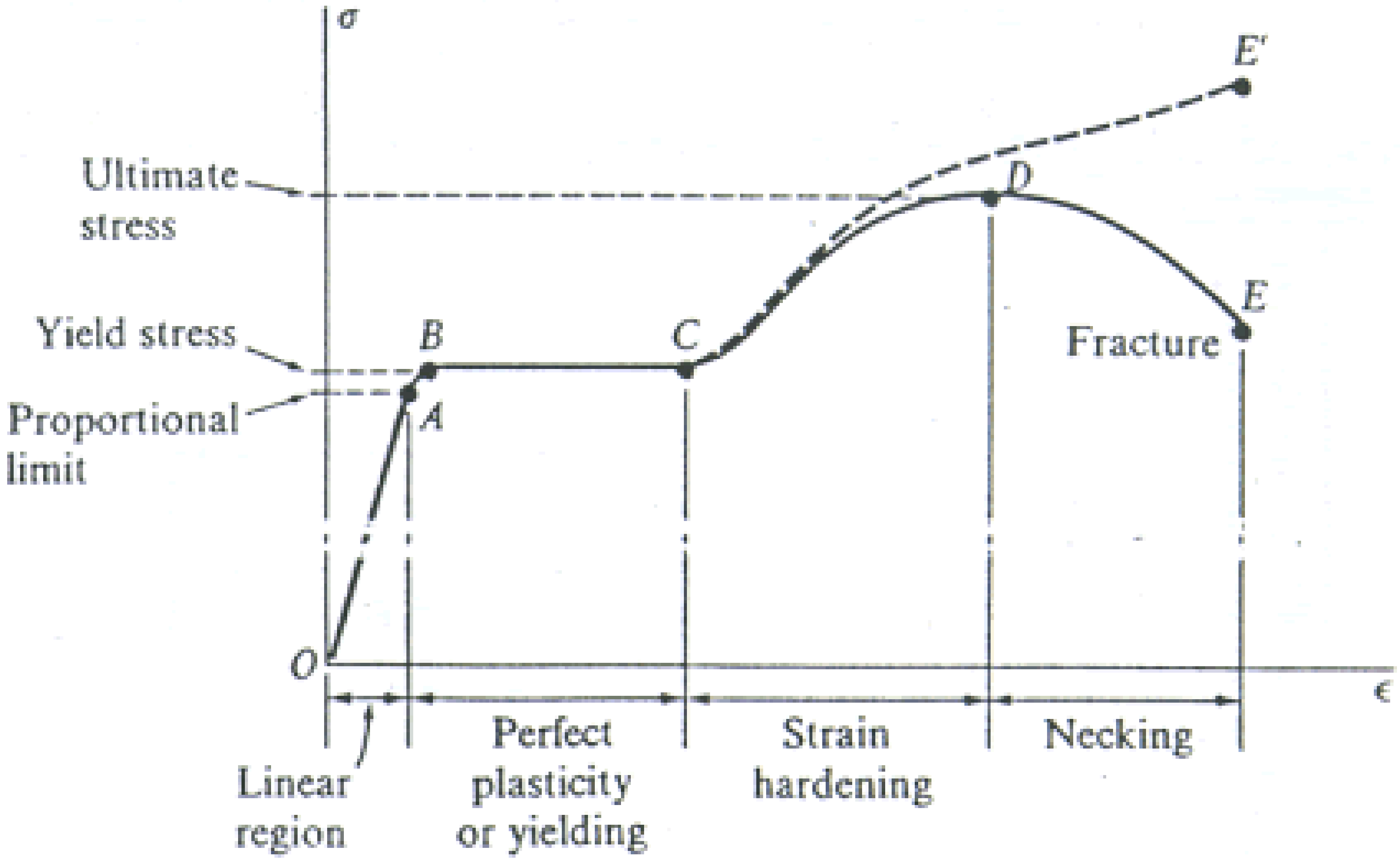
$$E = \frac{FL}{Ax} = \frac{\sigma}{\varepsilon}$$

- A graph of stress against strain will be straight line with gradient of E. The units of E are the same as the unit of stress.

ULTIMATE TENSILE STRESS

•If a material is stretched until it breaks, the tensile stress has reached the absolute limit and this stress level is called the ultimate tensile stress.

STRESS STRAIN DIAGRAM



STRESS STRAIN DIAGRAM

Elastic behaviour

- The curve is straight line trough out most of the region
- Stress is proportional with strain
- Material to be linearly elastic
- Proportional limit
  - The upper limit to linear line
  - The material still respond elastically
  - The curve tend to bend and flatten out
- Elastic limit
  - Upon reaching this point, if load is remove, the specimen still return to original shape

STRESS STRAIN DIAGRAM

Yielding

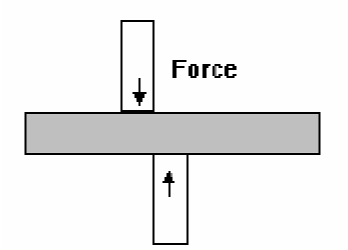
- A Slight increase in stress above the elastic limit will result in breakdown of the material and cause it to deform permanently.
- This behaviour is called yielding
- The stress that cause = YIELD STRESS@YIELD POINT
- Plastic deformation
- Once yield point is reached, the specimen will elongate (Strain) without any increase in load
- Material in this state = perfectly plastic

STRESS STRAIN DIAGRAM

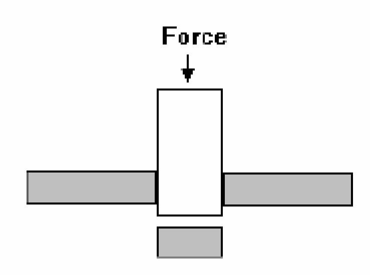
- STRAIN HARDENING
  - When yielding has ended, further load applied, resulting in a curve that rises continuously
  - Become flat when reached ULTIMATE STRESS
  - The rise in the curve = STRAIN HARDENING
  - While specimen is elongating, its cross sectional will decrease
  - The decrease is fairly uniform
- NECKING
  - At the ultimate stress, the cross sectional area begins its localised region of specimen
  - it is caused by slip planes formed within material
  - Actual strain produced by shear strain
  - As a result, “neck” tend to form
  - Smaller area can only carry lesser load, hence curve donward
  - Specimen break at FRACTURE STRESS

SHEAR STRESS τ

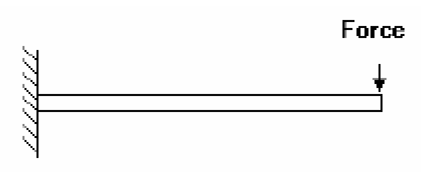
•Shear force is a force applied sideways on the material (transversely loaded).



When a pair of shears cut a material



When a material is punched

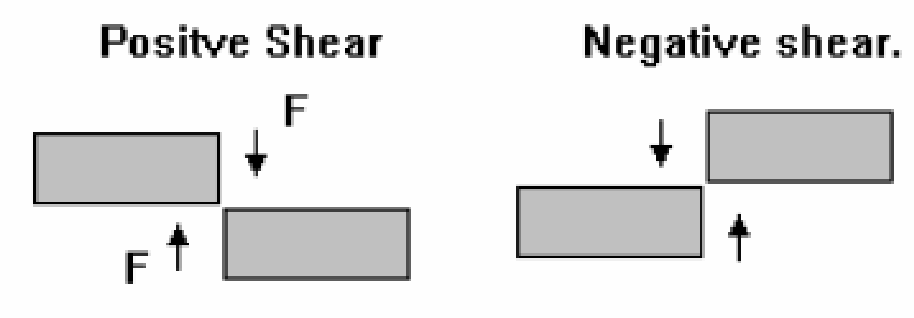


When a beam has a transverse load

➢Shear stress is the force per unit area carrying the load. This means the cross sectional area of the material being cut, the beam and pin.

•Shear stress, $\tau = \dfrac{F}{A}$ and symbol is called Tau

➢The sign convention for shear force and stress is based on how it shears the materials as shown below.



SHEAR STRAIN γ

- The force causes the material to deform as shown. The shear strain is defined as the ratio of the distance deformed to the height

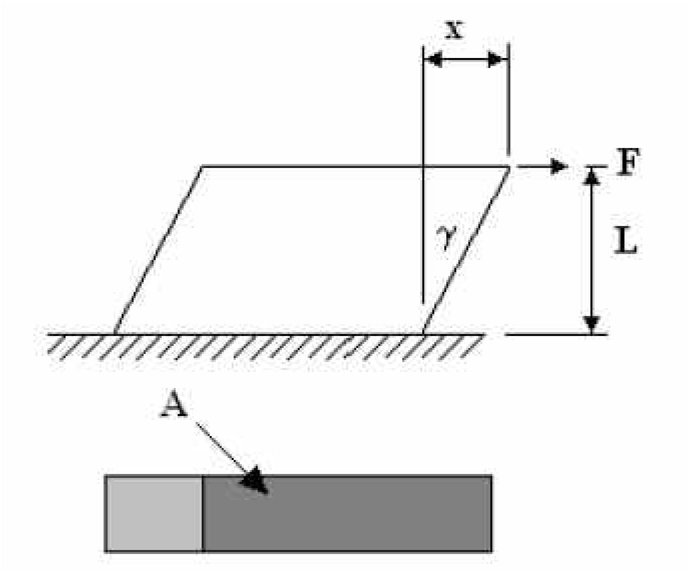
$$\frac{x}{L}$$

. Since this is a very small angle ,γ we can say that :

Shear strain

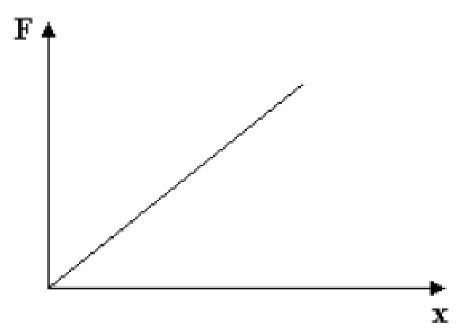
$$\gamma = \frac{x}{L}$$

(symbol called Gamma)



MODULUS OF RIGIDITY (G)

•If we conduct an experiment and measure x for various values of F, we would find that if the material is elastic, it behave like spring and so long as we do not damage the material by using too big force, the graph of F and x is straight line as shown.



•The gradient of the graph is constant so $\frac{F}{x} = constant$

and this is the spring stiffness of the block in N/m.

•If we divide F by area A and x by the height L, the relationship is still a constant and we get

• If we divide F by area A and x by the height L, the relationship is still a constant and we get

$$\frac{\frac{F}{A}}{\frac{x}{L}} = \frac{F}{A} x \frac{L}{x} = \frac{FL}{Ax} = con\tan t$$

Where $\frac{F}{A} = \tau$ then $\frac{F}{A} x \frac{L}{x} = \frac{FL}{Ax} = \frac{\tau}{\gamma} = con\tan t$

$\gamma = \frac{x}{L}$

This constant will have a special value for each elastic material and is called the Modulus of Rigidity (G).

$$\frac{\tau}{\gamma} = G$$

ULTIMATE SHEAR STRESS

- If a material is sheared beyond a certain limit and it becomes permanently distorted and does not spring all the way back to its original shape, the elastic limit has been exceeded.

- If the material stressed to the limit so that it parts into two, the ultimate limit has been reached.

- The ultimate shear stress has symbol τ and this value is used to calculate the force needed by shears and punches.

WORKED EXAMPLE No.1

A metal wire is 2.5 mm diameter and 2 m long. A force of 12 N is applied to it and it stretches 0.3 mm. Assume the material is elastic. Determine the following.

i. The stress in the wire σ.
ii. The strain in the wire ε.

SOLUTION

$$A = \frac{\pi d^2}{4} = \frac{\pi \text{ x } 2.5^2}{4} = 4.909 \text{ mm}^2$$

$$\sigma = \frac{F}{A} = \frac{12}{4.909} = 2.44 \text{ N/mm}^2$$

Answer (i) is hence 2.44 MPa

$$\varepsilon = \frac{x}{L} = \frac{0.3 \text{ mm}}{2000} = 0.00015 \text{ or } 150\ \mu\varepsilon$$

SELF ASSESSMENT EXERCISE No.1

1. A steel bar is 10 mm diameter and 2 m long. It is stretched with a force of 20 kN and extends by 0.2 mm. Calculate the stress and strain.

   (Answers 254.6 MPa and 100 $\mu\varepsilon$)

2. A rod is 0.5 m long and 5 mm diameter. It is stretched 0.06 mm by a force of 3 kN. Calculate the stress and strain.

   (Answers 152.8 MPa and 120$\mu\varepsilon$)

WORKED EXAMPLE No.2

A steel tensile test specimen has a cross sectional area of 100 mm^2 and a gauge length of 50 mm, the gradient of the elastic section is 410 x 10^3 N/mm. Determine the modulus of elasticity.

SOLUTION

The gradient gives the ratio F/A = and this may be used to find E.

$$E = \frac{\sigma}{\varepsilon} = \frac{F}{x} x \frac{L}{A} = 410 \text{ x } 10^3 \text{ x } \frac{50}{100} = 205\,000 \text{ N/mm}^2 \text{ or } 205\,000 \text{ MPa or } 205 \text{ GPa}$$

WORKED EXAMPLE No.3

A Steel column is 3 m long and 0.4 m diameter. It carries a load of 50 MN. Given that the modulus of elasticity is 200 GPa, calculate the compressive stress and strain and determine how much the column is compressed.

SOLUTION

$$A = \frac{\pi d^2}{4} = \frac{\pi \text{ x } 0.4^2}{4} = 0.126 \text{ m}^2$$

$$\sigma = \frac{F}{A} = \frac{50 \text{ x} 10^6}{0.126} = 397.9 \text{ x} 10^6 \text{ Pa}$$

$$E = \frac{\sigma}{\varepsilon} \quad \text{so} \quad \varepsilon = \frac{\sigma}{E} = \frac{397.9 \text{ x} 10^6}{200 \text{ x} 10^9} = 0.001989$$

$$\varepsilon = \frac{x}{L} \quad \text{so} \;\; x = \varepsilon L = 0.001989 \text{ x } 3000 \text{ mm} = 5.97 \text{ mm}$$

SELF ASSESSMENT EXERCISE No.2

1. A bar is 500 mm long and is stretched to 505 mm with a force of 50 kN. The bar is 10 mm diameter. Calculate the stress and strain.

   The material has remained within the elastic limit. Determine the modulus of elasticity.
   (Answers 636.6 MPa, 0.01 and 63.66 GPa.

2. A steel bar is stressed to 280 MPa. The modulus of elasticity is 205 GPa. The bar is 80 mm diameter and 240 mm long.

   Determine the following.

   i. The strain. (0.00136)
   ii. The force. (1.407 MN)

3. A circular metal column is to support a load of 500 Tonne and it must not compress more than 0.1 mm. The modulus of elasticity is 210 GPa. the column is 2 m long.

   Calculate the cross sectional area and the diameter. (0.467 m^2 and 0.771 m)

   Note 1 Tonne is 1000 kg.

WORKED EXAMPLE No.4

Calculate the force needed to guillotine a sheet of metal 5 mm thick and 0.8 m wide given that the ultimate shear stress is 50 MPa.

SOLUTION

The area to be cut is a rectangle 800 mm x 5 mm

$A = 800 \times 5 = 4000 \text{ mm}^2$ The ultimate shear stress is 50 N/mm^2

$$\tau = \frac{F}{A} \quad \text{so} \quad F = \tau \times A = 50 \times 4000 = 200\,000 \text{ N or } 200 \text{ kN}$$

WORKED EXAMPLE No.5

Calculate the force needed to punch a hole 30 mm diameter in a sheet of metal 3 mm thick given that the ultimate shear stress is 60 MPa.

SOLUTION

The area to be cut is the circumference x thickness = $\pi d \times t$

$A = \pi \times 30 \times 3 = 282.7 \text{ mm}^2$ The ultimate shear stress is 60 N/mm^2

$\tau = \dfrac{F}{A}$ so $F = \tau \times A = 60 \times 282.7 = 16965 \text{ N}$ or 16.965 kN

WORKED EXAMPLE No.6

Calculate the force needed to shear a pin 8 mm diameter given that the ultimate shear stress is 60 MPa.

SOLUTION

The area to be sheared is the circular area $A = \frac{\pi d}{4}$

$A = \frac{\pi \times 8^2}{4} = 50.26\text{mm}^2$ The ultimate shear stress is 60 N/mm^2

$\tau = \frac{F}{A}$ so $F = \tau \times A = 60 \times 50.26 = 3016 \text{ N or } 3.016 \text{ kN}$

SELF ASSESSMENT EXERCISE No.3

1. A guillotine must shear a sheet of metal 0.6 m wide and 3 mm thick. The ultimate shear stress is 45 MPa. Calculate the force required. (Answer 81 kN)

2. A punch must cut a hole 30 mm diameter in a sheet of steel 2 mm thick. The ultimate shear stress is 55 MPa. Calculate the force required. (Answer10.37 kN)

3. Two strips of metal are pinned together as shown with a rod 10 mm diameter. The ultimate shear stress for the rod is 60 MPa. Determine the maximum force required to break the pin. (Answer 4.71 kN)

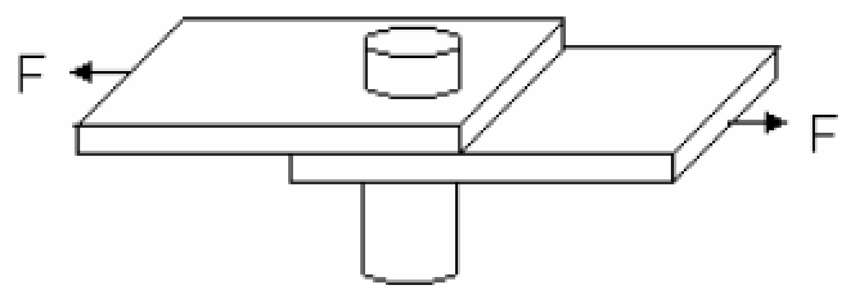
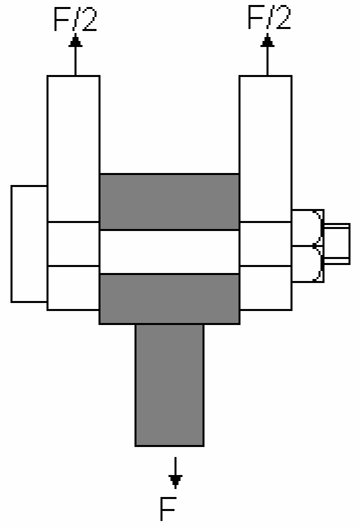


Figure 11

DOUBLE SHEAR

- Consider a pin joint with a support on both ends as shown. This is called CLEVIS and CLEVIS PIN
- By balance of force, the force in the two supports is F/2 each
- The area sheared is twice the cross section of the pin
- So it takes twice as much force to break the pin as for a case of single shear
- Double shear arrangements doubles the maximum force allowed in the pin



WORKED EXAMPLE No.7

A pin is used to attach a clevis to a rope. The force in the rope will be a maximum of 60 kN. The maximum shear stress allowed in the pin is 40 MPa. Calculate the diameter of a suitable pin.

SOLUTION

The pin is in double shear so the shear sress is $\tau = \dfrac{F}{2A}$

$$A = \frac{F}{2\tau} = \frac{60000}{2 \text{ x } 40\text{x}10^6} = 750 \text{ x } 10^{-6}\,\text{m}^2$$

$$A = 750\,\text{mm}^2 = \frac{\pi d^2}{4}$$

$$d = \sqrt{\frac{4 \text{ x } 750}{\pi}} = 30.9\,\text{mm}$$

SELF ASSESSMENT EXERCISE No.4

1. A clevis pin joint as shown above uses a pin 8 mm diameter. The shear stress in the pin must not exceed 40 MPa. Determine the maximum force that can be exerted. (Answer 4.02 kN)

2.

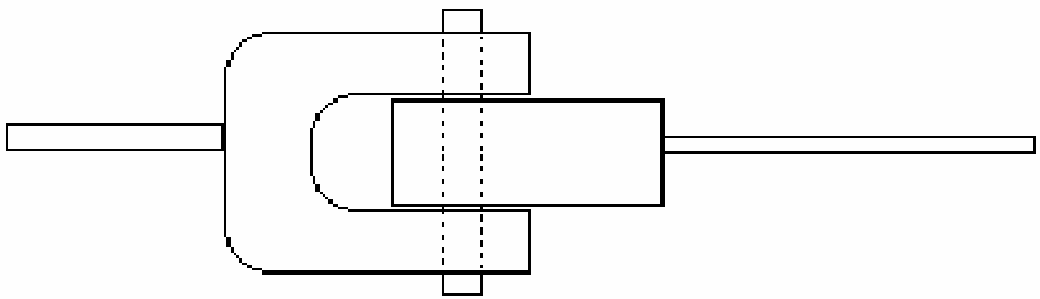


Figure 13

A rope coupling device shown uses a pin 5 mm diameter to link the two parts. If the shear stress in the pin must not exceed 50 MPa, determine the maximum force allowed in the ropes. (Answer 1.96 kN)

LOAD AND STRESS LIMIT

- DESIGN CONSIDERATION
  - Will help engineers with their important task in Designing structural/machine that is SAFE and ECONOMICALLY perform for a specified function

- DETERMINATION OF ULTIMATE STRENGTH
  - An important element to be considered by a designer is how the material that has been selected will behave under a load
  - This is determined by performing specific test (e.g. Tensile test)
  - ULTIMATE FORCE (P_U)= The largest force that may be applied to the specimen is reached, and the specimen either breaks or begins to carry less load
  - ULTIMATE NORMAL STRESS

$$(\sigma_U) = \text{ULTIMATE FORCE}(P_U)/\text{AREA}$$

ALLOWABLE LOAD / ALLOWABLE STRESS

- Max load that a structural member/machine component will be allowed to carry under normal conditions of utilisation is considerably smaller than the ultimate load
- This smaller load = Allowable load / Working load / Design load
- Only a fraction of ultimate load capacity of the member is utilised when allowable load is applied
- The remaining portion of the load-carrying capacity of the member is kept in reserve to assure its safe performance
- The ratio of the ultimate load/allowable load is used to define FACTOR OF SAFETY

- *FACTOR OF SAFETY = ULTIMATE LOAD/ALLOWABLE LOAD*

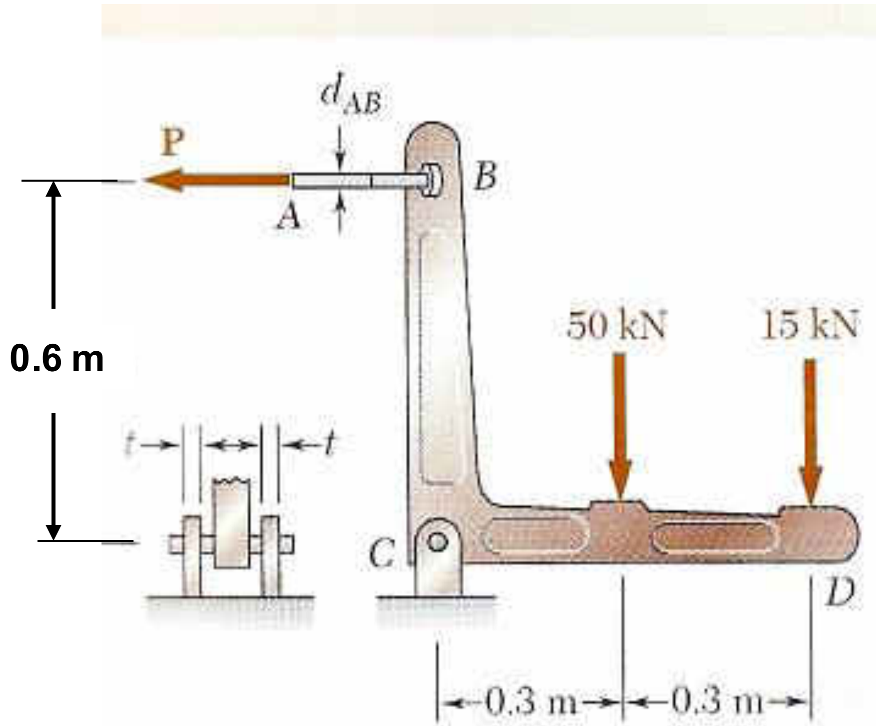
@

- *FACTOR OF SAFETY = ULTIMATE STRESS/ALLOWABLE STRESS*

SELECTION OF F.S.

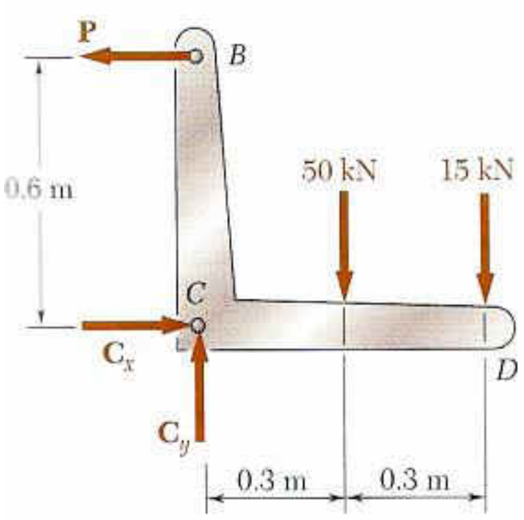
1. Variations that may occur in the properties of the member under considerations
2. The number of loading that may be expected during the life of the structural/machine
3. The type of loading that are planned for in the design, or that may occur in the future
4. The type of failure that may occur
5. Uncertainty due to the methods of analysis
6. Deterioration that may occur in the future because of poor maintenance / because of unpreventable natural causes
7. The importance of a given member to the integrity of the whole structure

WORKED EXAMPLE 8



Two forces are applied to the bracket *BCD* as shown. (*a*) Knowing that the control rod *AB* is to be made of a steel having an ultimate normal stress of 600 MPa, determine the diameter of the rod for which the factor of safety with respect to failure will be 3.3. (*b*) The pin at *C* is to be made of a steel having an ultimate shearing stress of 350 MPa. Determine the diameter of the pin *C* for which the factor of safety with respect to shear will also be 3.3. (*c*) Determine the required thickness of the bracket supports at *C* knowing that the allowable bearing stress of the steel used is 300 MPa.

SOLUTION



Free Body: Entire Bracket. The reaction at C is represented by its components $\mathbf{C}_x$ and $\mathbf{C}_y$.

$$+\circlearrowleft \Sigma M_C = 0: \quad P(0.6 \text{ m}) - (50 \text{ kN})(0.3 \text{ m}) - (15 \text{ kN})(0.6 \text{ m}) = 0 \qquad P = 40 \text{ kN}$$

$$\Sigma F_x = 0: \qquad C_x = 40 \text{ k}$$

$$\Sigma F_y = 0: \qquad C_y = 65 \text{ kN} \qquad C = \sqrt{C_x^2 + C_y^2} = 76.3 \text{ kN}$$

a. Control Rod *AB*. Since the factor of safety is to be 3.3, the allowable stress is

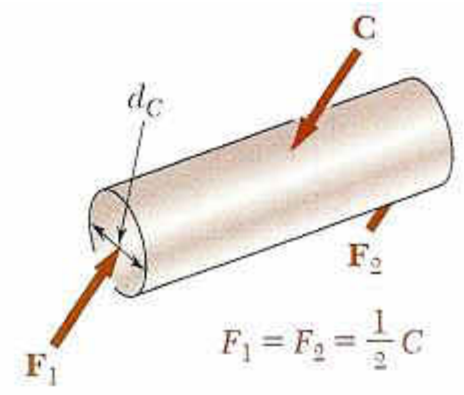
$$\sigma_{\text{all}} = \frac{\sigma_U}{F.S.} = \frac{600 \text{ MPa}}{3.3} = 181.8 \text{ MPa}$$

For $P = 40$ kN the cross-sectional area required is

$$A_{\text{req}} = \frac{P}{\sigma_{\text{all}}} = \frac{40 \text{ kN}}{181.8 \text{ MPa}} = 220 \times 10^{-6} \text{ m}^2$$

$$A_{\text{req}} = \frac{\pi}{4} d_{AB}^2 = 220 \times 10^{-6} \text{ m}^2 \qquad d_{AB} = 16.74 \text{ mm} \blacktriangleleft$$

SOLUTION



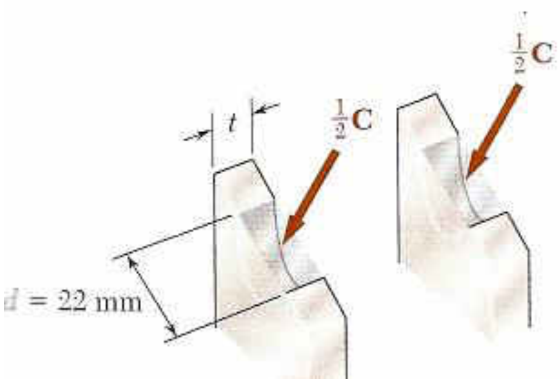
b. Shear in Pin C. For a factor of safety of 3.3, we have

$$\tau_{all} = \frac{\tau_U}{F.S.} = \frac{350\text{ MPa}}{3.3} = 106.1\text{ MPa}$$

Since the pin is in double shear, we write

$$A_{req} = \frac{C/2}{\tau_{all}} = \frac{(76.3\text{ kN})/2}{106.1\text{ MPa}} = 360\text{ mm}^2$$

$$A_{req} = \frac{\pi}{4} d_C^2 = 360\text{ mm}^2 \qquad d_C = 21.4\text{ mm} \qquad \text{Use: } d_C = 22\text{ mm}$$

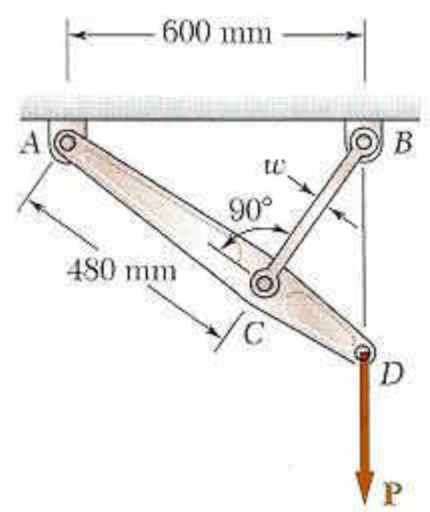


c. Bearing at C. Using $d = 22$ mm, the nominal bearing area of each bracket is $22t$. Since the force carried by each bracket is $C/2$ and the allowable bearing stress is 300 MPa, we write

$$A_{req} = \frac{C/2}{\sigma_{all}} = \frac{(76.3\text{ kN})/2}{300\text{ MPa}} = 127.2\text{ mm}^2$$

Thus $22t = 127.2 \qquad t = 5.78$ mm $\qquad$ Use: $t = 6$ mm ◀

SELF ASSESSMENT NO. 5



1.37 Link BC is 6 mm thick, has a width $w = 25$ mm, and is made of a steel with a 480-MPa ultimate strength in tension. What was the safety factor used if the structure shown was designed to support a 16-kN load **P**?

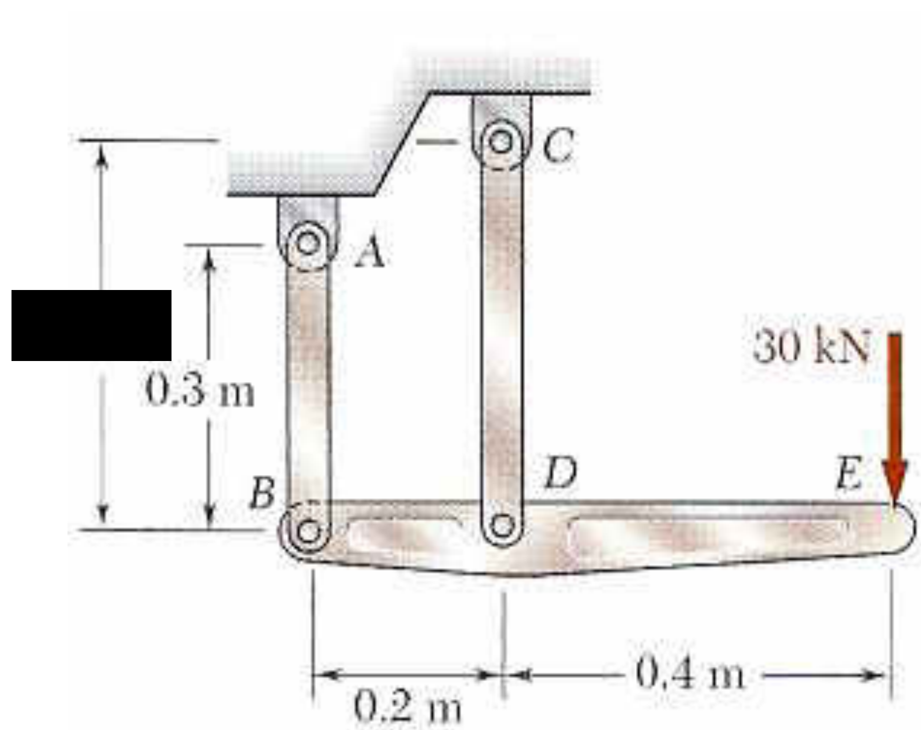
1.38 Link BC is 6 mm thick and is made of a steel with a 450-MPa ultimate strength in tension. What should be its width w if the structure shown is being designed to support a 20-kN load **P** with a factor of safety of 3?

AXIAL FORCE & DEFLECTION OF BODY

- Deformations of members under axial loading
  - If the resulting axial stress does not exceed the proportional limit of the material, Hooke's Law may be applied $\sigma = E\varepsilon$
  - Then deformation (x / δ) can be written as

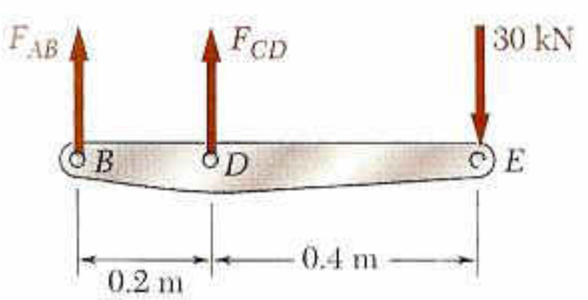
$$\delta = \frac{FL}{AE}$$

WORKED EXAMPLE 9



The rigid bar *BDE* is supported by two links *AB* and *CD*. Link *AB* is made of aluminum (E = 70 GPa) and has a cross-sectional area of 500 mm^2; link *CD* is made of steel (E = 200 GPa) and has a cross-sectional area of 600 mm^2. For the 30-kN force shown, determine the deflection (*a*) of *B*, (*b*) of *D*, (*c*) of *E*.

WORKED EXAMPLE 9



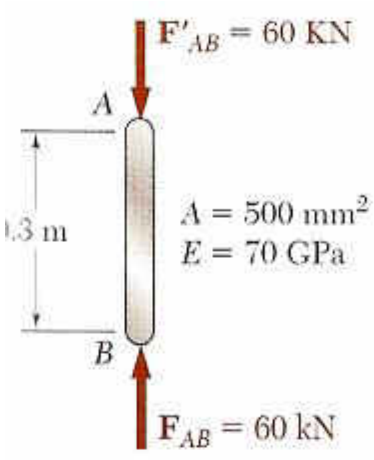
Free Body: Bar *BDE*

$+\circlearrowleft \Sigma M_B = 0$: $\quad -(30 \text{ kN})(0.6 \text{ m}) + F_{CD}(0.2 \text{ m}) = 0$

$F_{CD} = +90 \text{ kN} \qquad F_{CD} = 90 \text{ kN}$ *tension*

$+\circlearrowleft \Sigma M_D = 0$: $\quad -(30 \text{ kN})(0.4 \text{ m}) - F_{AB}(0.2 \text{ m}) = 0$

$F_{AB} = -60 \text{ kN} \qquad F_{AB} = 60 \text{ kN}$ *compression*



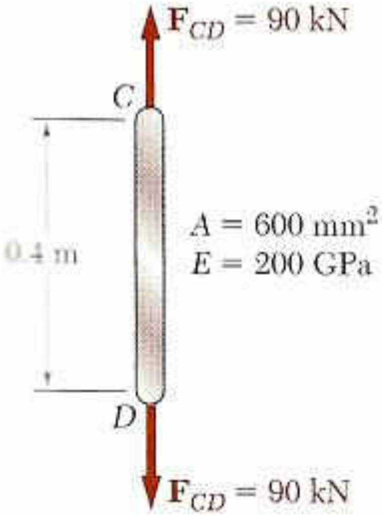
***a*. Deflection of *B*.** Since the internal force in link *AB* is compressive, we have $P = -60$ kN

$$\delta_B = \frac{PL}{AE} = \frac{(-60 \times 10^3 \text{ N})(0.3 \text{ m})}{(500 \times 10^{-6} \text{ m}^2)(70 \times 10^9 \text{ Pa})} = -514 \times 10^{-6} \text{ m}$$

The negative sign indicates a contraction of member *AB*, and, thus, an upward deflection of end *B*:

$$\delta_B = 0.514 \text{ mm} \uparrow \blacktriangleleft$$

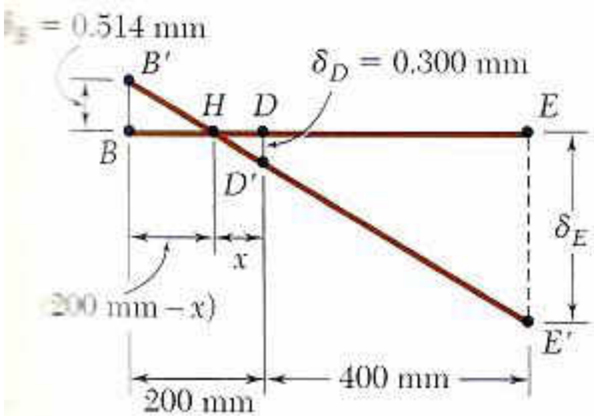
WORKED EXAMPLE 9



b. Deflection of *D*. Since in rod CD, $P = 90$ kN, we write

$$\delta_D = \frac{PL}{AE} = \frac{(90 \times 10^3 \text{ N})(0.4 \text{ m})}{(600 \times 10^{-6} \text{ m}^2)(200 \times 10^9 \text{ Pa})}$$

$$= 300 \times 10^{-6} \text{ m} \qquad \delta_D = 0.300 \text{ mm} \downarrow \blacktriangleleft$$



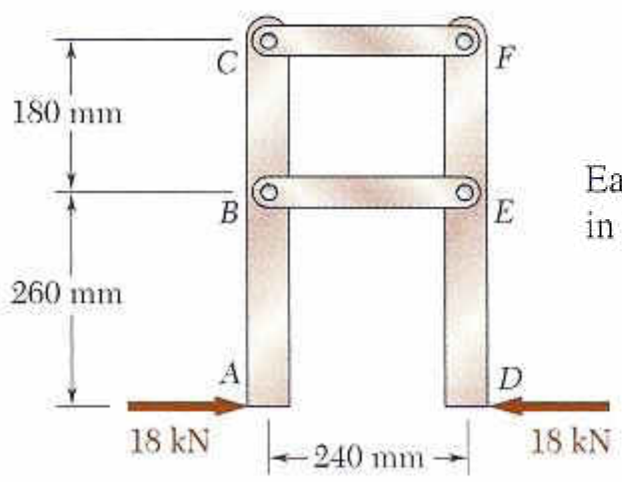
c. Deflection of *E*. We denote by B' and D' the displaced positions of points B and D. Since the bar BDE is rigid, points B', D', and E' lie in a straight line and we write

$$\frac{BB'}{DD'} = \frac{BH}{HD} \qquad \frac{0.514 \text{ mm}}{0.300 \text{ mm}} = \frac{(200 \text{ mm}) - x}{x} \qquad x = 73.7 \text{ mm}$$

$$\frac{EE'}{DD'} = \frac{HE}{HD} \qquad \frac{\delta_E}{0.300 \text{ mm}} = \frac{(400 \text{ mm}) + (73.7 \text{ mm})}{73.7 \text{ mm}}$$

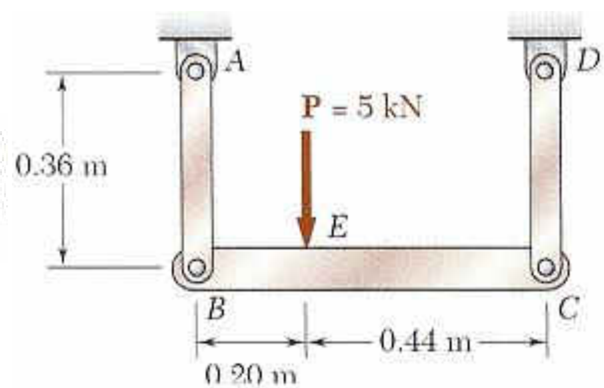
$$\delta_E = 1.928 \text{ mm} \downarrow \blacktriangleleft$$

SELF ASSESSMENT NO. 6



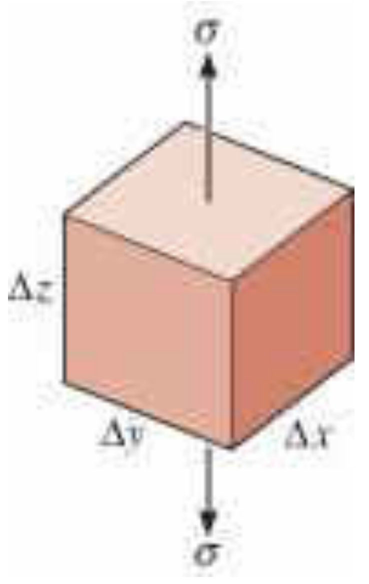
2.26 Members ABC and DEF are joined with steel links ($E = 200$ GPa). Each of the links is made of a pair of 25×35-mm plates. Determine the change in length of (a) member BE, (b) member CF.

2.27 Each of the links AB and CD is made of aluminum ($E = 75$ GPa) and has a cross-sectional area of 125 mm^2. Knowing that they support the rigid member BC, determine the deflection of point E.



- When material is deformed by external loading, energy is stored *internally* throughout its volume
- Internal energy is also referred to as strain energy
- Stress develops a force,

$$\Delta F = \sigma \, \Delta A = \sigma \, (\Delta x \, \Delta y)$$



Resilience

- Strain-energy density is strain energy per unit volume of material

$$u = \frac{\Delta U}{\Delta V} = \frac{\sigma\varepsilon}{2}$$

- If material behavior is linear elastic, Hooke's law applies,

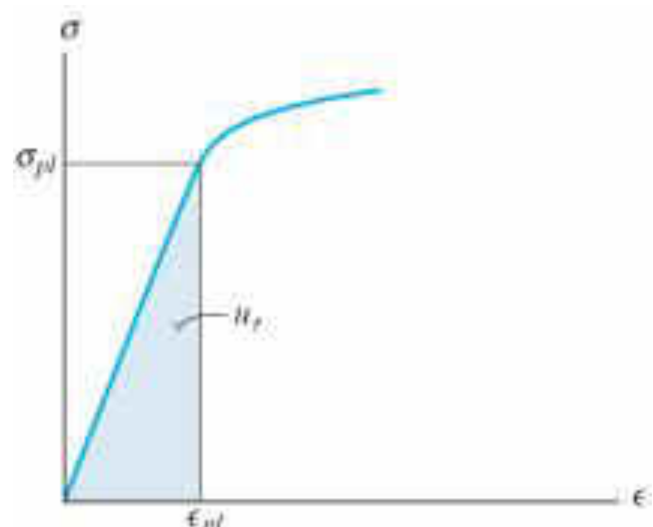
$$u = \frac{\sigma}{2}\left(\frac{\sigma}{\varepsilon}\right) = \frac{\sigma^2}{2E}$$

Modulus of resilience

- When stress reaches proportional limit, strain-energy-energy density is called modulus of resilience

$$u_r = \frac{\sigma_{pl}\,\varepsilon_{pl}}{2} = \frac{\sigma_{pl}^{\,2}}{2E}$$

- A material's resilience represents its ability to absorb energy without any permanent damage

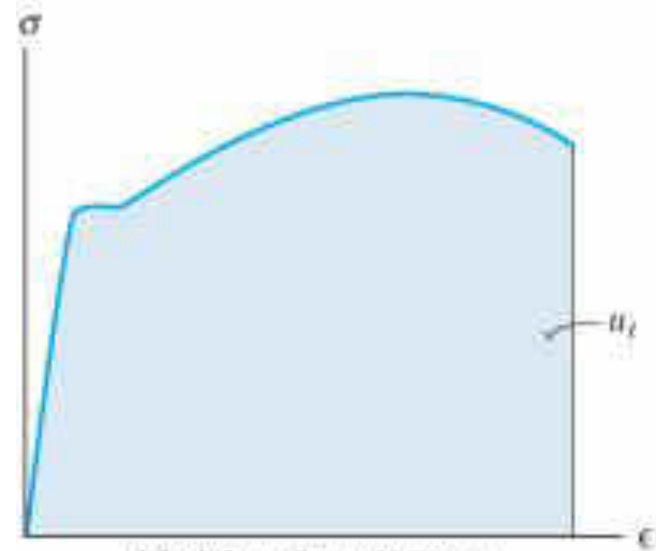


Modulus of resilience u_r

(a)

Modulus of toughness

- Modulus of toughness u_t, indicates the strain-energy density of material before it fractures
- Shaded area under stress-strain diagram is the modulus of toughness



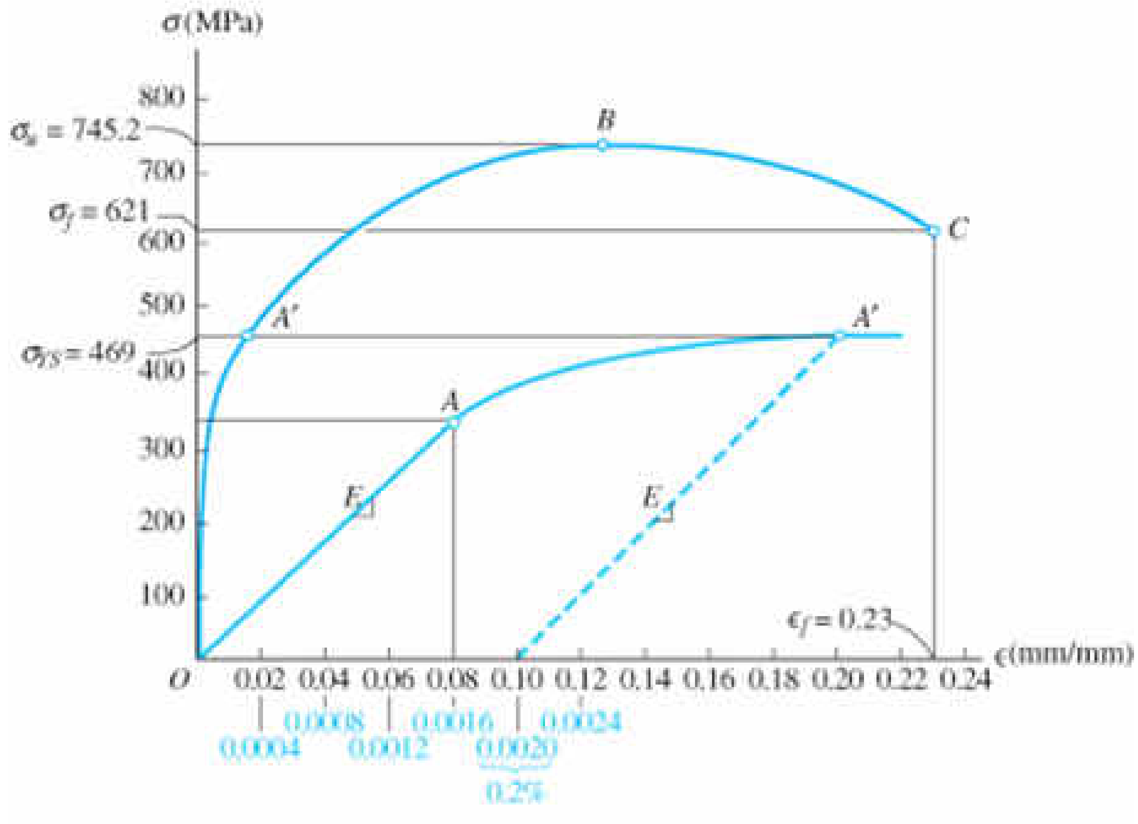
Modulus of toughness u_t

(b)

- Used for designing members that may be accidentally overloaded
- Higher u_t is preferable as distortion is noticeable before failure

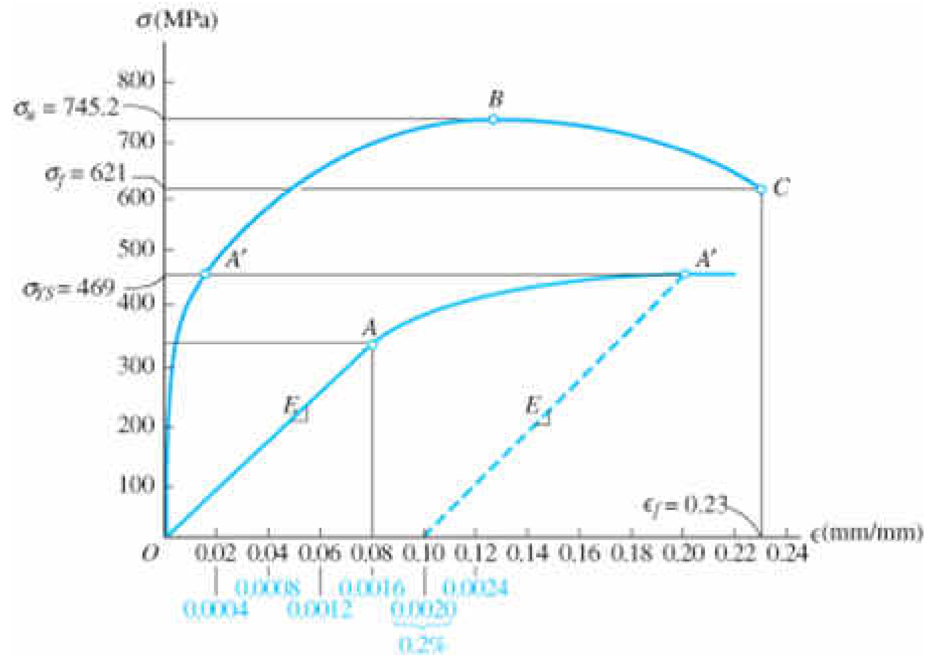
Tension test for a steel alloy results in the stress-strain diagram below.

Calculate the modulus of elasticity and the yield strength based on a 0.2%.



Modulus of elasticity

Calculate the slope of initial straight-line portion of the graph. Use magnified curve and scale shown in light blue, line extends from O to A, with coordinates (0.0016 mm, 345 MPa)



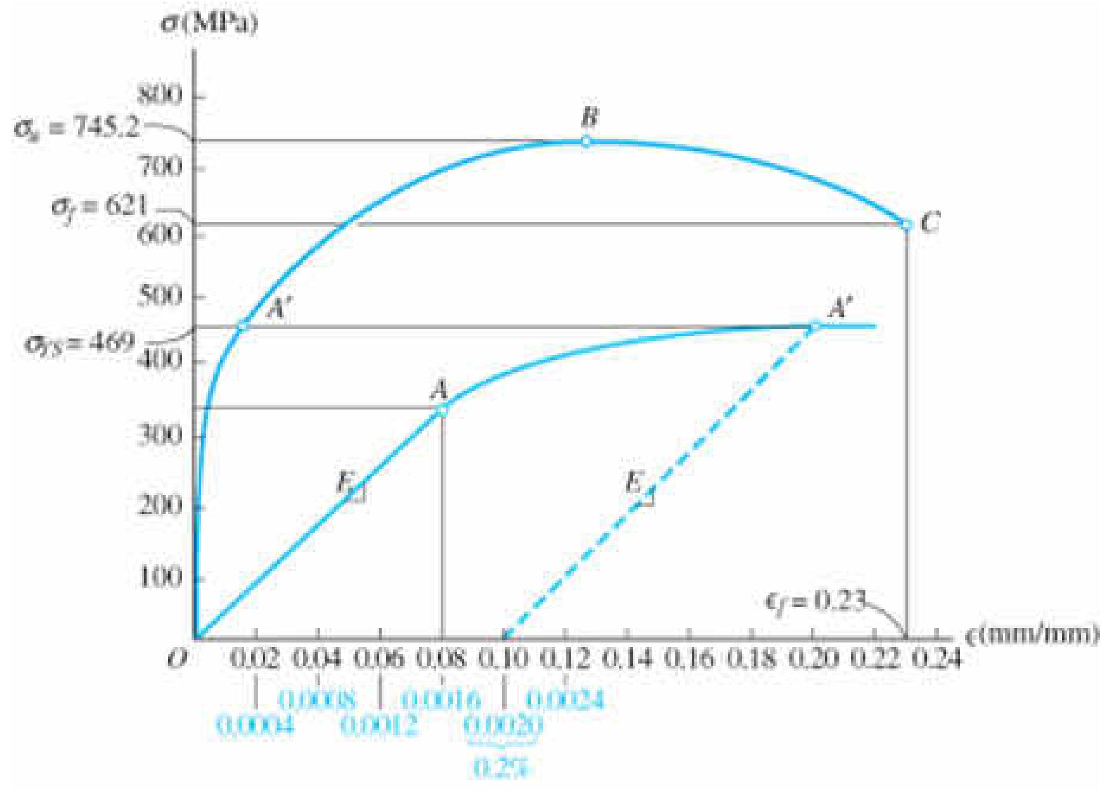
$$E = \frac{345 \text{ MPa}}{0.0016 \text{ mm/mm}}$$

$$= 215 \text{ GPa}$$

Yield strength

At 0.2% strain, extrapolate line (dashed) parallel to *OA* till it intersects stress-strain curve at *A'*

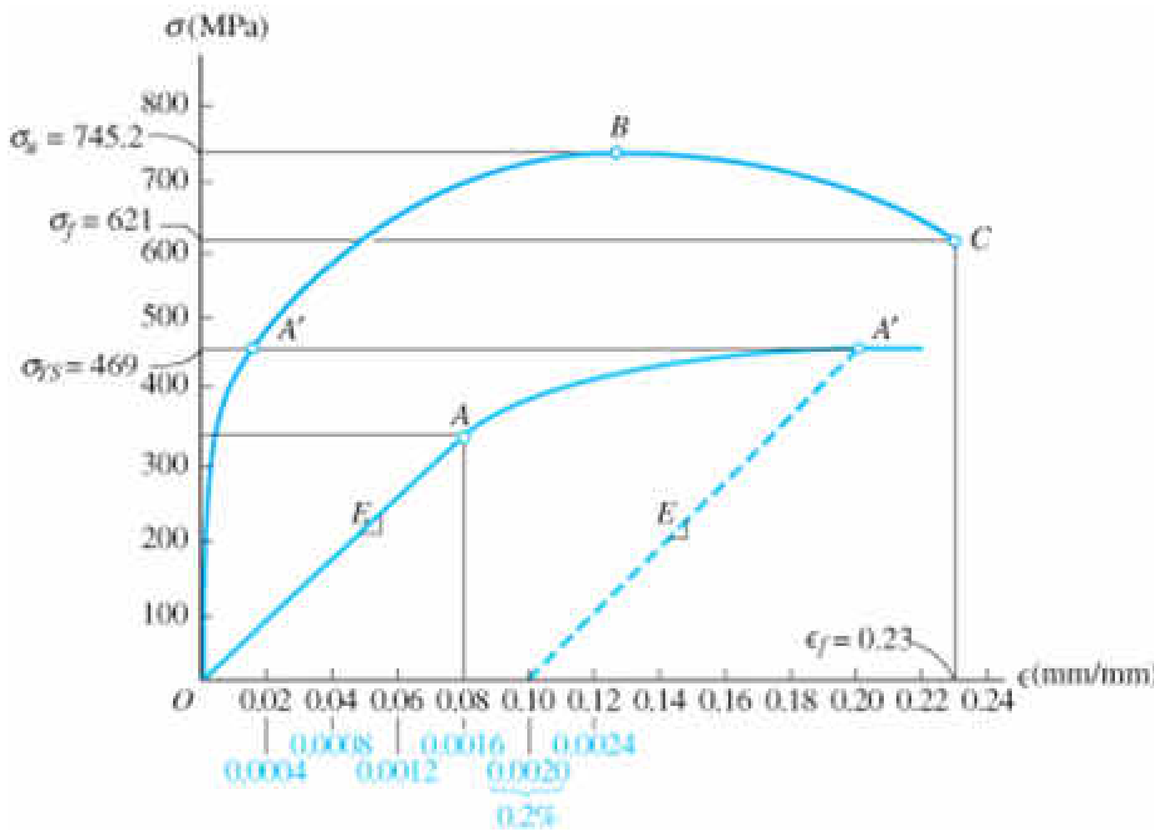
$$\sigma_{YS} = 469 \text{ MPa}$$



Ultimate stress

Defined at peak of graph, point B,

$$\sigma_u = 745.2 \text{ MPa}$$

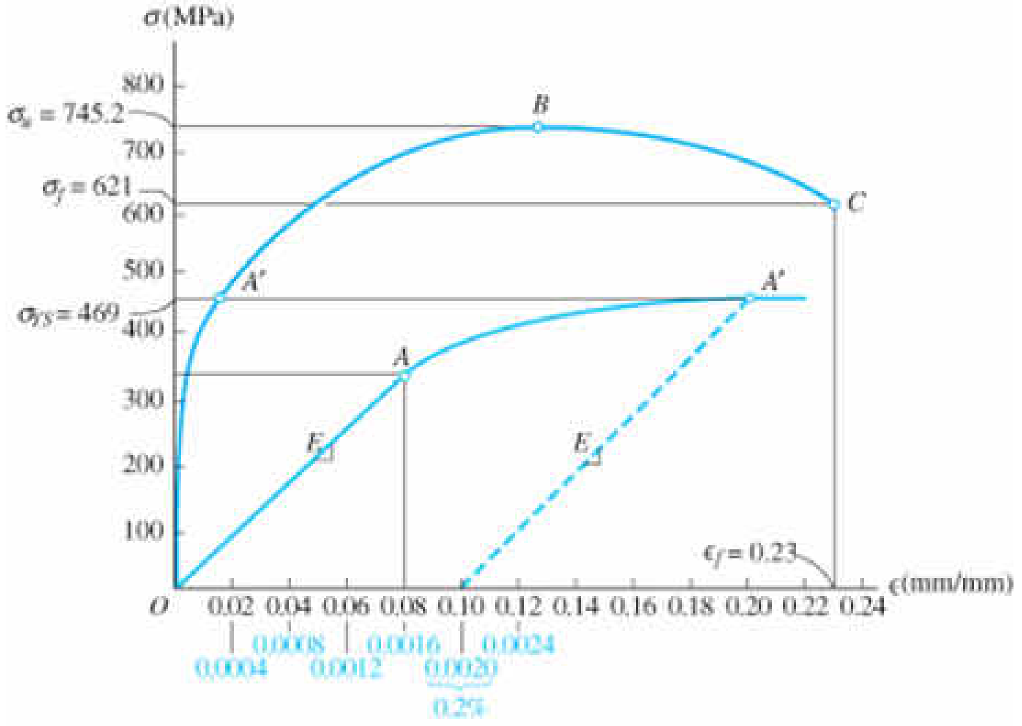


Fracture stress

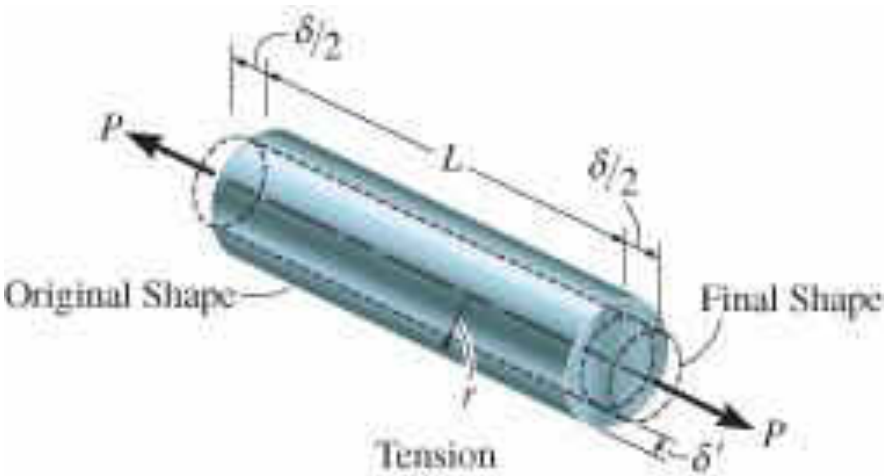
When specimen strained to maximum of ε_f = 0.23 mm/mm, fractures occur at C.

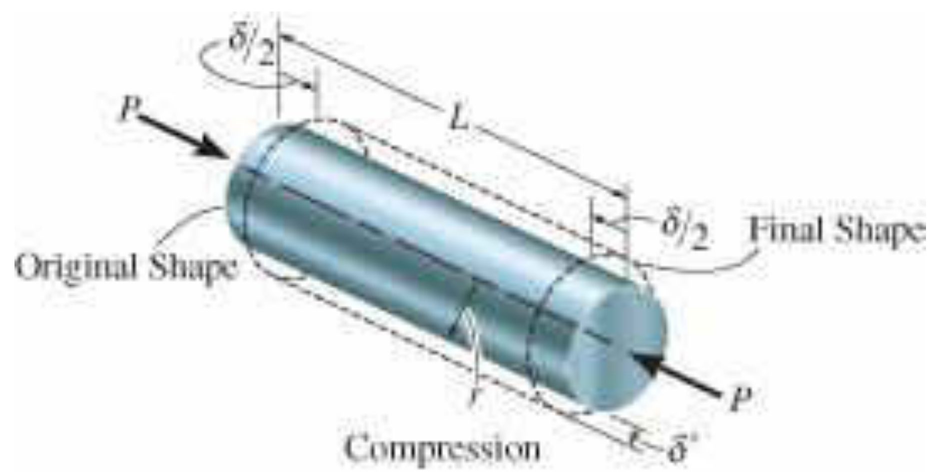
Thus,

$$\sigma_f = 621 \text{ MPa}$$



- When body subjected to axial tensile force, it elongates and contracts laterally
- Similarly, it will contract and its sides expand laterally when subjected to an axial compressive force





- Strains of the bar are:

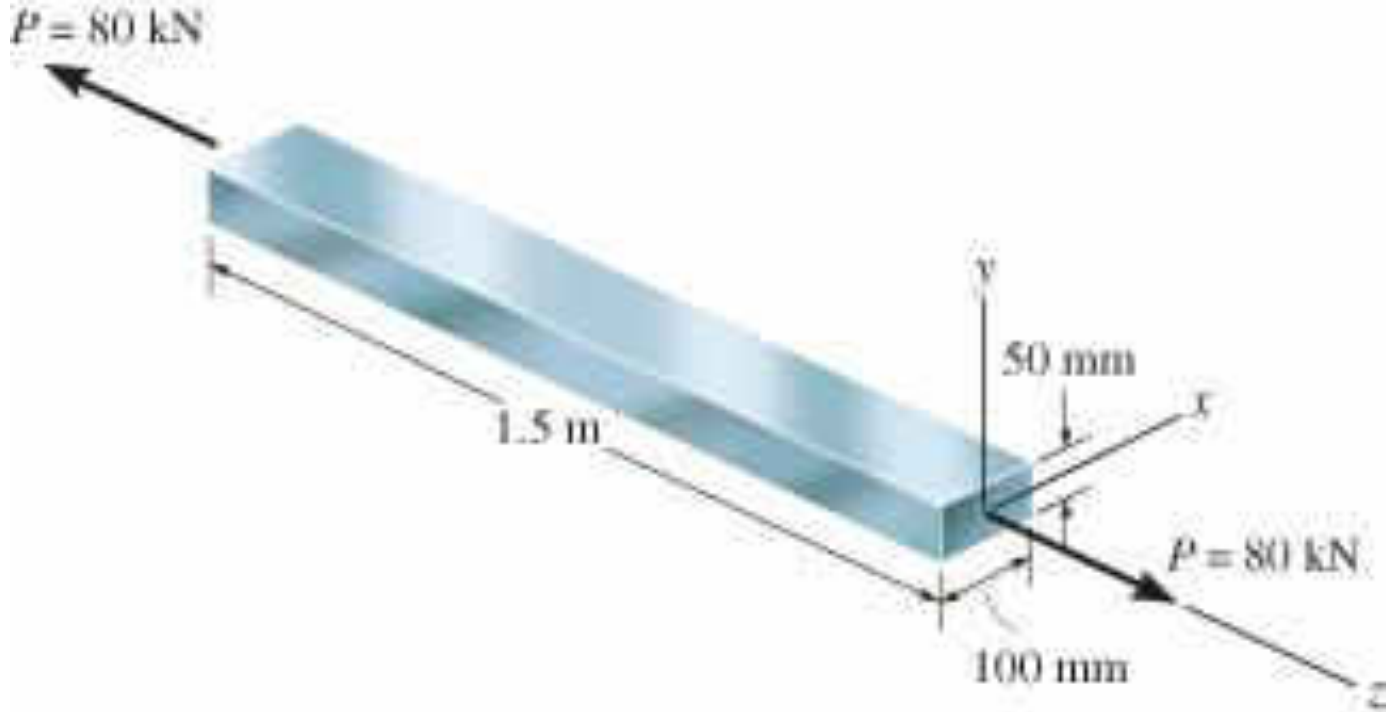
$$\varepsilon_{\text{long}} = \frac{\delta}{L} \qquad \varepsilon_{\text{lat}} = \frac{\delta'}{r}$$

- Early 1800s, S.D. Poisson realized that within elastic range, ration of the two strains is a constant value, since both are proportional.

$$Poisson's\ ratio,\ v = -\frac{\varepsilon_{\text{lat}}}{\varepsilon_{\text{long}}}$$

- *v* is unique for *homogenous* and *isotropic* material
- Why negative sign? Longitudinal elongation cause lateral contraction (-ve strain) and vice versa
- Lateral strain is the same in all lateral (radial) directions
- Poisson's ratio is dimensionless, $0 \leq \nu \leq 0.5$

Bar is made of A-36 steel and behaves elastically. Determine change in its length and change in dimensions of its cross section after load is applied.



Normal stress in the bar is

$$\sigma_z = \frac{P}{A} = 16.0(10^6)\ \text{Pa}$$

From tables, E_{st} = 200 GPa, strain in z-direction is

$$\varepsilon_z = \frac{\sigma_z}{E_{st}} = 80(10^{-6})\ \text{mm/mm}$$

Axial elongation of the bar is,

$$\delta_z = \varepsilon_z L_z = [80(10^{-6})](1.5\ \text{m}) = -25.6\ \mu\text{m/m}$$

Using $v_{st} = 0.32$, contraction strains in both x and y directions are

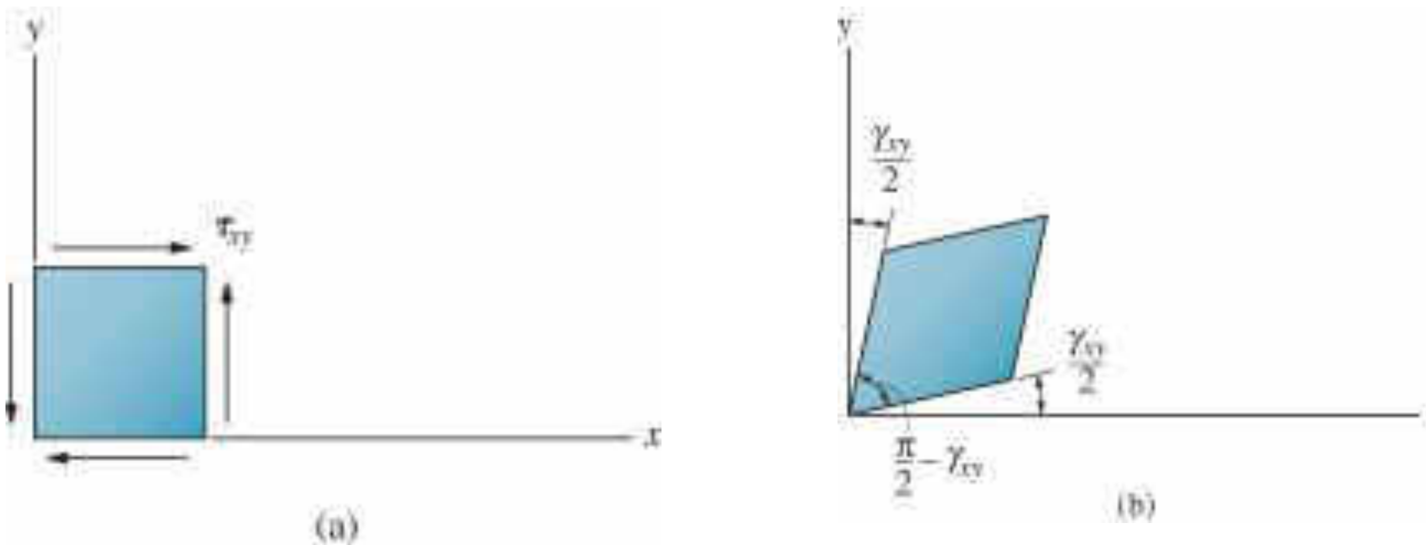
$$\varepsilon_x = \varepsilon_y = -v_{st}\varepsilon_z = -0.32[80(10^{-6})] = -25.6\ \mu\text{m/m}$$

Thus changes in dimensions of cross-section are

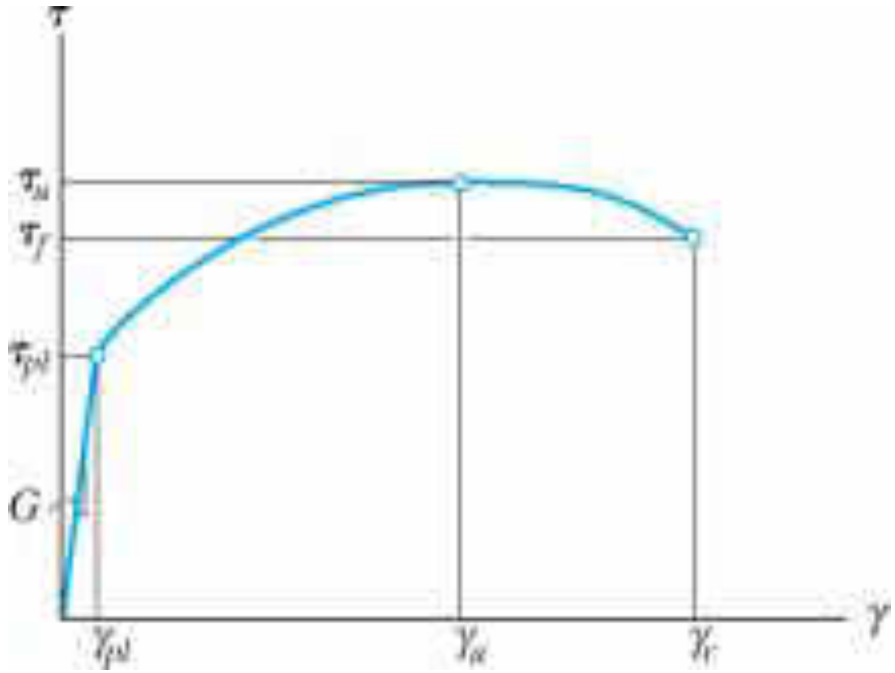
$$\delta_x = \varepsilon_x L_x = -[25.6(10^{-6})](0.1\ \text{m}) = -25.6\ \mu\text{m}$$

$$\delta_y = \varepsilon_y L_y = -[25.6(10^{-6})](0.05\ \text{m}) = -1.28\ \mu\text{m}$$

- Use thin-tube specimens and subject it to torsional loading
- Record measurements of applied torque and resulting angle of twist

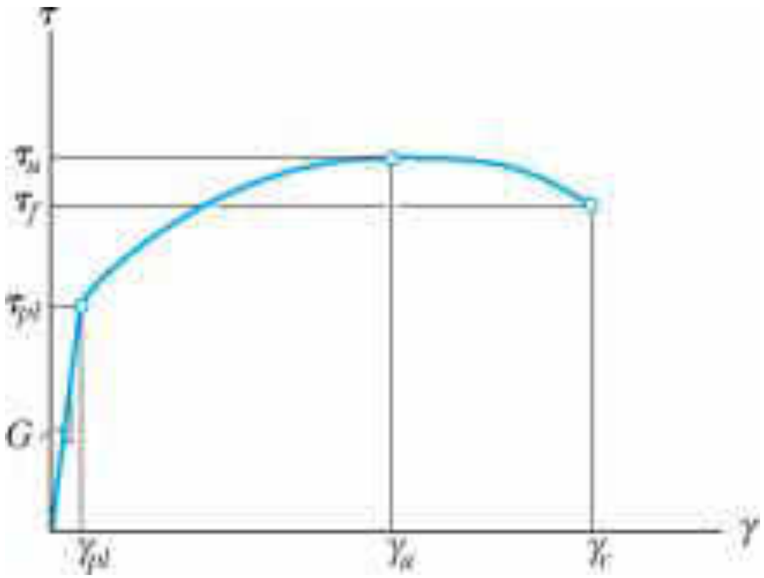


- Material will exhibit linear-elastic behavior till its proportional limit, τ_{pl}
- Strain-hardening continues till it reaches ultimate shear stress, τ_u
- Material loses shear strength till it fractures, at stress of τ_f



- Hooke's law for shear

$$\tau = G\gamma$$



G is shear modulus of elasticity or modulus of rigidity

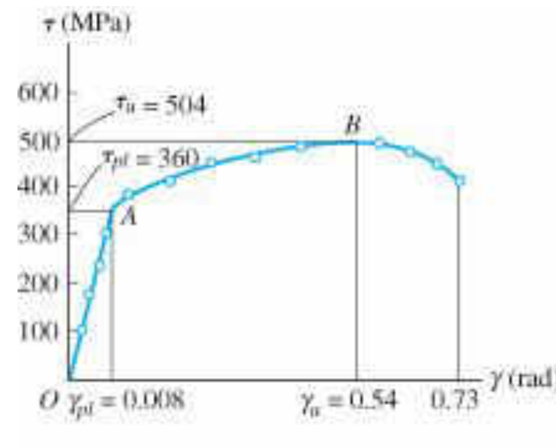
- G can be measured as slope of line on τ-γ diagram, $G = \tau_{pl}/\ \gamma_{pl}$
- The three material constants E, v, and G is related by

$$G = \frac{E}{2(1 + v)}$$

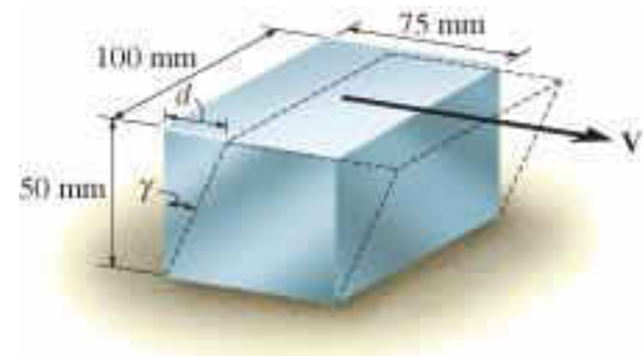
Specimen of titanium alloy tested in torsion & shear stress-strain diagram shown below.

Determine shear modulus G, proportional limit, and ultimate shear stress.

Also, determine the maximum distance d that the top of the block shown, could be displaced horizontally if material behaves elastically when acted upon by $\mathbf{V}$. Find magnitude of $\mathbf{V}$ necessary to cause this displacement.



(a)



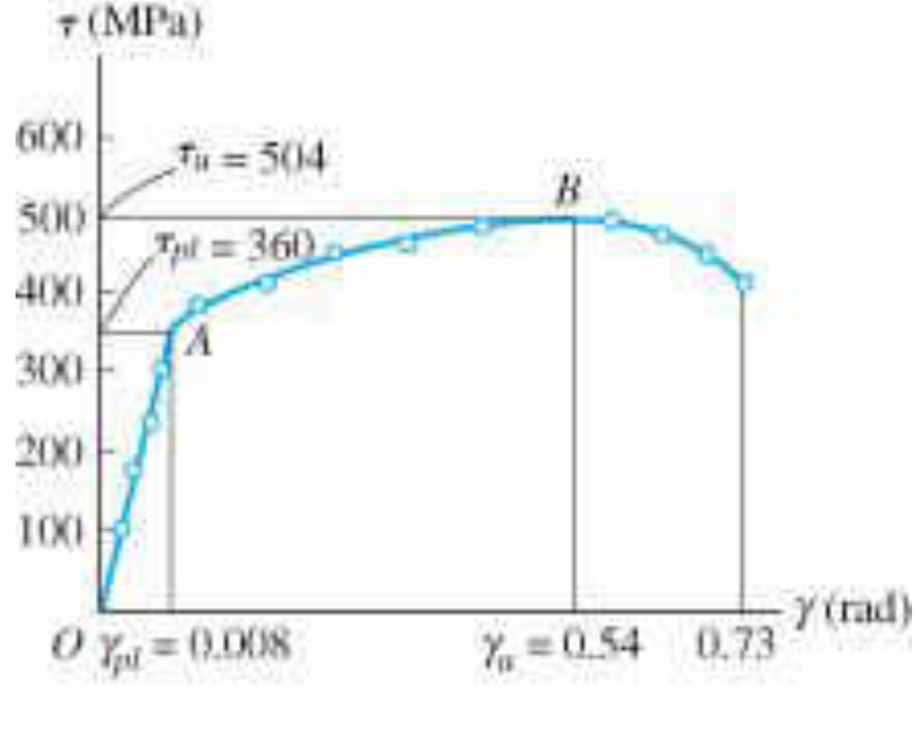
(b)

Shear modulus

Obtained from the slope of the straight-line portion OA of the τ-γ diagram. Coordinates of A are (0.008 rad, 360 MPa)

$$G = \frac{360 \text{ MPa}}{0.008 \text{ rad}}$$

$$= 45(10^3) \text{ MPa}$$

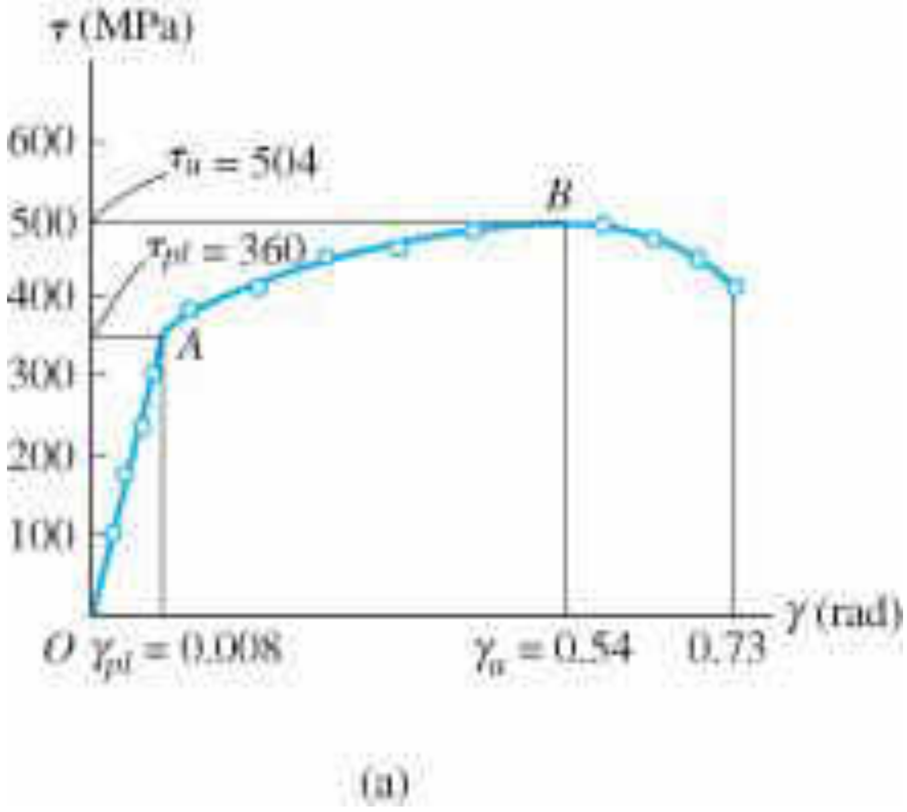


(a)

Proportional limit

By inspection, graph ceases to be linear at point A, thus,

$$\tau_{pl} = 360 \text{ MPa}$$



Ultimate stress

From graph,

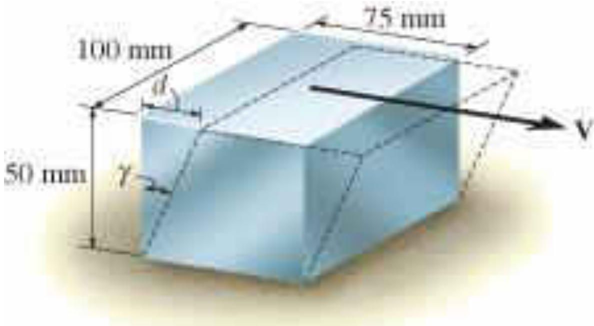
$$\tau_u = 504 \text{ MPa}$$

Maximum elastic displacement and shear force

By inspection, graph ceases to be linear at point A, thus,

$$\tan(0.008\text{ rad}) \approx 0.008\text{ rad} = \frac{d}{50\text{ mm}}$$

$$d = 0.4\text{ mm}$$



(b)

$$\tau_{avg} = \frac{V}{A} \qquad 360\text{ MPa} = \frac{V}{(75\text{ mm})(100\text{ mm})}$$

$$V = 2700\text{ kN}$$

Creep

- Occurs when material supports a load for very long period of time, and continues to deform until a sudden fracture or usefulness is impaired
- Is only considered when metals and ceramics are used for structural members or mechanical parts subjected to high temperatures
- Other materials (such as polymers & composites) are also affected by creep without influence of temperature

Creep

- Stress and/or temperature significantly affects the rate of creep of a material
- Creep strength represents the *highest initial stress* the material can withstand during *given time* without causing *specified creep strain*

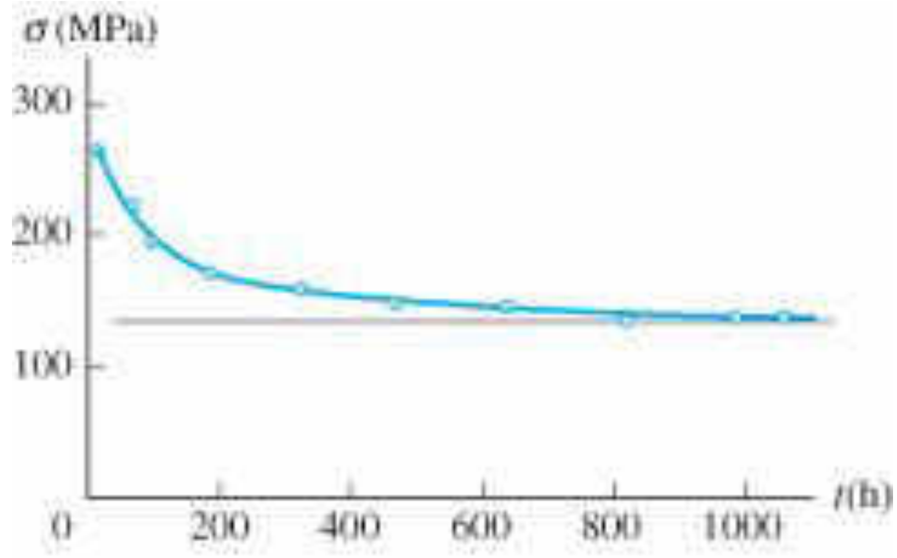
Simple method to determine creep strength

- Test several specimens simultaneously
  - At constant temperature, but
  - Each specimen subjected to different axial stress

Creep

Simple method to determine creep strength

- Measure time taken to produce allowable strain or rupture strain for each specimen
- Plot stress vs. strain
- Creep strength inversely proportional to temperature and applied stresses



σ–t diagram for stainless steel at 650°C and creep strain at 1%

Fatigue

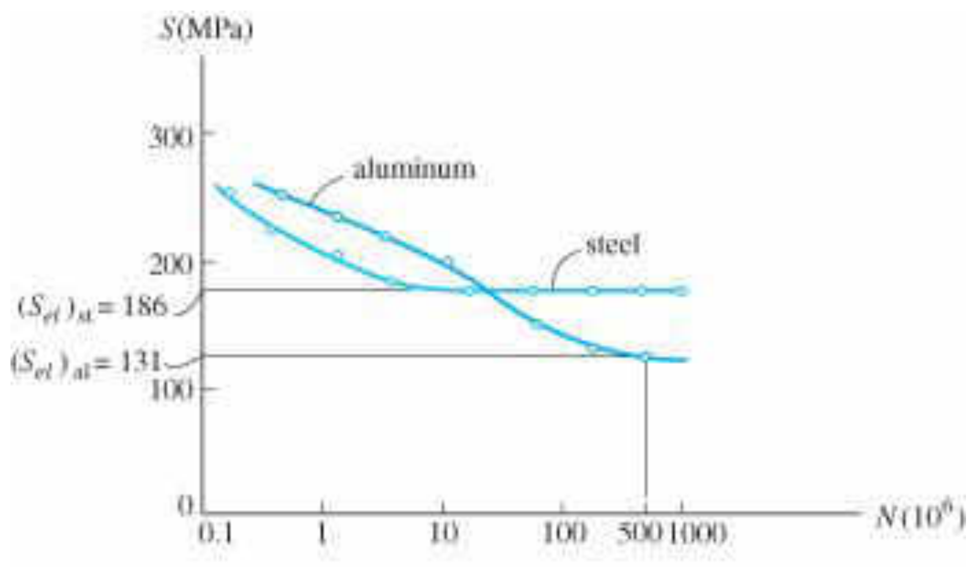
- Defined as a metal subjected to repeated cycles of stress and strain, breaking down structurally, before fracturing
- Needs to be accounted for in design of connecting rods (e.g. steam/gas turbine blades, connections/supports for bridges, railroad wheels/axles and parts subjected to cyclic loading)
- Fatigue occurs at a stress *lesser* than the material's yield stress

Fatigue

- Also referred to as the endurance or fatigue limit

Method to get value of fatigue

- Subject series of specimens to specified stress and cycled to failure



S–N diagram for steel and aluminum alloys (N axis has a logarithmic scale)

- Plot stress (S) against number of cycles-to-failure N (S-N diagram) on logarithmic scale

- *Tension test* is the most important test for determining material strengths. Results of normal stress and normal strain can then be plotted.
- Many engineering materials behave in a *linear-elastic* manner, where stress is proportional to strain, defined by Hooke's law, $\sigma = E\varepsilon$. E is the modulus of elasticity, and is measured from slope of a stress-strain diagram
- When material stressed beyond yield point, permanent deformation will occur.

- Strain hardening causes further yielding of material with increasing stress
- At ultimate stress, localized region on specimen begin to constrict, and starts *"necking"*. Fracture occurs.
- *Ductile* materials exhibit both plastic and elastic behavior. *Ductility* specified by permanent elongation to failure or by the permanent reduction in cross-sectional area
- *Brittle* materials exhibit little or no yielding before failure

- Yield point for material can be increased by *strain hardening*, by applying load great enough to cause increase in stress causing yielding, then releasing the load. The larger stress produced becomes the new yield point for the material
- Deformations of material under load causes *strain energy* to be stored. Strain energy per unit volume/strain energy density is equivalent to area under stress-strain curve.

- The area up to the yield point of stress-strain diagram is referred to as the *modulus of resilience*
- The entire area under the stress-strain diagram is referred to as the *modulus of toughness*
- *Poisson's ratio* (v), a dimensionless property that measures the lateral strain to the longitudinal strain [$0 \leq v \leq 0.5$]
- For shear stress vs. strain diagram: within elastic region, $\tau = G\gamma$, where G is the *shearing modulus*, found from the slope of the line within elastic region

- G can also be obtained from the relationship of $G = \mathrm{E}/[2(1+ v)]$
- When materials are in service for long periods of time, *creep* and *fatigue* are important.
- Creep is the time rate of deformation, which occurs at high stress and/or high temperature. Design the material not to exceed a predetermined stress called the *creep strength*

- *Fatigue* occur when material undergoes a large number of cycles of loading. Will cause micro-cracks to occur and lead to brittle failure.
- Stress in material must not exceed specified *endurance or fatigue limit*

MOMENT OF INERTIA

WHAT IS MOMENT OF INERTIA?

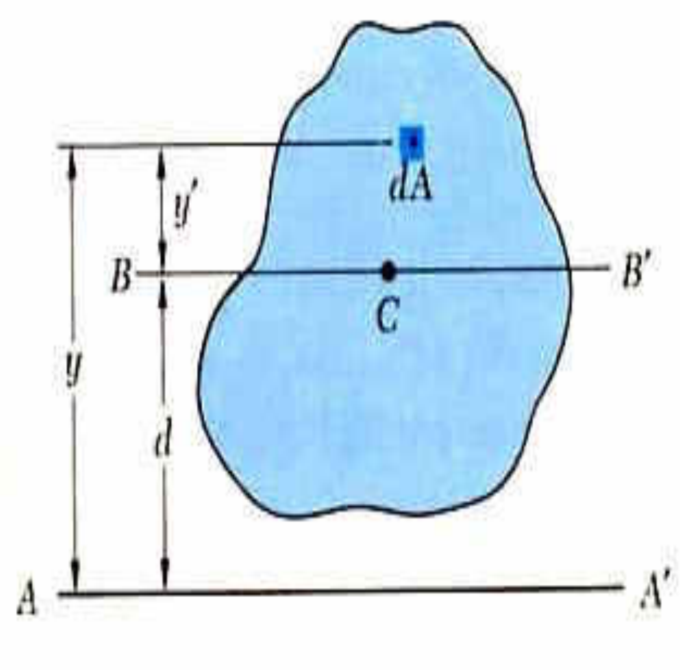
- IT IS THE MOMENT REQUIRED BY A SOLID BODY TO OVERCOME IT'S RESISTANCE TO ROTATION
- IT IS RESISTANCE OF BENDING MOMENT OF A BEAM
- IT IS THE SECOND MOMENT OF MASS (mr^2) OR SECOND MOMENT OF AREA (Ar^2)
- IT'S UNIT IS m^4 OR kgm^2

PERPENDICULAR AXIS THEOREM

- The moment of inertia of a plane area about an axis normal to the plane is equal to the sum of the moments of inertia about any two mutually perpendicular axes lying in the plane and passing through the given axis.
  Moment of Inertia: $I_z = I_x + I_y$

PARALLEL (TRANSFER)AXIS THEOREM

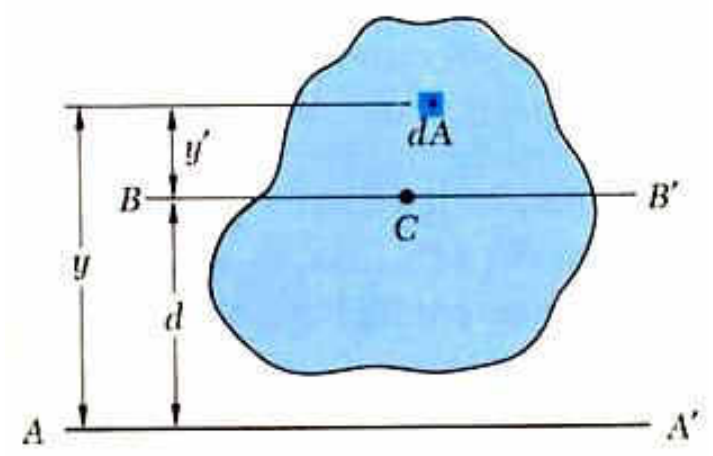
- ***THE MOMENT OF AREA OF AN OBJECT ABOUT ANY AXIS PARALLEL TO THE CENTROIDAL AXIS IS THE SUM OF MI ABOUT IT'S CENTRODAL AXIS AND THE PRODUCT OF AREA WITH THE SQUARE OF DISTANCE OF CG FROM THE REF AXIS***
- $I_{XX} = I_G + Ad^2$
- A is the cross-sectional area.
  : is the perpendicuar distance between the centroidal axis and the parallel axis.



Moment of Inertia - Parallel Axis Theorem

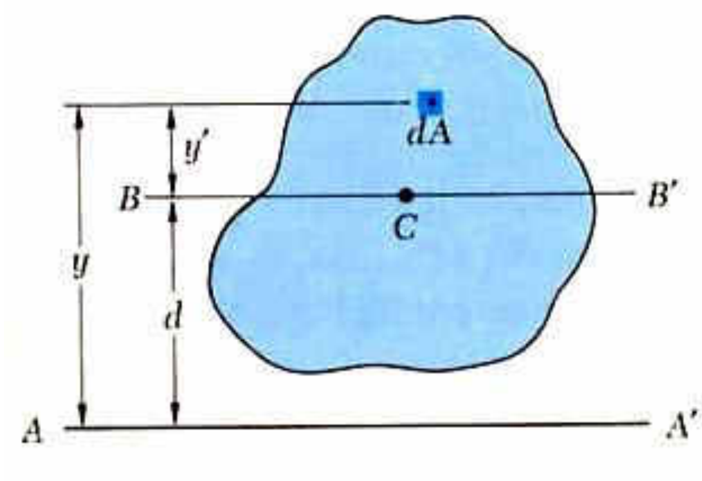
Parallel axis theorem:

Consider the moment of inertia Ix of an area A with respect to an axis AA'. Denote by y the distance from an element of area dA to AA'.



$$I_x = \int y^2 dA$$

Moment of Inertia - Parallel Axis Theorem

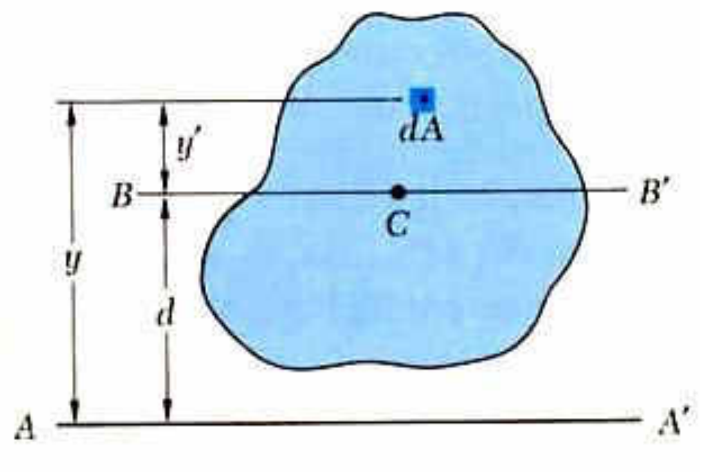


Consider an axis BB' parallel to AA' through the centroid C of the area, known as the centroidal axis. The equation of the moment inertia becomes

$$I_x = \int y^2 dA = \int (y' + d)^2 dA$$

$$= \int y'^2 dA + 2\int y' dA + d^2 \int dA$$

Moment of Inertia - Parallel Axis Theorem



The first integral is the moment of inertia about the centroid.

$$\overline{I_x} = \int y'^2 dA$$

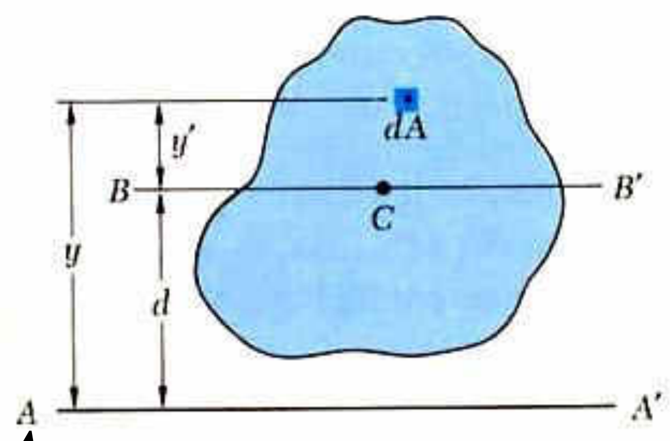
The second component is the first moment area about the centroid

$$\overline{y'} A = \int y' dA \Rightarrow \overline{y'} = 0$$

$$\Rightarrow \int y' dA = 0$$

Moment of Inertia - Parallel Axis Theorem

Modify the equation obtained with the parallel axis theorem.

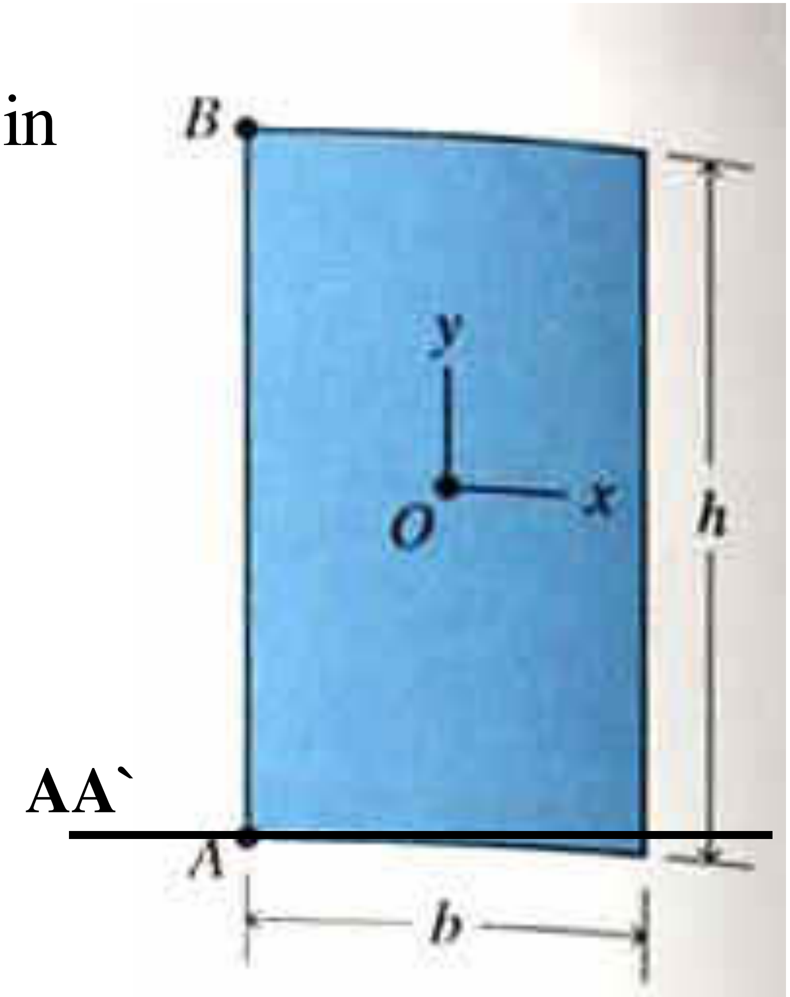


$$I_x = \int y'^2 dA + 2\cancel{\int y' dA} + d^2 \int dA$$

$$= \overline{I}_x + d^2 A$$

Example – Moment of Inertia

Compute the moment of inertia in the x about the AA` plane.

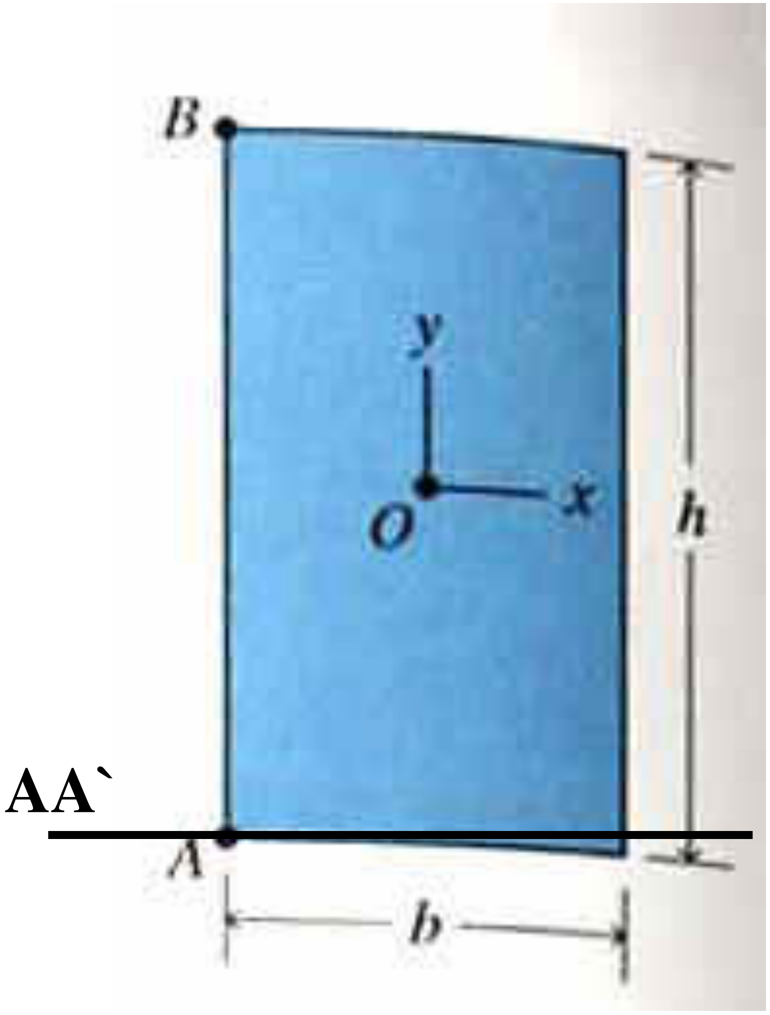


Example – Moment of Inertia

Compute the moment of inertia in the x about the AA` plane.

$$I_x = \int_{\text{Area}} y^2 dA = \int_0^h \int_0^b y^2 dx dy$$

$$= \left[b \frac{y^3}{3} \right]_0^h = \frac{bh^3}{3}$$

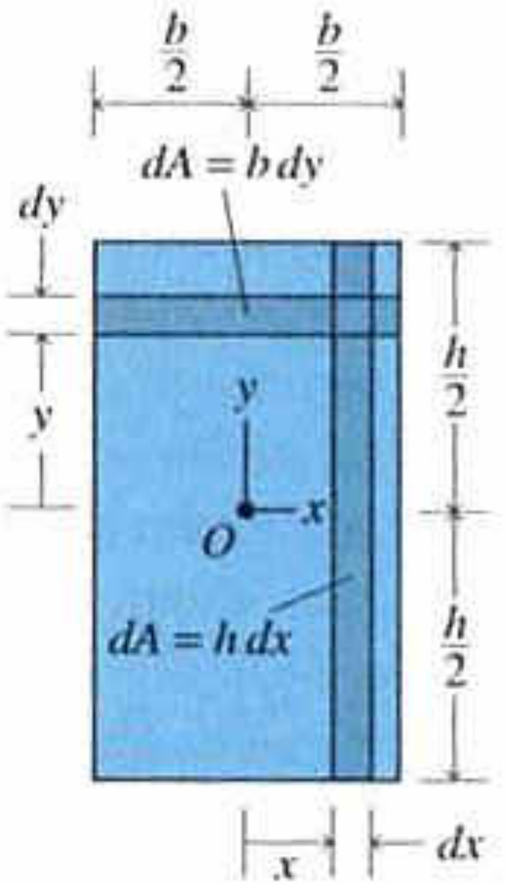


Example – Moment of Inertia

From earlier lecture, the moment of inertia about the centroid

$$\overline{I_x} = \int_{\text{Area}} y^2 dA = \int_{-h/2}^{h/2} y^2 b\,dy$$

$$= \left[b\frac{y^3}{3} \right]_{-h/2}^{h/2} = \frac{bh^3}{12}$$



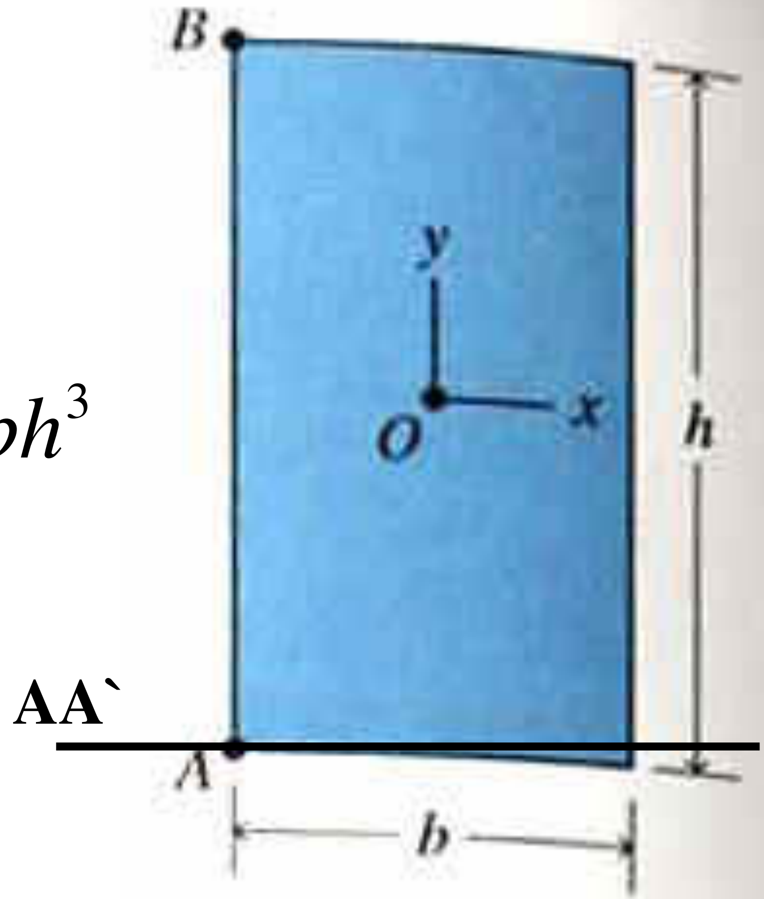
Example – Moment of Inertia

Using the parallel axis theorem

$$I_x = \overline{I_x} + d^2 A$$

$$= \frac{bh^3}{12} + \left(\frac{h}{2}\right)^2 (bh) = \left(\frac{4}{12}\right) bh^3$$

$$= \frac{bh^3}{3}$$



Parallel Axis - Why?

Recall that the method of finding centroids of composite bodies? -

Follow a Table technique

How would you be able to find the moment of inertia of the body. Use a similar technique, table method, to find the moment of inertia of the body.

Parallel Axis - Why?

Use a similar technique, table method, to find the moment of inertia of the body.

Bodies	A_i	y_i	y_i*A_i	I_i	d_i=y_i-ybar	$d_i^2A_i$

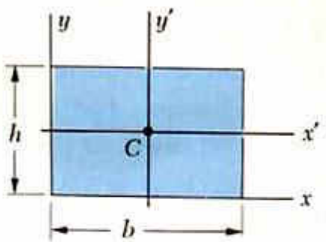
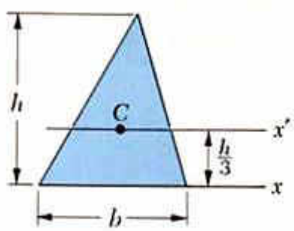
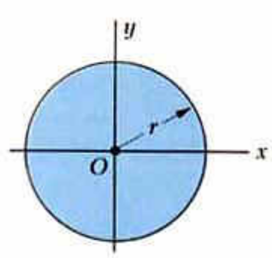
$$\bar{y} = \frac{\sum y_i A_i}{\sum A_i}$$

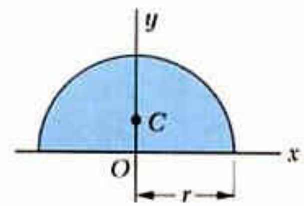
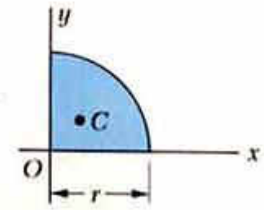
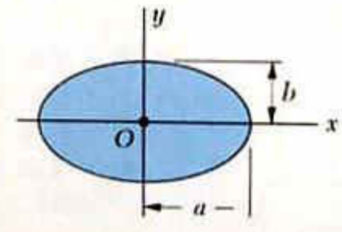
$$I_x = \sum\left(\bar{I}_x + d^2 A\right)_i$$

$$= \sum \bar{I}_{xi} + \sum\left(y_i - \bar{y}\right)^2 A_i$$

Moment of Inertia

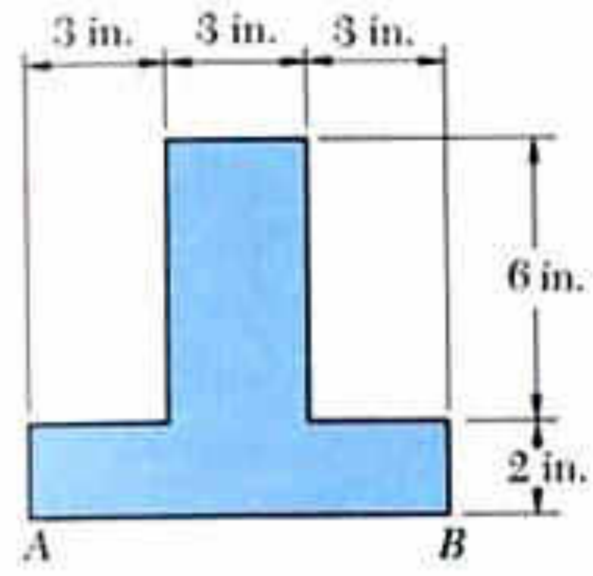
Use a set of standard tables:

Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$

Semicircle		$I_x = I_y = \frac{1}{8}\pi r^4$ $J_O = \frac{1}{4}\pi r^4$
Quarter circle		$I_x = I_y = \frac{1}{16}\pi r^4$ $J_O = \frac{1}{8}\pi r^4$
Ellipse		$\bar{I}_x = \frac{1}{4}\pi ab^3$ $\bar{I}_y = \frac{1}{4}\pi a^3b$ $J_O = \frac{1}{4}\pi ab(a^2 + b^2)$

Example - Moment of Inertia

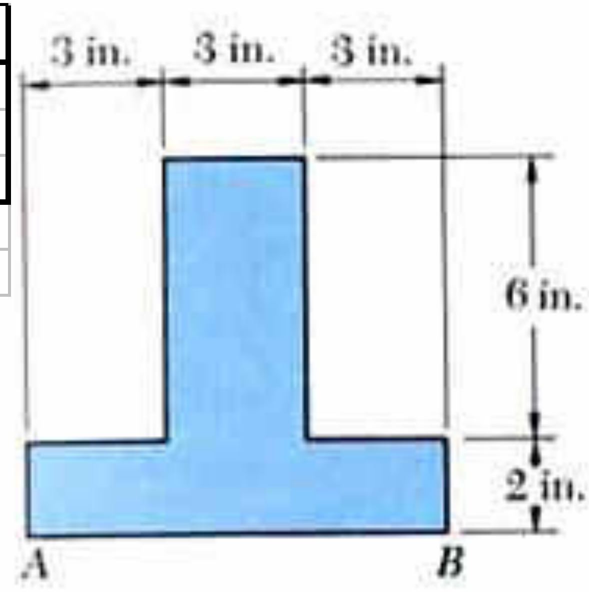
Find the moment of inertia of the body, I_x and the radius of gyration, k_x (r_x)



Example - Moment of Inertia

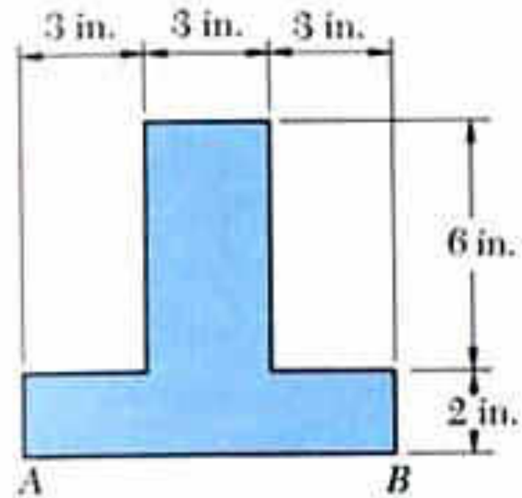
Set up the reference axis at AB and find the centroid

Bodies	A_i	y_i	$y_i{*}A_i$	I_i	$d_i=y_i\text{-ybar}$	$d_i^2A_i$
1	18	1	18			
2	18	5	90			
	36		108			



$$\bar{y} = \frac{\sum y_i A_i}{\sum A_i} = \frac{108 \text{ in}^3}{36 \text{ in}^2} = 3.0 \text{ in.}$$

Example - Moment of Inertia



From the table to find the moment of inertia

Bodies	A_i	y_i	$y_i{}^*A_i$	I_i	$d_i=y_i\text{-ybar}$	$d_i^2A_i$
1	18	1	18	6	-2	72
2	18	5	90	54	2	72
	36		108	60		144
ybar	3	in.				
I	204	in^4				

$$I_x = \sum \overline{I_{xi}} + \sum \left(y_i - \bar{y} \right)^2 A_i$$

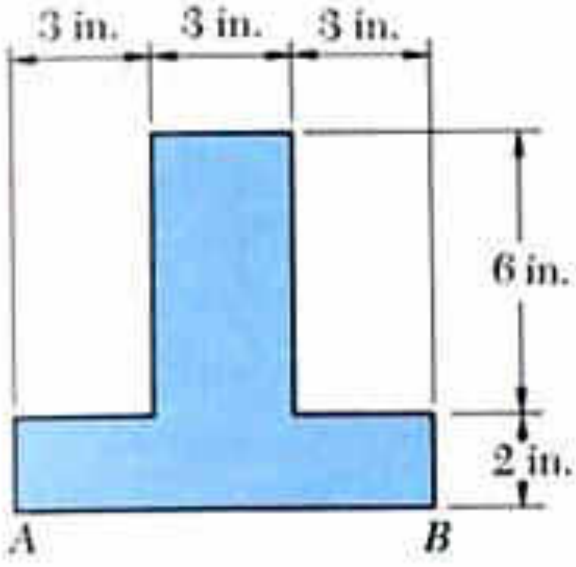
$$= 60 \text{ in}^4 + 144 \text{ in}^4 = 204 \text{ in}^4$$

Example - Moment of Inertia

Compute the radius of gyration, r_x.

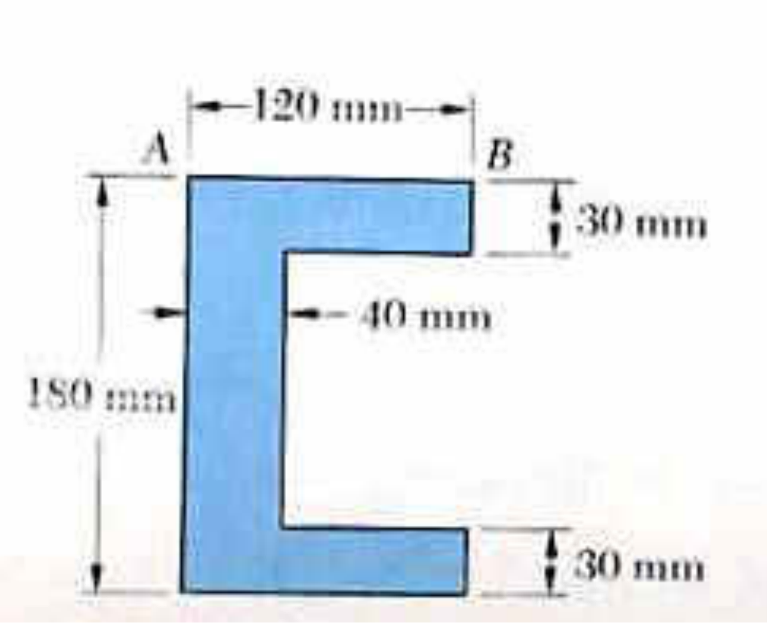
$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{204 \text{ in}^4}{36 \text{ in}^2}}$$

$$= 2.38 \text{ in.}$$



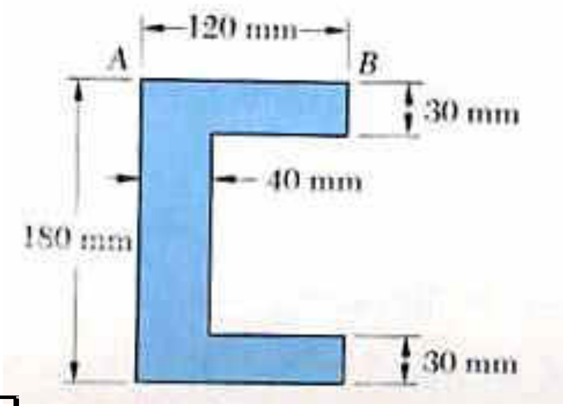
Example - Moment of Inertia

Find the moment of inertia of the body, I_x and the radius of gyration, k_x (r_x)



Example - Moment of Inertia

The components of the two bodies and subtract the center area from the total area.



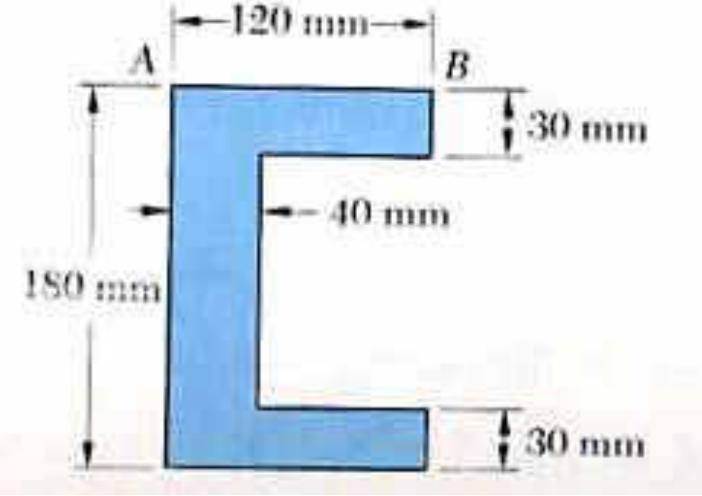
Bodies	A_i	y_i	y_i*A_i	I_i	$d_i=y_i$-ybar	$d_i^2A_i$
1	21600	90	1944000	58320000	0	0
2	-9600	90	-864000	-11520000	0	0
	12000		1080000	46800000		0
ybar	90	mm				
I	46800000	mm^4				

Example - Moment of Inertia

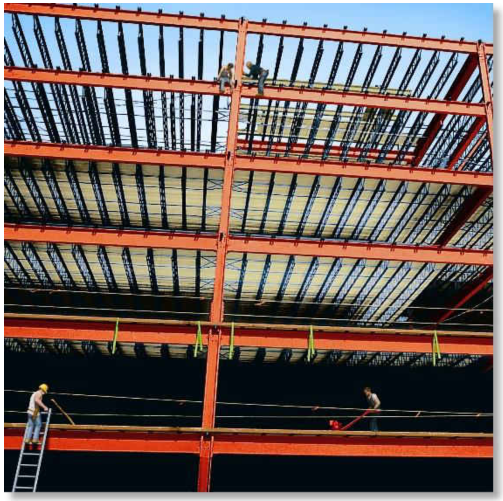
Compute the radius of gyration, r_x.

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{46800000 \text{ mm}^4}{12000 \text{ mm}^2}}$$

$$= 62.45 \text{ mm}$$

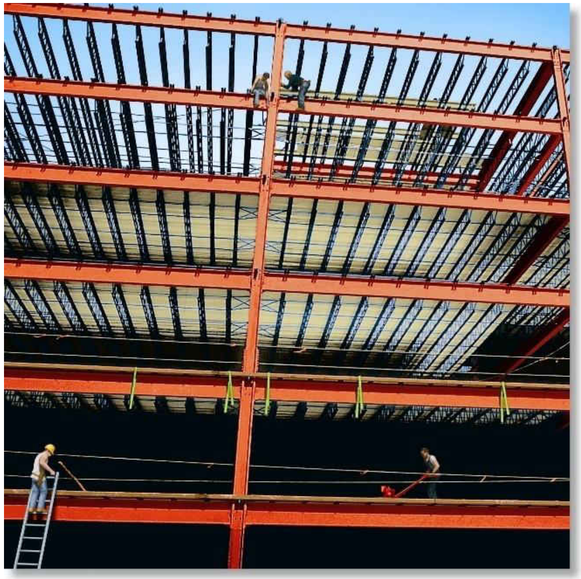


- Determine stress in members caused by bending
- Discuss how to establish shear and moment diagrams for a beam or shaft



- Determine largest shear and moment in a member, and specify where they occur
- Consider members that are straight, symmetric x-section and homogeneous linear-elastic material
- Consider special cases of unsymmetrical bending and members made of composite materials

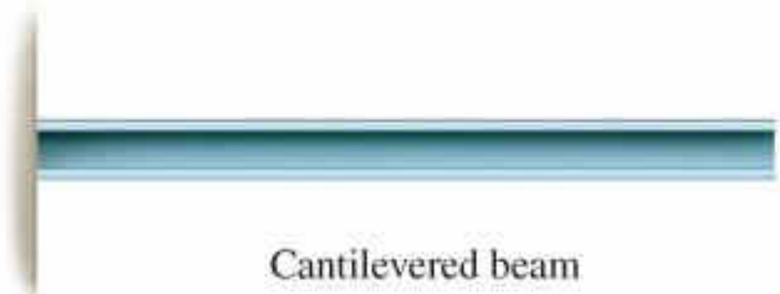
- Consider curved members, stress concentrations, inelastic bending, and residual stresses



Bending Moment and Shearing Force

- Members that are slender and support loadings applied perpendicular to their longitudinal axis are called *beams*

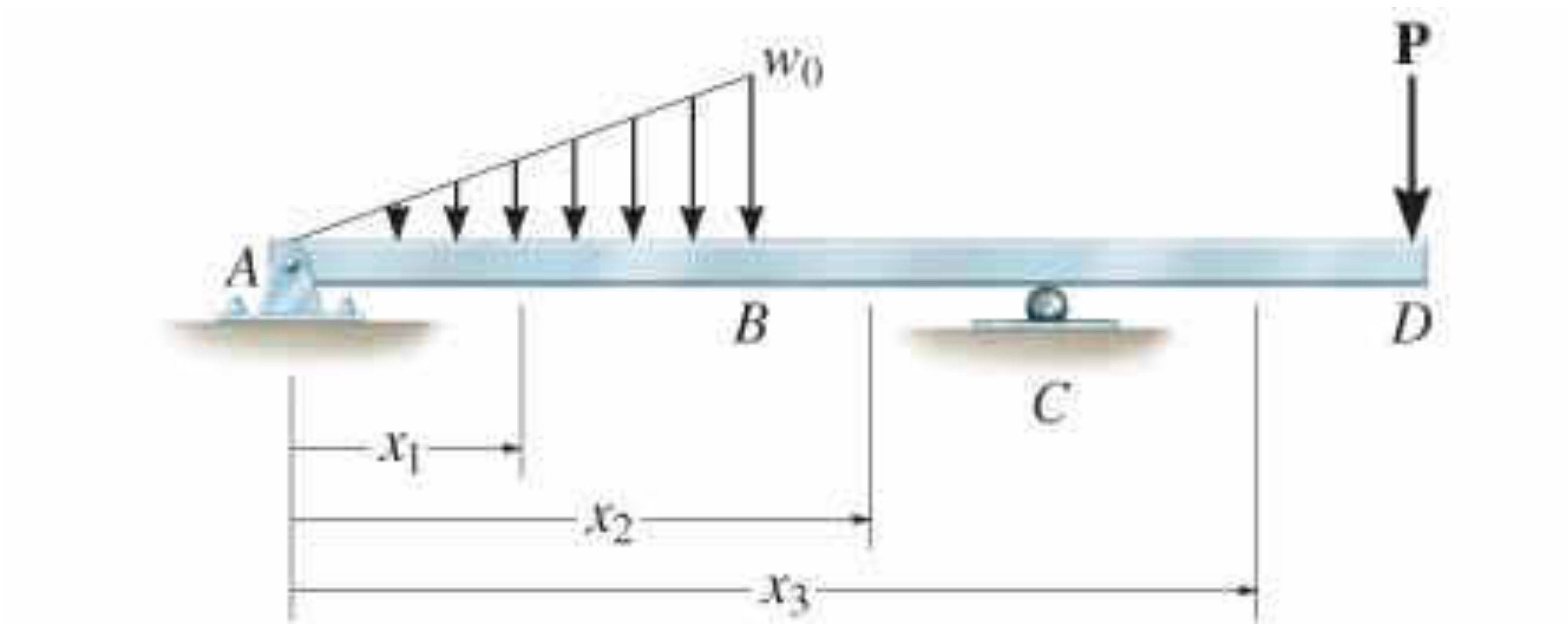
Simply supported beam



Cantilevered beam

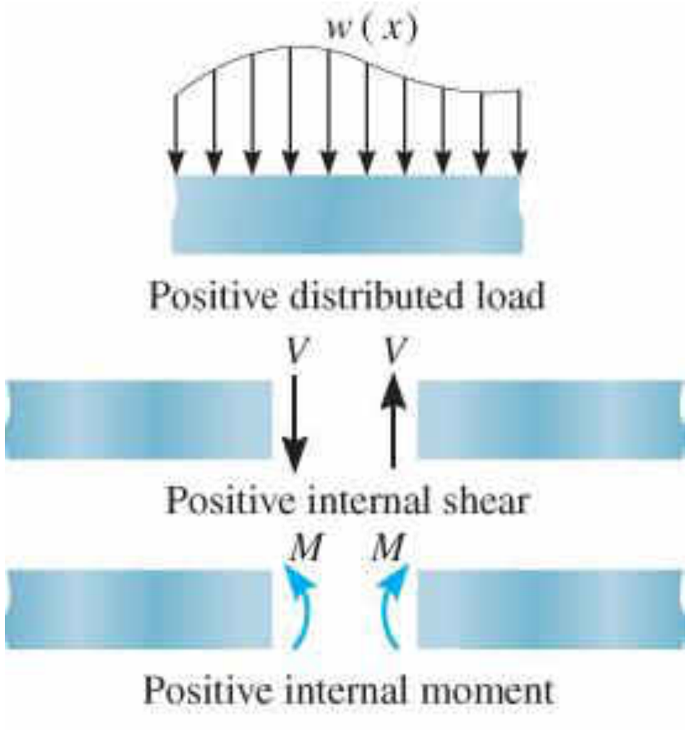
Overhanging beam

- Shear and bending-moment functions must be determined for each *region* of the beam *between* any two discontinuities of loading



Beam sign convention

- Although choice of sign convention is arbitrary, in this course, we adopt the one often used by engineers:



Beam sign convention

IMPORTANT

- Beams are long straight members that carry loads perpendicular to their longitudinal axis. They are classified according to how they are supported
- To design a beam, we need to know the variation of the shear and moment along its axis in order to find the points where they are maximum
- Establishing a sign convention for positive shear and moment will allow us to draw the shear and moment diagrams

Procedure for analysis

Support reactions

- Determine all reactive forces and couple moments acting on beam
- Resolve all forces into components acting perpendicular and parallel to beam's axis

Shear and moment functions

- Specify separate coordinates x having an origin at beam's left end, and extending to regions of beam between concentrated forces and/or couple moments, or where there is no discontinuity of distributed loading

Procedure for analysis

Shear and moment functions

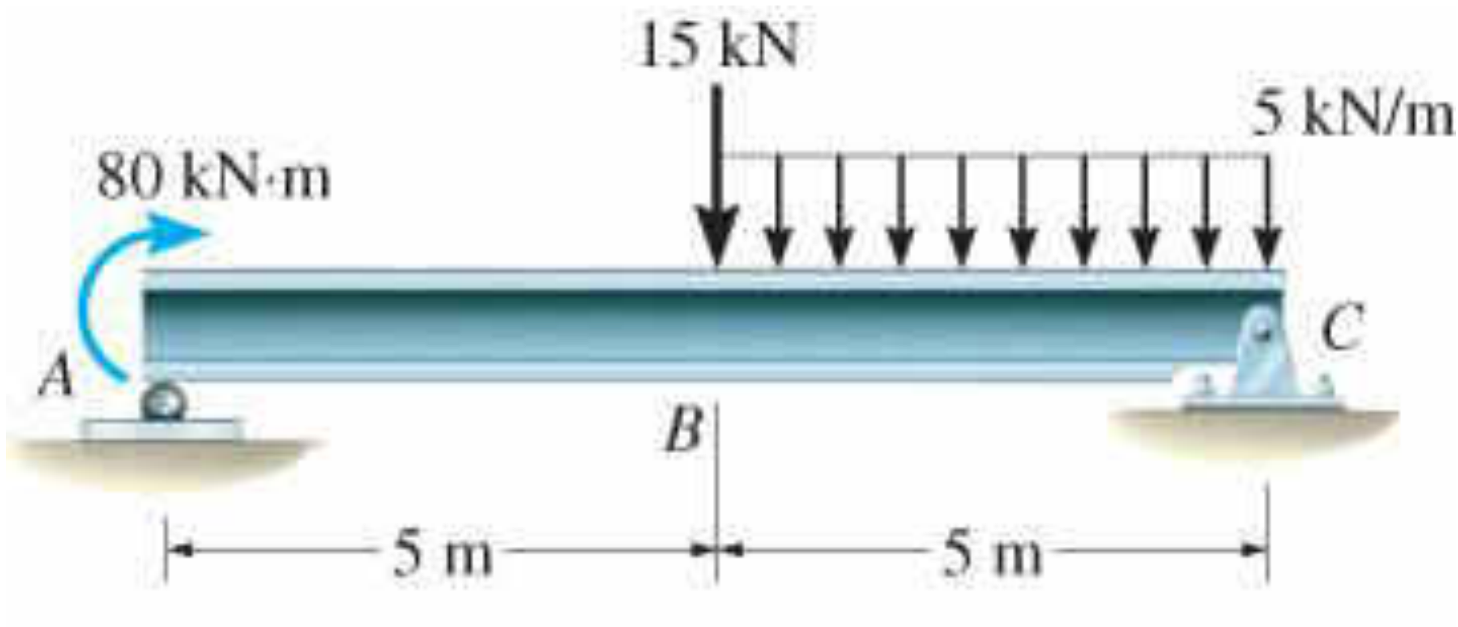
- Section beam perpendicular to its axis at each distance *x*
- Draw free-body diagram of one segment
- Make sure **V** and **M** are shown acting in positive sense, according to sign convention
- Sum forces perpendicular to beam's axis to get shear
- Sum moments about the sectioned end of segment to get moment

Procedure for analysis

Shear and moment diagrams

- Plot shear diagram (**V** vs. *x*) and moment diagram (**M** vs. *x*)
- If numerical values are positive, values are plotted above axis, otherwise, negative values are plotted below axis
- It is convenient to show the shear and moment diagrams directly below the free-body diagram

Draw the shear and moment diagrams for beam shown below.

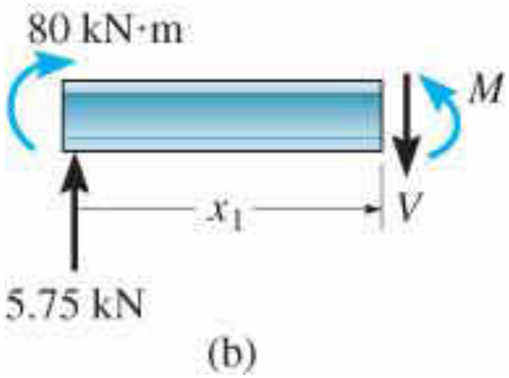


(a)

Support reactions: Shown in free-body diagram.

Shear and moment functions

Since there is a discontinuity of distributed load and a concentrated load at beam's center, two regions of *x* must be considered.

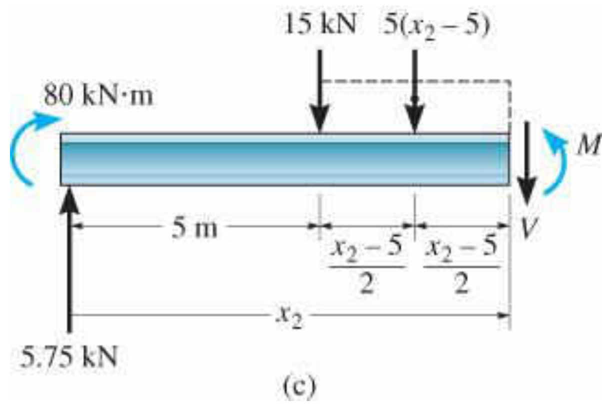


$0 \leq x_1 \leq 5$ m,

$$+\uparrow \Sigma F_y = 0; \quad \ldots \quad V = 5.75\ \text{N}$$

$$+\Sigma M = 0; \quad \ldots \quad M = (5.75x_1 + 80)\ \text{kN}\cdot\text{m}$$

Shear and moment functions



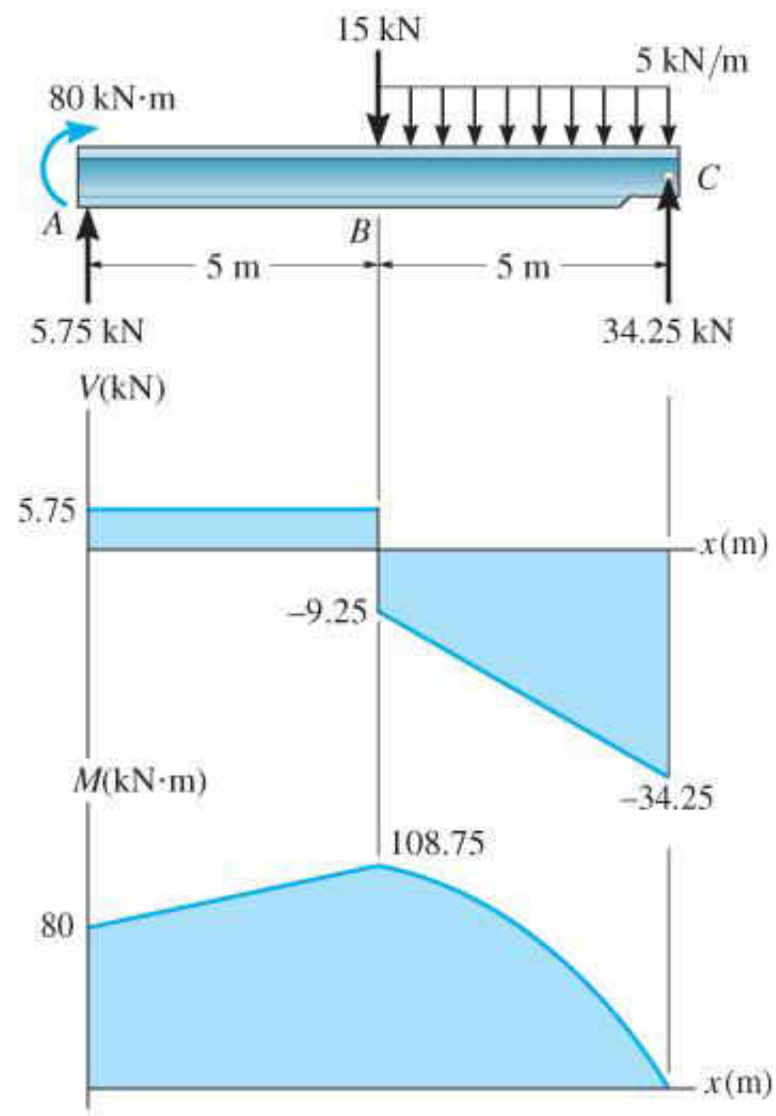
$5 \text{ m} \le x_2 \le 10 \text{ m},$

$+\uparrow \Sigma F_y = 0;$ ... $V = (15.75 - 5x_2)\ \text{kN}$

$+ \Sigma M = 0;$ ... $M = (-5.75x_2^2 + 15.75x_2 + 92.5)\ \text{kN·m}$

Check results by applying $w = dV/dx$ and $V = dM/dx$.

Shear and moment diagrams



(d)

- A simpler method to construct shear and moment diagram, one that is based on two differential equations that exist among distributed load, shear and moment

Regions of distributed load

$$\frac{dV}{dx} = -w(x)$$

Slope of shear diagram at each point $= -$distributed load intensity at each point

$$\frac{dM}{dx} = V$$

Slope of moment diagram at each point = shear at each point

Regions of distributed load

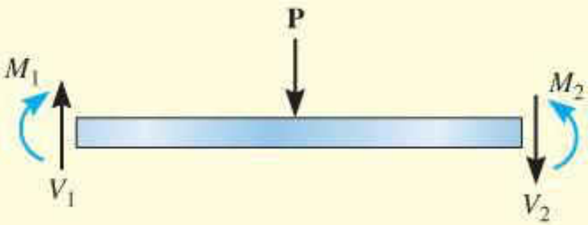
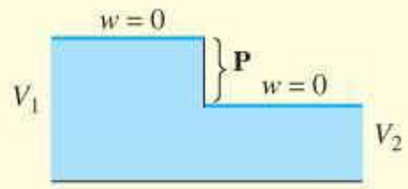
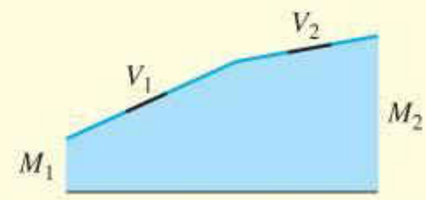
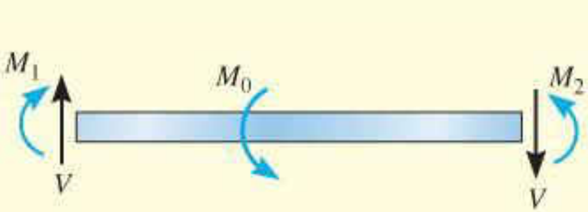
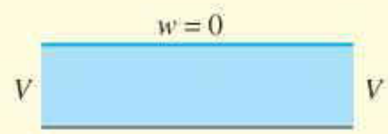
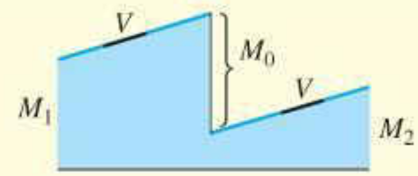
$$\Delta V = -\int w(x)\, dx$$

Change in shear = $-$area under distributed loading

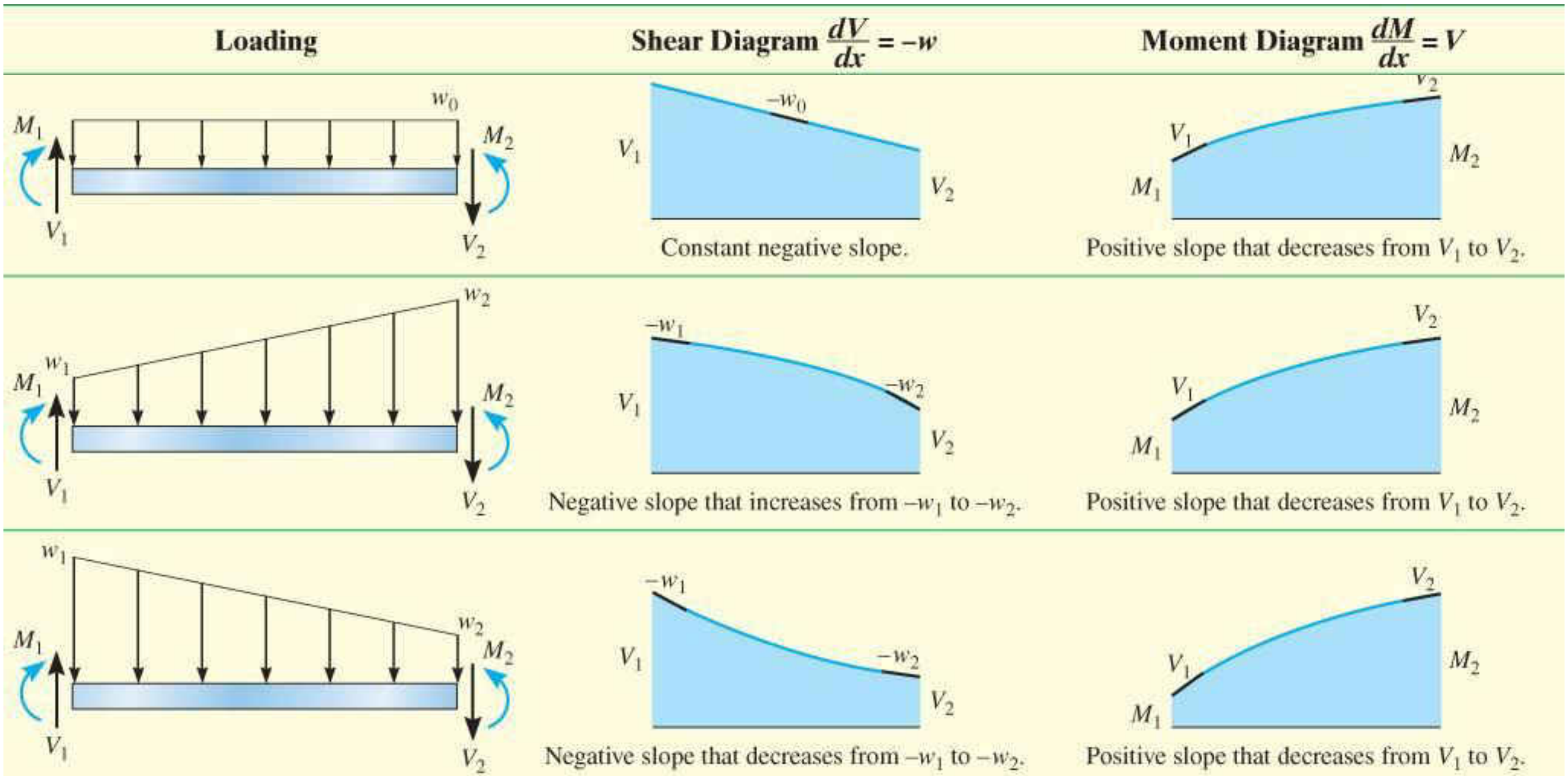
$$\Delta M = \int V(x)\, dx$$

Change in moment = area under shear diagram

Regions of concentrated force and moment

Loading	Shear Diagram $\frac{dV}{dx} = -w$	Moment Diagram $\frac{dM}{dx} = V$
	 Downward force **P** causes V to jump downward from V_1 to V_2.	 Constant slope changes from V_1 to V_2.
	 No change in shear since slope $w = 0$.	 Constant positive slope. Counterclockwise $\mathbf{M}_0$ causes M to jump downward.

Regions of concentrated force and moment



Procedure for analysis

Support reactions

- Determine support reactions and resolve forces acting on the beam into components that are perpendicular and parallel to beam's axis

Shear diagram

- Establish **V** and *x* axes
- Plot known values of shear at two ends of the beam

Procedure for analysis

Shear diagram

- Since $dV/dx = -w$, slope of the shear diagram at any point is equal to the (-ve) intensity of the distributed loading at that point
- To find numerical value of shear at a point, use method of sections and equation of equilibrium or by using $\Delta V = -\int w(x)\, dx$, i.e., change in the shear between any two points is equal to (-ve) area under the load diagram between the two points

Procedure for analysis

Shear diagram

- Since $w(x)$ *must be integrated to obtain* ΔV*, then if* $w(x)$ is a curve of degree n, $V(x)$ will be a curve of degree $n+1$

Moment diagram

- Establish **M** and x axes and plot known values of the moment at the ends of the beam
- Since $dM/dx = V$, slope of the moment diagram at any point is equal to the shear at the point

Procedure for analysis

Moment diagram

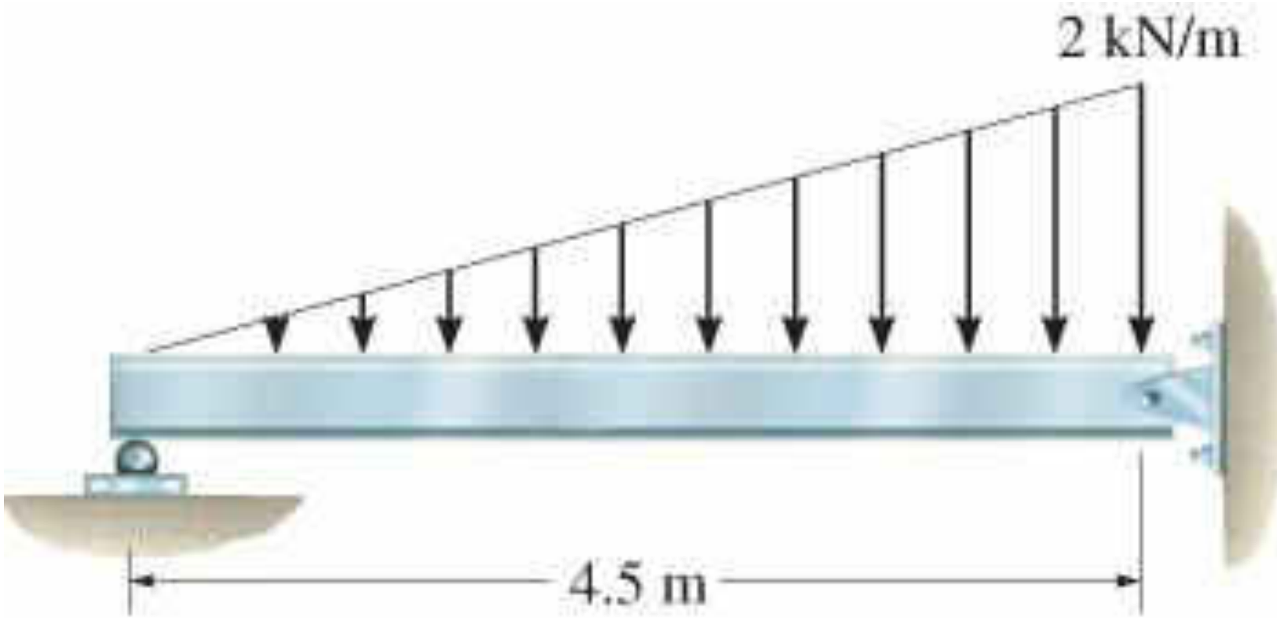
- At point where shear is zero, $dM/dx = 0$ and therefore this will be a point of maximum or minimum moment
- If numerical value of moment is to be determined at the point, use method of sections and equation of equilibrium, or by using $\Delta M = \int V(x)\, dx$, i.e., change in moment between any two pts is equal to area under shear diagram between the two pts

Procedure for analysis

Moment diagram

- Since $V(x)$ must be integrated to obtain ΔM, then if $V(x)$ is a curve of degree n, M(x) will be a curve of degree $n+1$

Draw the shear and moment diagrams for beam shown below.



(a)

Support reactions:

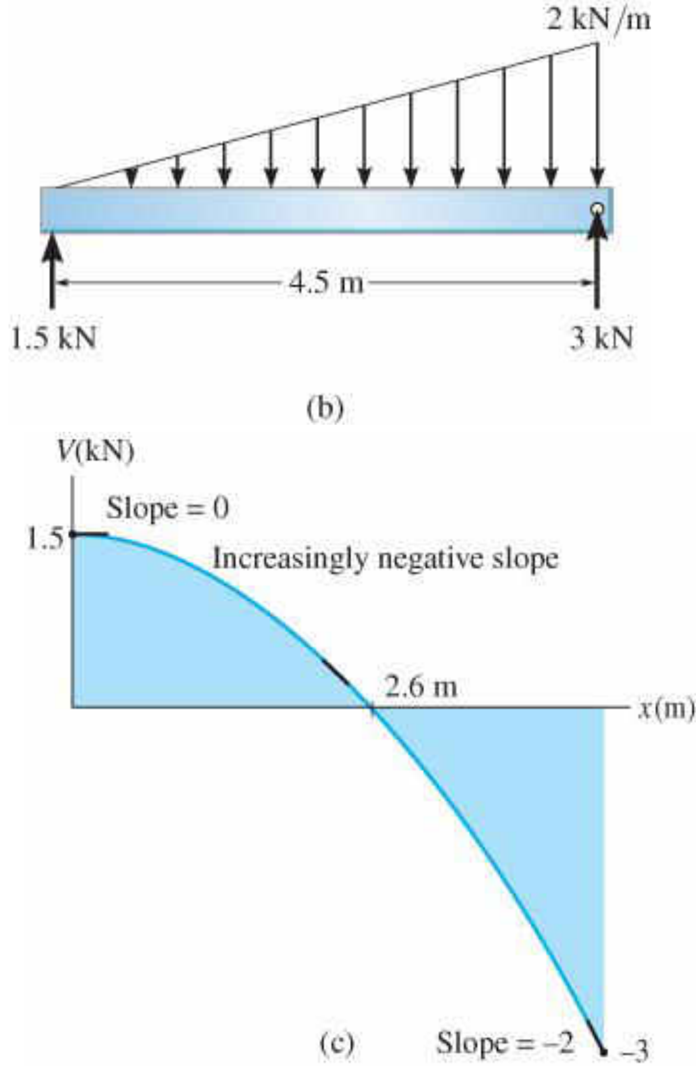
See free-body diagram.

Shear diagram

From behavior of distributed load, slope of shear diagram varies from zero at $x = 0$ to -2 at $x = 4.5$. Thus, its parabolic shape.

Use method of sections to find point of zero shear:

$$+\uparrow \Sigma F_y = 0; \qquad \ldots \qquad x = 2.6\ \text{m}$$

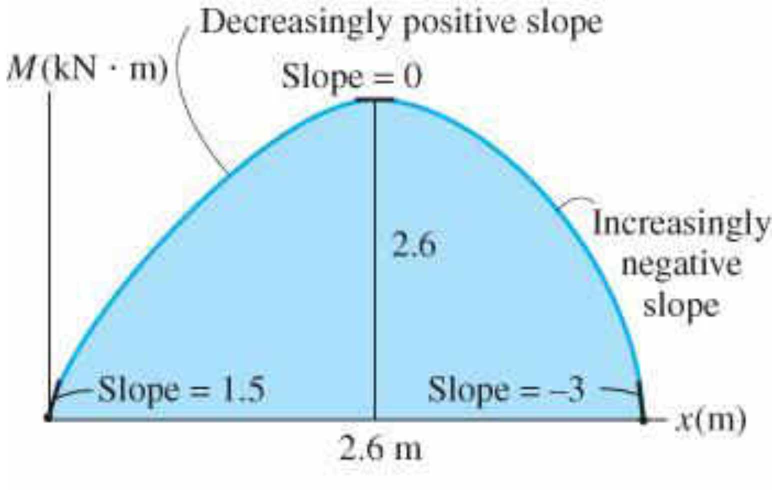


Moment diagram

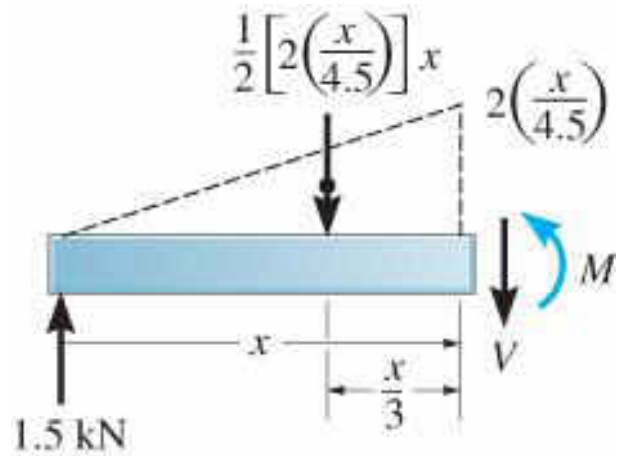
From shear diagram, slope of moment diagram begin at +1.5, then decreases positively till it reaches zero at 2.6 m. Then it increases negatively and reaches –3 at *x* = 4.5 m. Moment diagram is a cubic function of *x*.

$$\curvearrowleft + \Sigma M = 0; \quad . . .$$

$$M = 2.6 \text{ kN}\cdot\text{m}$$

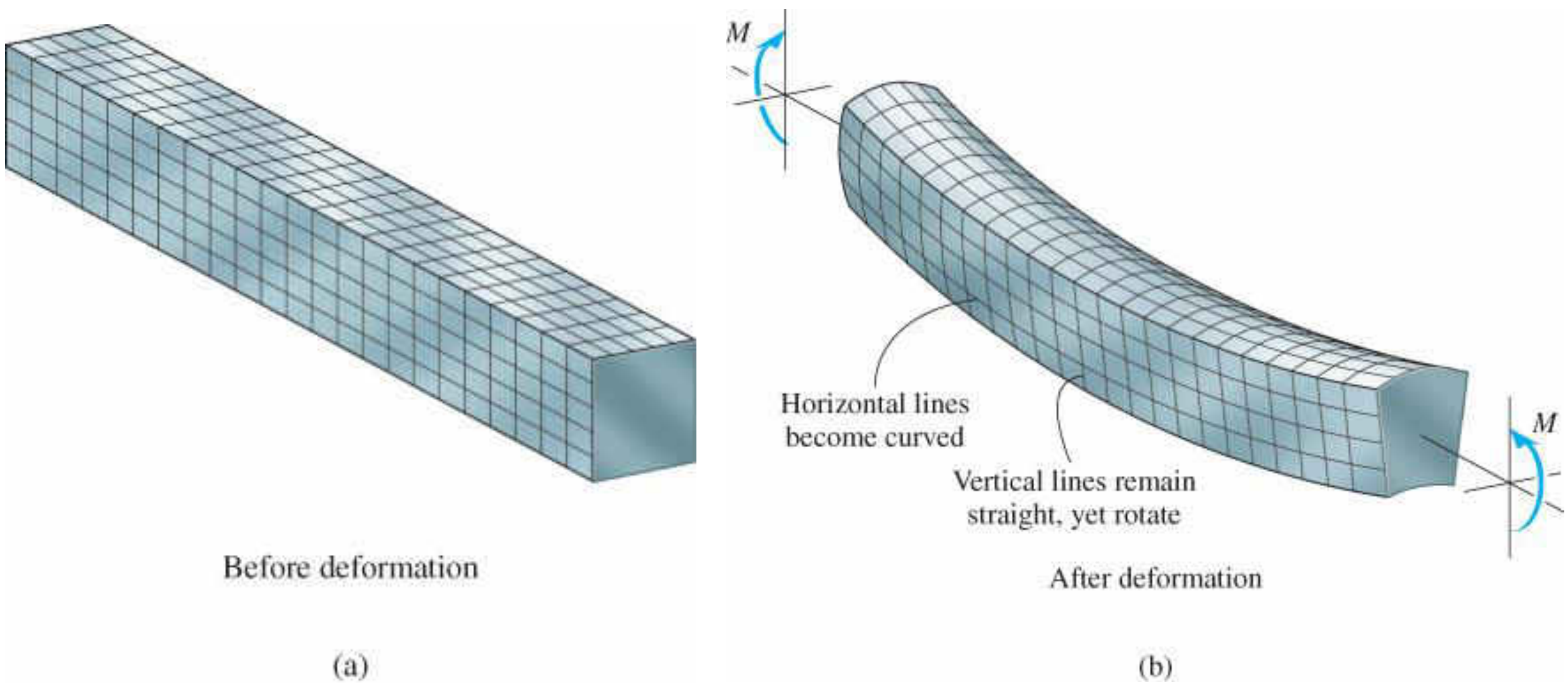


(d)

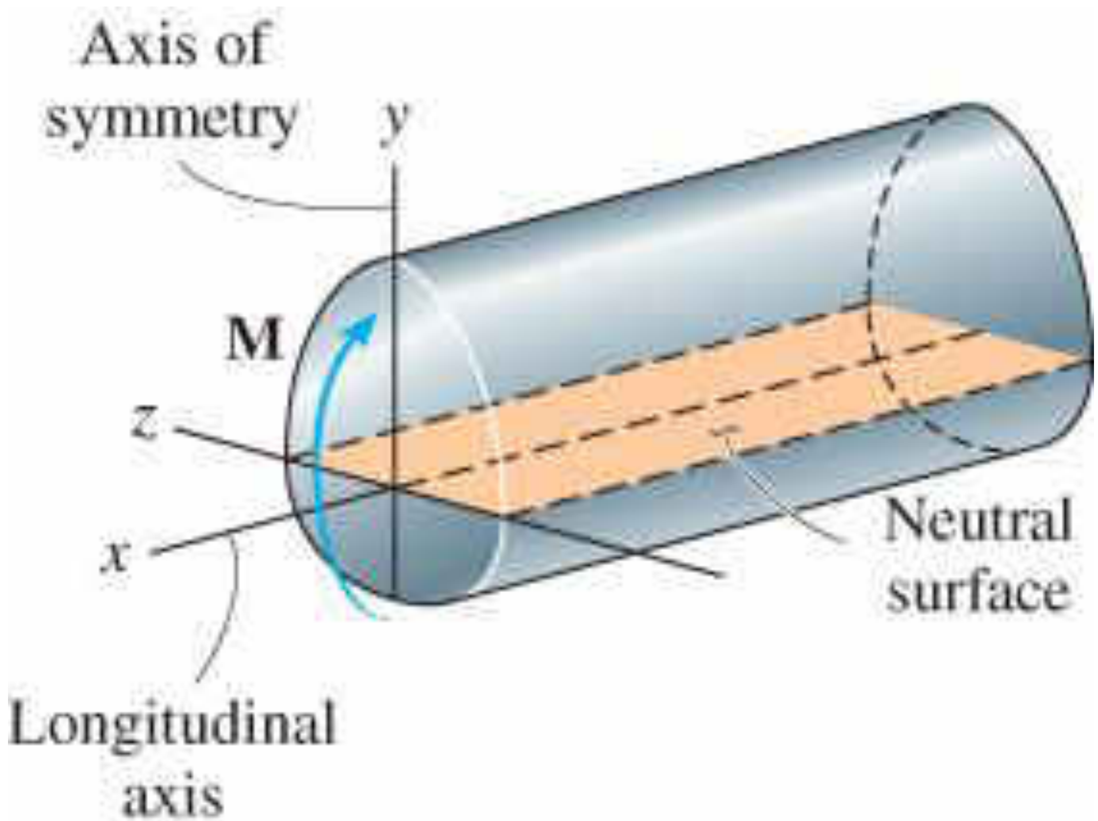


(e)

- When a bending moment is applied to a straight prismatic beam, the longitudinal lines become *curved* and vertical transverse lines *remain straight* and yet undergo a *rotation*

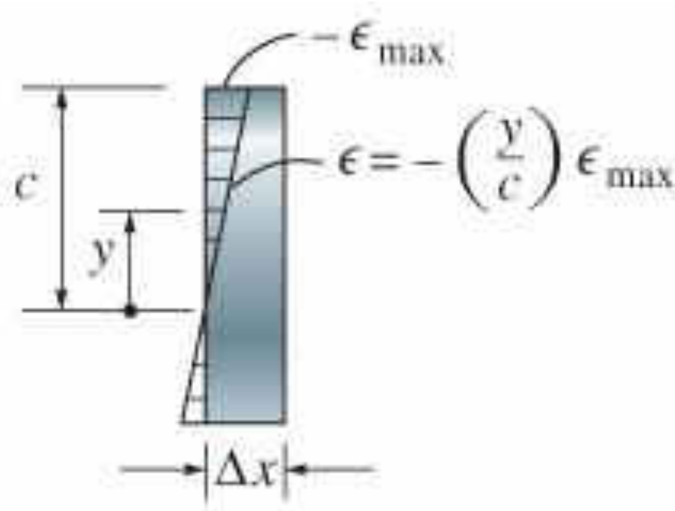


- A *neutral surface* is where longitudinal fibers of the material will not undergo a change in length.



- Thus, we make the following assumptions:
  1. Longitudinal axis *x* (within neutral surface) does not experience any *change in length*
  2. All *cross sections* of the beam *remain plane* and perpendicular to longitudinal axis during the deformation
  3. Any *deformation* of the *cross-section* within its own plane will be *neglected*
- In particular, the *z* axis, in plane of x-section and about which the x-section rotates, is called the *neutral axis*

- For any specific x-section, the longitudinal normal strain will vary linearly with *y* from the neutral axis
- A contraction will occur ($-\varepsilon$) in fibers located above the neural axis (+*y*)
- An elongation will occur ($+\varepsilon$) in fibers located below the axis ($-y$)

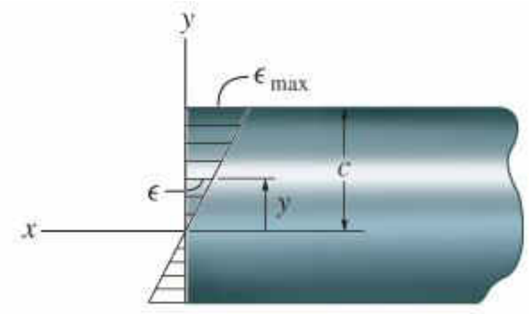


Normal strain distribution

Equation 6-8

$$\varepsilon = -(y/c)\varepsilon_{max}$$

- Assume that material behaves in a linear-elastic manner so that Hooke's law applies.
- A *linear variation of normal strain must* then be the consequence of a *linear variation in normal stress*
- Applying Hooke's law to Eqn 6-8,

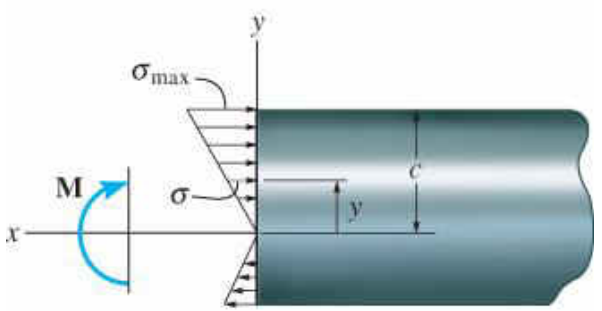


Normal strain variation
(profile view)

(a)

Equation 6-9

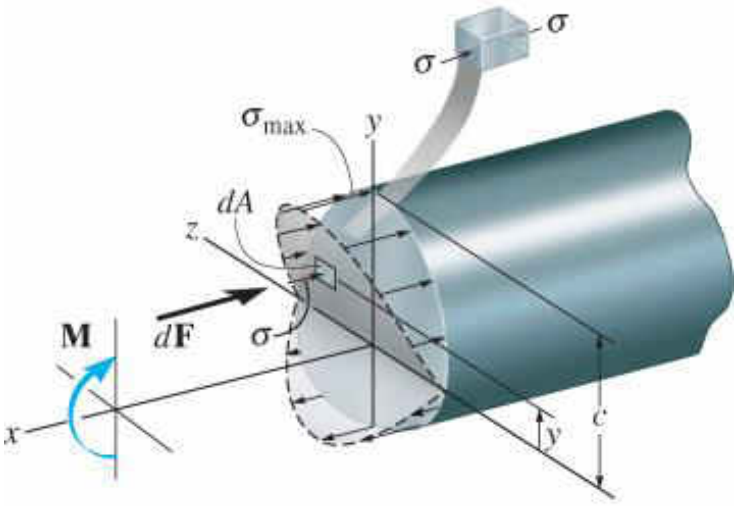
$$\sigma = -(y/c)\sigma_{max}$$



Bending stress variation
(profile view)

(b)

- By mathematical expression, equilibrium equations of moment and forces, we get



Bending stress variation

(c)

Equation 6-10

$$\int_A y \, dA = 0$$

Equation 6-11

$$M = \frac{\sigma_{max}}{c} \int_A y^2 \, dA$$

- The integral represents the *moment of inertia* of x-sectional area, computed about the neutral axis. We symbolize its value as I.

- Hence, Eqn 6-11 can be solved and written as

Equation 6-12

$$\sigma_{max} = \frac{Mc}{I}$$

$\sigma_{\max}$ = maximum normal stress in member, at a pt on x-sectional area farthest away from neutral axis

M = resultant internal moment, computed about neutral axis of x-section

I = moment of inertia of x-sectional area computed about neutral axis

c = perpendicular distance from neutral axis to a pt farthest away from neutral axis, where $\sigma_{\max}$ acts

- Normal stress at intermediate distance y can be determined from

Equation 6-13

$$\sigma = -\frac{My}{I}$$

- σ is -ve as it acts in the -ve direction (compression)
- Equations 6-12 and 6-13 are often referred to as the *flexure formula*.

IMPORTANT

- X-section of straight beam *remains plane* when beam deforms due to bending.
- The *neutral axis* is subjected to *zero stress*
- Due to deformation, *longitudinal strain* varies *linearly* from zero at neutral axis to maximum at outer fibers of beam
- Provided material is homogeneous and Hooke’s law applies, *stress* also varies *linearly* over the x-section

IMPORTANT

- For linear-elastic material, neutral axis passes through *centroid* of x-sectional area. This is based on the fact that resultant normal force acting on x-section must be zero
- Flexure formula is based on requirement that resultant moment on the x-section is equal to moment produced by linear normal stress distribution about neutral axis

Procedure for analysis

Internal moment

- Section member at pt where bending or normal stress is to be determined and obtain internal moment **M** at the section
- Centroidal or neutral axis for x-section must be known since **M** is computed about this axis
- If absolute maximum bending stress is to be determined, then draw moment diagram in order to determine the maximum moment in the diagram

Procedure for analysis

Section property

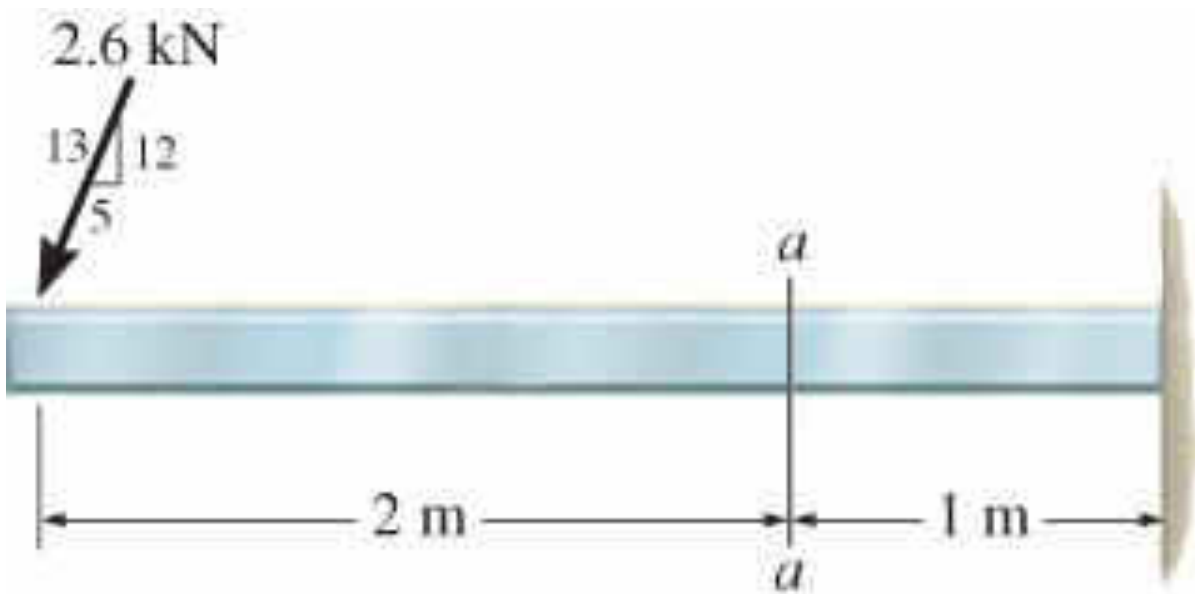
- Determine moment of inertia I, of x-sectional area about the neutral axis
- Methods used are discussed in Textbook Appendix A
- Refer to the course book's inside front cover for the values of I for several common shapes

Procedure for analysis

Normal stress

- Specify distance y, measured perpendicular to neutral axis to pt where normal stress is to be determined
- Apply equation $\sigma = My/I$, or if maximum bending stress is needed, use $\sigma_{\max} = Mc/I$
- Ensure units are consistent when substituting values into the equations

Beam shown has x-sectional area in the shape of a channel. Determine the maximum bending stress that occurs in the beam at section *a-a*.

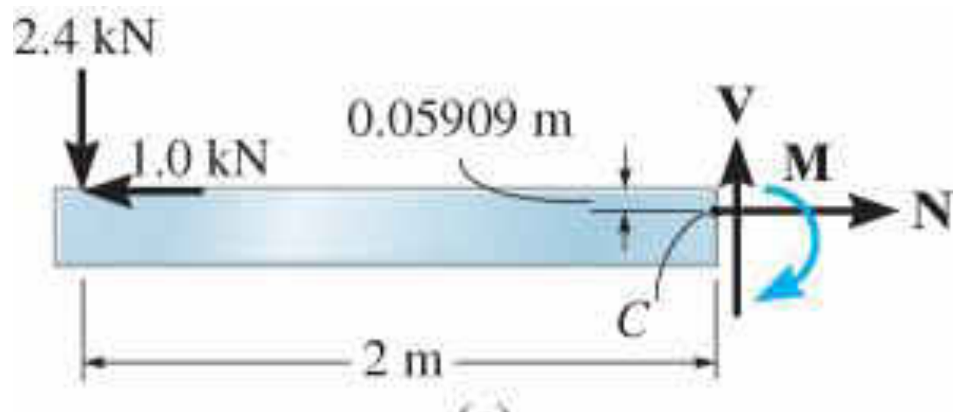


(a)

Internal moment

Beam support reactions need not be determined. Instead, use method of sections, the segment to the left of *a*-*a*. Note that resultant internal axial force **N** passes through centroid of x-section.

The resultant internal moment must be computed about the beam's neutral axis a section *a*-*a*.

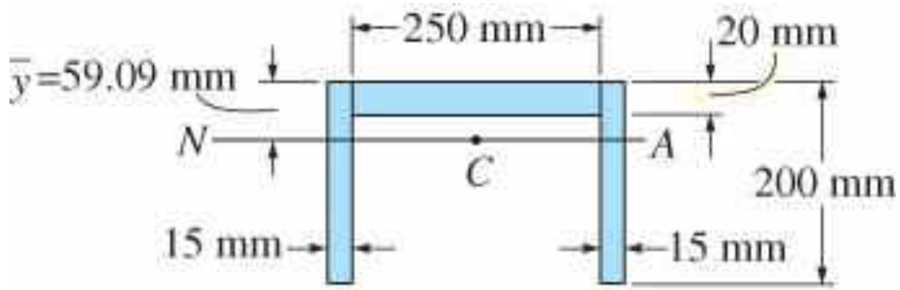


(c)

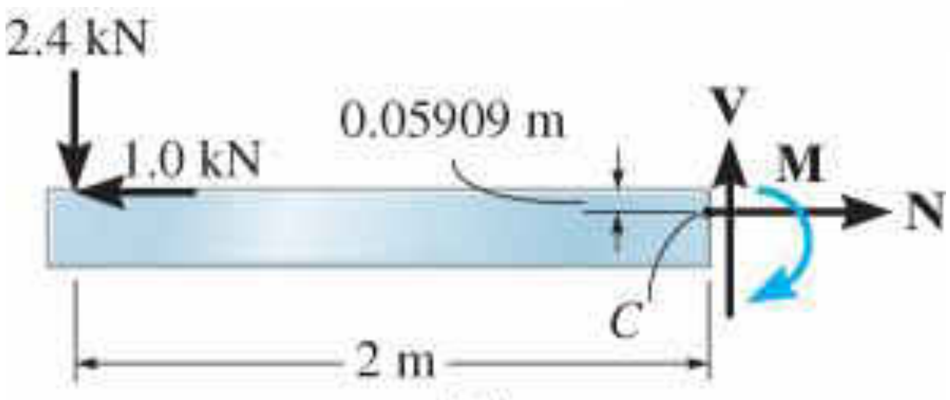
Internal moment

To find location of neutral axis, x-sectional area divided into 3 composite parts as shown. Then using Eqn. A-2 of Appendix A:

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \ldots = 59.09 \text{ mm}$$



(b)



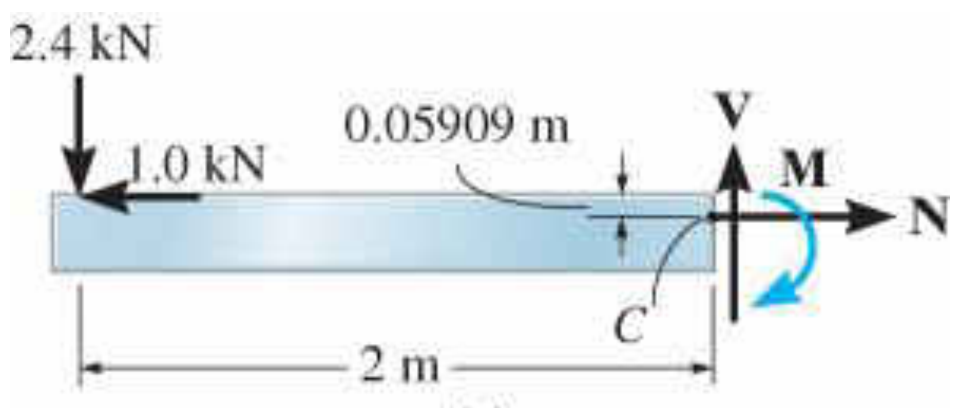
(c)

Internal moment

Apply moment equation of equilibrium about neutral axis,

$$+\Sigma M_{NA} = 0; \quad 24\text{ kN}(2\text{ m}) + 1.0\text{ kN}(0.05909\text{ m}) - M = 0$$

$$M = 4.859\text{ kN}\cdot\text{m}$$



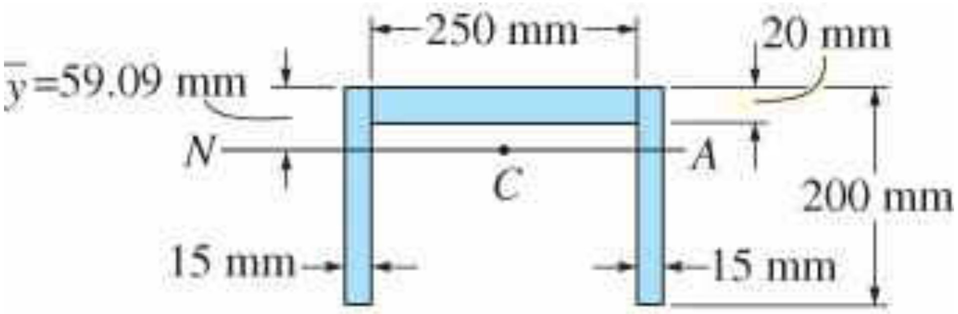
(c)

Section property

Moment of inertia about neutral axis is determined using parallel-axis theorem applied to each of the three composite parts of the x-sectional area.

$$
\begin{aligned}
I = {} & [1/12(0.250\,\text{m})(0.020\,\text{m})^3 \\
& \quad + (0.250\,\text{m})(0.020\,\text{m})(0.05909\,\text{m} - 0.010\,\text{m})^2] \\
& + 2[1/12(0.015\,\text{m})(0.200\,\text{m})^3 \\
& \quad + (0.015\,\text{m})(0.200\,\text{m})(0.100\,\text{m} - 0.05909\,\text{m})^2]
\end{aligned}
$$

$$I = 42.26(10^{-6})\,\text{m}^4$$



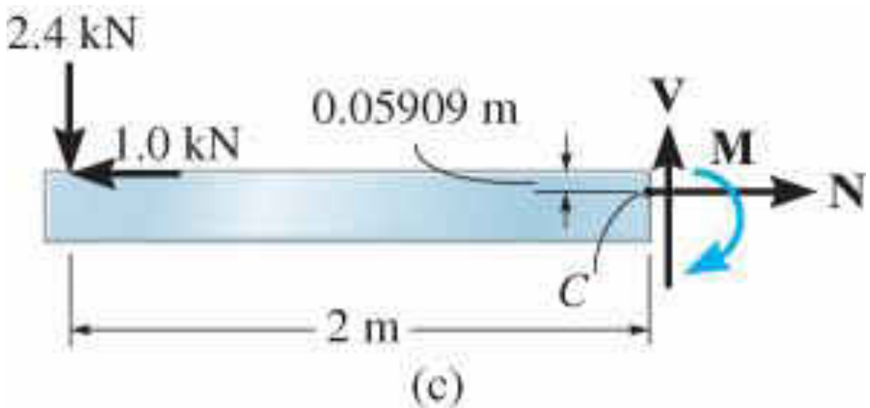
(b)

Maximum bending stress

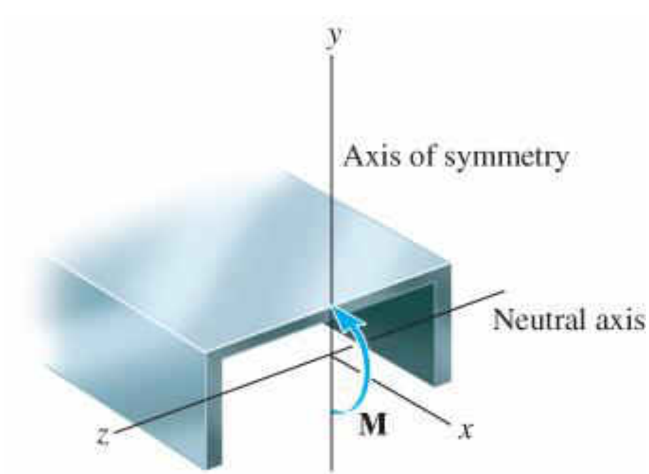
It occurs at points farthest away from neutral axis. At bottom of beam, c = 0.200 m – 0.05909 m = 0.1409 m. Thus,

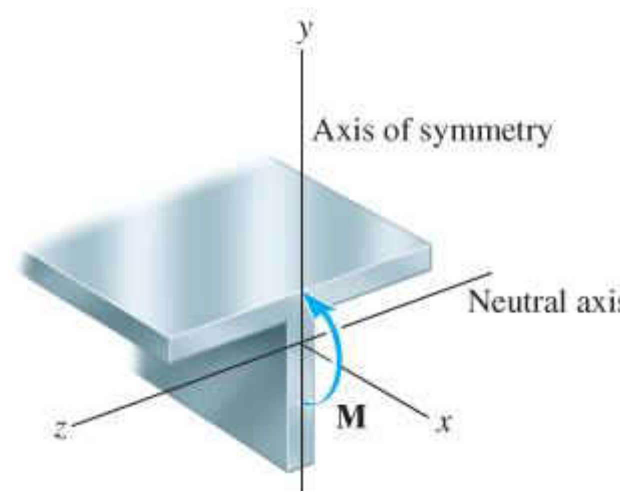
$$\sigma_{max} = \frac{Mc}{I} = \frac{4.859\ \text{kN}\cdot\text{m}(0.1409\ \text{m})}{42.26(10^{-6})\ \text{m}^4} = 16.2\ \text{MPa}$$

At top of beam, σ' = 6.79 MPa. In addition, normal force of N = 1 kN and shear force V = 2.4 kN will also contribute additional stress on x-section.



- A condition for flexure formula is the symmetric x-sectional area of beam about an axis perpendicular to neutral axis
- However, the flexure formula can also be applied either to a beam having x-sectional area of any shape OR to a beam having a resultant moment that acts in any direction





Moment applied along principal axis

- Consider a beam with unsymmetrical shape
- Establish coordinate system as per usual and that resultant moment **M** acts along +*z* axis
- Conditions:

1. Stress distribution acting over entire x-sectional area to be a zero force resultant,
2. Resultant internal moment about *y* axis to be zero
3. Resultant internal moment about *z* axis to be equal to **M**

Moment applied along principal axis

- Express the 3 conditions mathematically by considering force acting on differential element *dA* located at (0, *y*, *z*). Force is $dF = \sigma\, dA$, therefore

$$F_R = F_y; \qquad 0 = \int_A \sigma\, dA$$

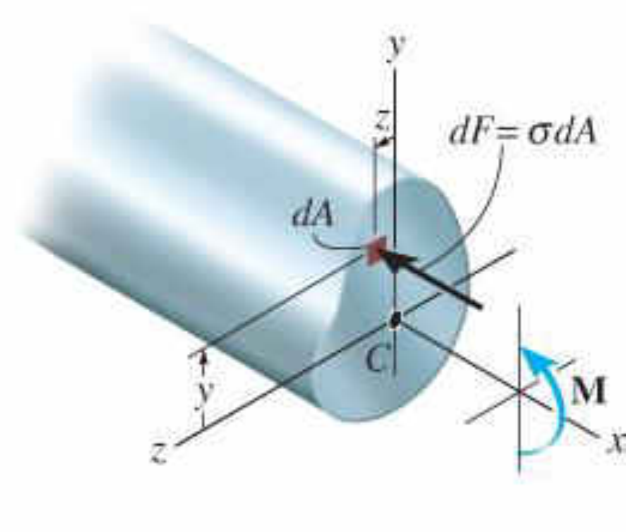
Equation 6-14

$$(M_R)_y = \Sigma M_y; \qquad 0 = \int_A z\, \sigma\, dA$$

Equation 6-15

$$(M_R)_z = \Sigma M_z; \qquad 0 = \int_A -y\, \sigma\, dA$$

Equation 6-16

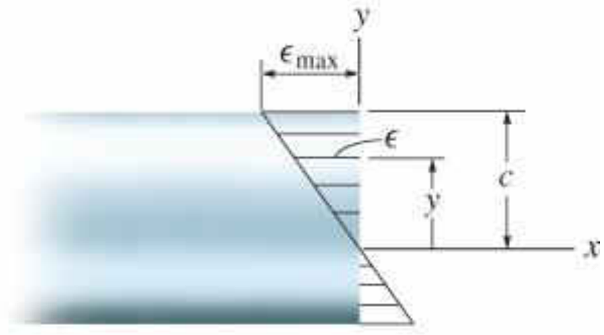


(a)

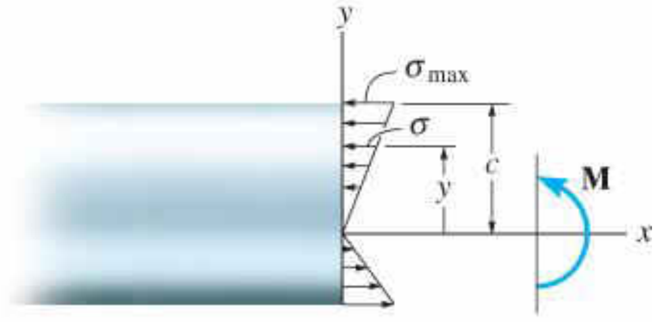
Moment applied along principal axis

- Eqn 6.14 is satisfied since *z* axis passes through *centroid* of x-sectional area
- If material has linear-elastic behavior, then we can substitute $\sigma = -(y/c)\sigma_{max}$ into Eqn 6-16 and after integration, we get

$$\int_A yz\, dA = 0$$



Normal strain distribution
(profile view)
(b)



Bending stress distribution
(profile view)
(c)

Moment applied along principal axis

$$\int_A yz \, dA = 0$$

- This integral is the product of inertia for the area. It will be zero if y and z axes are chosen as principal axes of inertia for the area.
- Thus, Eqns 6-14 to 6-16 will always be satisfied regardless of the direction of applied moment **M**

Moment arbitrarily applied

- If a member is loaded such that resultant internal moment does not act about one of the principal axes of x-section, resolve the moment into components directed along the principal axes
- Use flexure formula to determine normal stress caused by each moment component
- Use principle of superposition to determine resultant normal stress at the pt

Moment arbitrarily applied

- Resultant general normal stress at any pt on x-section is

$$\sigma = \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$ Equation 6-17

σ = normal stress at the pt

y, z = coordinates of pt measured from x, y, z axes having origin at centroid of x-sectional area and forming a right-handed coordinate system

Moment arbitrarily applied

- Resultant general normal stress at any pt on x-section is

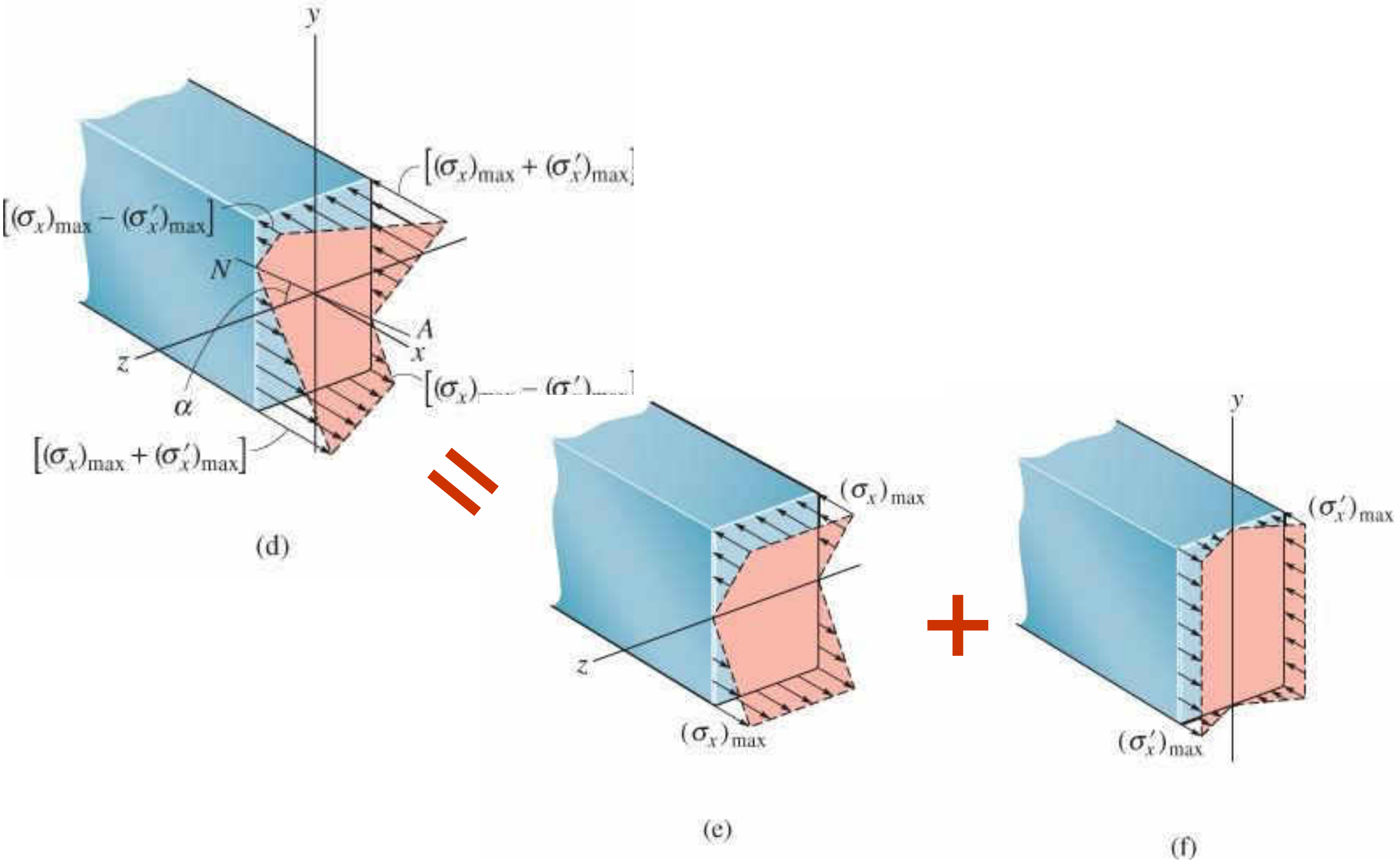
$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

Equation 6-17

M_y, M_z = resultant internal moment components along principal y and z axes. Positive if directed along $+y$ and $+z$ axes. Can also be stated as $M_y = M \sin\theta$ and $M_z = M \cos\theta$, where θ is measured positive from $+z$ axis toward $+y$ axis

I_y, I_z = principal moments of inertia computed about the y and z axes, respectively

Moment arbitrarily applied



Orientation of neutral axis

- Angle α of neutral axis can be determined by applying Eqn 6-17 with $\sigma = 0$, since no normal stress acts on neutral axis. Finally, we get

$$\tan \alpha = \frac{I_z}{I_y} \tan \theta$$

Equation 6-19

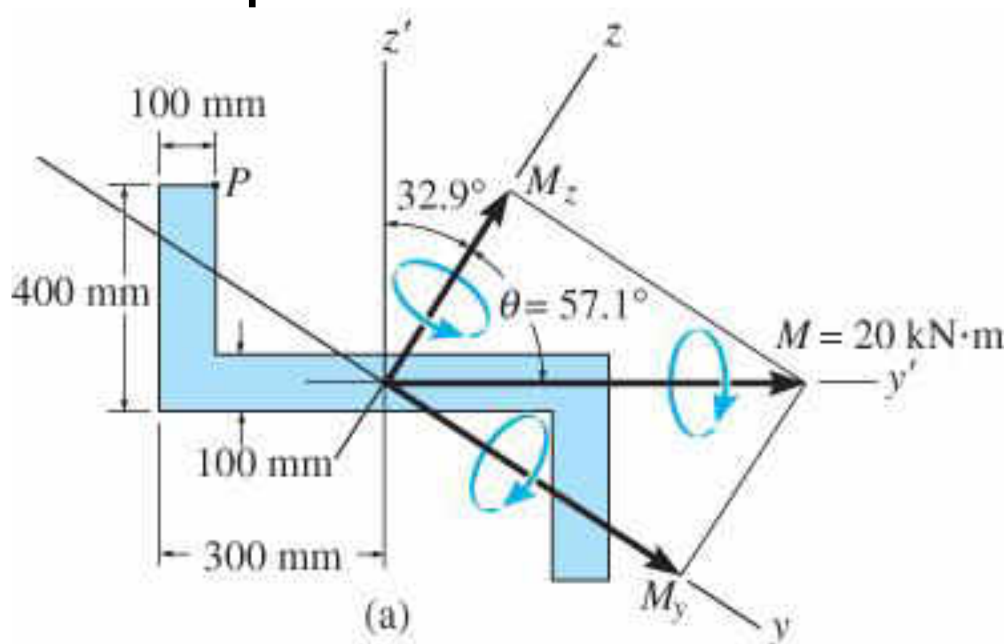
- For unsymmetrical bending, angle θ defining direction of moment **M** is not equal to angle α, angle defining inclination of neutral axis *unless* $I_z = I_y$.
- *Thus,* $\theta \leq \alpha \leq 90^{\mathrm{o}}$

IMPORTANT

- Flexure formula applied only when bending occurs about axes that represent the principal axes of inertia for x-section
- These axes have their origin at centroid and are orientated along an axis of symmetry and perpendicular to it
- If moment applied about arbitrary axis, then resolve moment into components along each of the principal axes, and stress at a pt is determined by superposition of stress caused by each moment component.

Z-section shown is subjected to bending moment of M = 20 kN·m. Using methods from Appendix A, the principal axes y and z are oriented as shown such that they represent the maximum and minimum principal moments of inertia, $I_y = 0.960(10^{-3})\ m^4$ and $I_z = 7.54(10^{-3})\ m^4$ respectively.

Determine normal stress at point P and orientation of neutral axis.

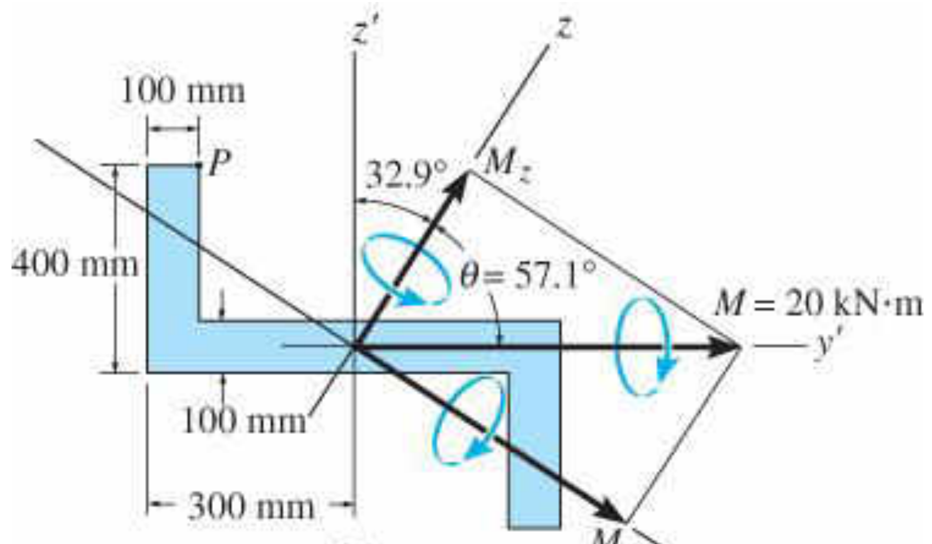


Internal moment components

To use Eqn 6-19, *z* axis needs to be principal axis for the maximum moment of inertia, as most of the area if located furthest away from this axis

$$M_y = 20\ \text{kN·m} \sin 57.1^\circ = 16.79\ \text{kN·m}$$

$$M_z = 20\ \text{kN·m} \cos 57.1^\circ = 10.86\ \text{kN·m}$$



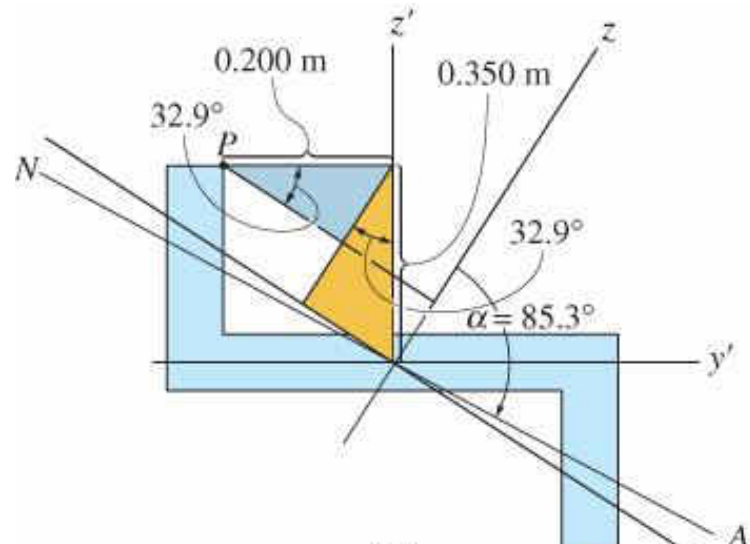
(a)

Bending stress

The *y* and *z* coordinates of *P* must be determined first. Note that *y'*, *z'* coordinates of *P* are (–0.2 m, 0.35 m). Using colored and shaded triangles from construction shown below,

$$y_P = -0.35 \sin 32.9^{\circ} - 0.2 \cos 32.9^{\circ} = 0.3580\,\text{m}$$

$$z_P = 0.35 \cos 32.9^{\circ} - 0.2 \sin 32.9^{\circ} = 0.1852\,\text{m}$$

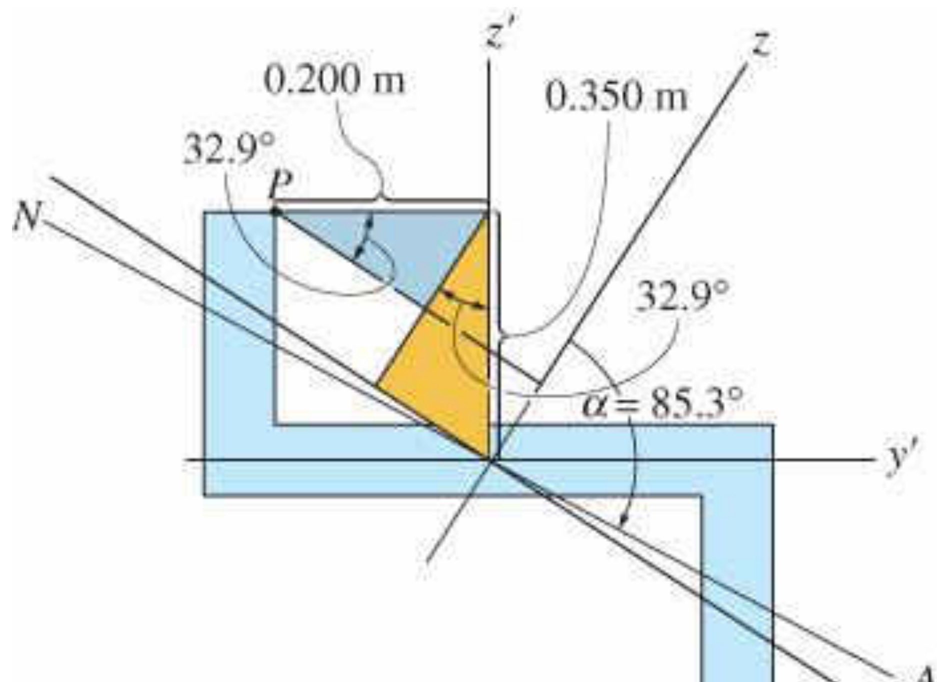


(b)

Bending stress

Applying Eqn 6-17, we have

$$\sigma = \frac{M_z y}{I_z} + \frac{M_y z}{I_y} = \ldots = 3.76\,\mathrm{MPa}$$



(b)

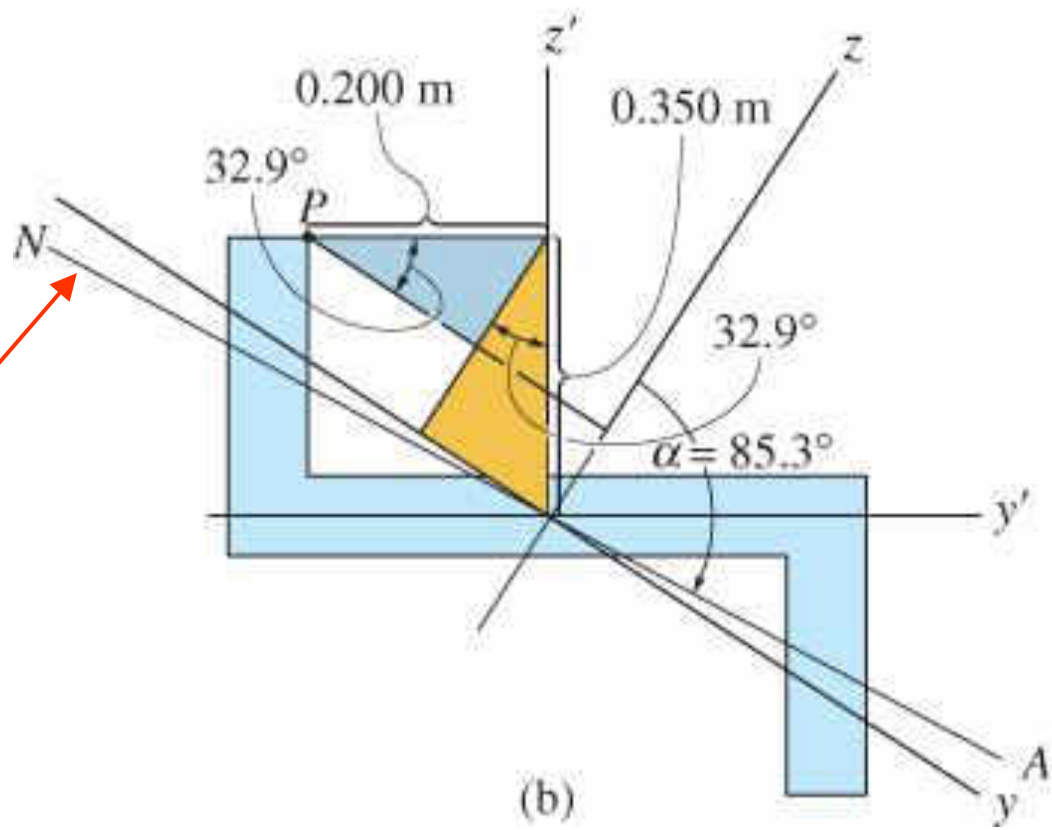
Orientation of neutral axis

Angle =57.1° is shown, Thus,

$$\tan\alpha = \frac{I_z}{I_y}\tan\theta$$

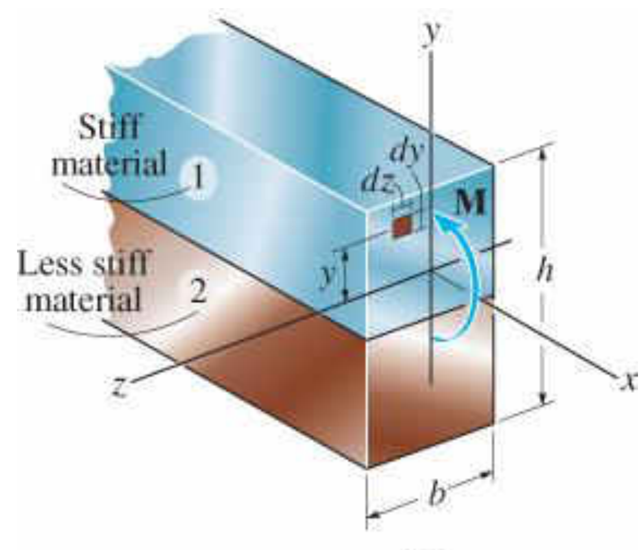
$$\alpha = 85.3^{\circ}$$

Neutral Axis



- Beams constructed of two or more different materials are called composite beams
- Engineers design beams in this manner to develop a more efficient means for carrying applied loads
- Flexure formula cannot be applied directly to determine normal stress in a composite beam
- Thus a method will be developed to “transform” a beam’s x-section into one made of a single material, then we can apply the flexure formula

- Trial-and-error procedure requires the stress distribution produce a zero resultant force on x-section and moment of stress distribution about neutral axis must be equal to **M**
- A simpler way to satisfy the conditions is to "transform" the beam into one made of a *single material*

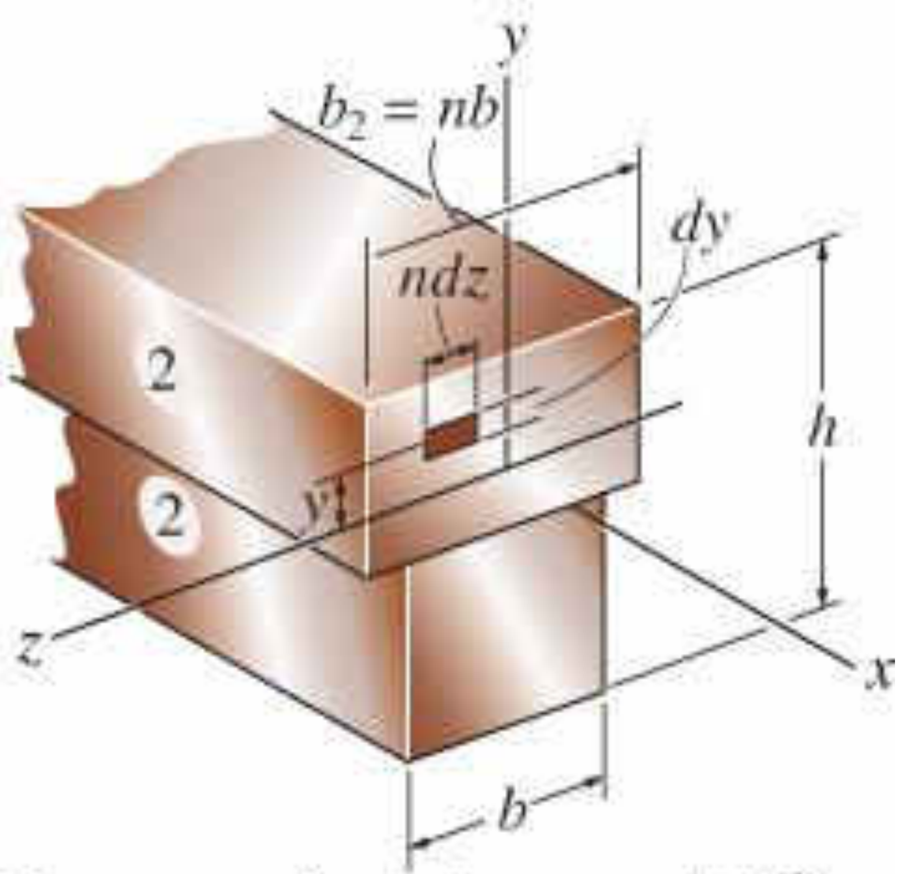


(a)

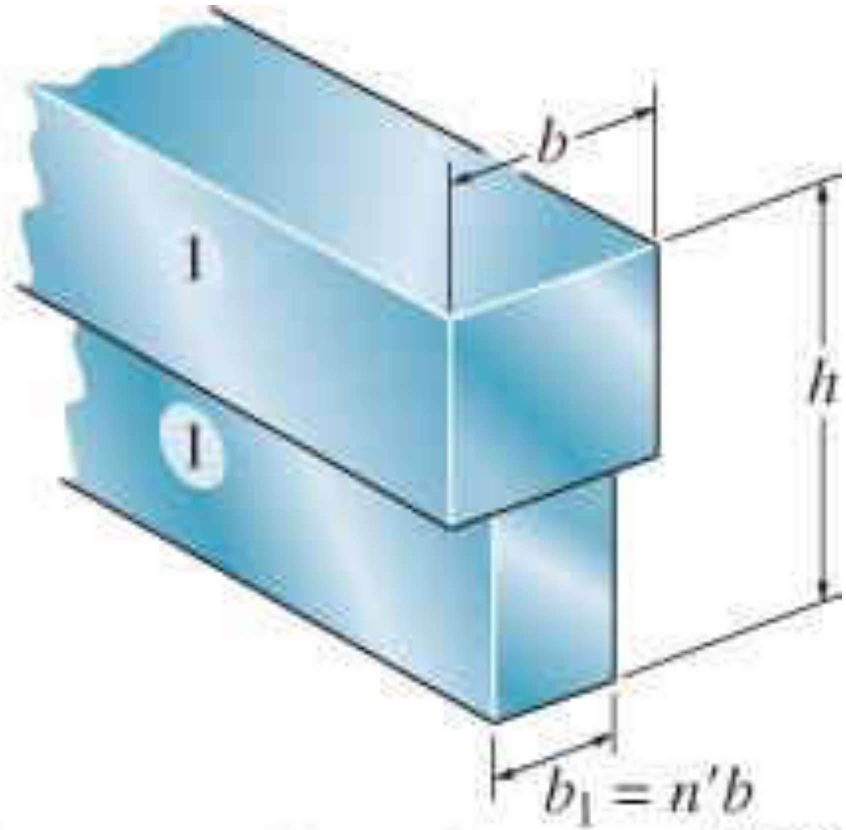
- Height remains the same, but upper portion of beam widened to carry equivalent load to that carried by material 1.

$$n = \frac{E_1}{E_2}$$ Equation 6-20

- Dimensionless factor n, is called the transformation factor. It indicates that x-section, with a width b on original beam, be increased to a width of $b_2 = nb$ in region where material 1 is being transformed into material 2.

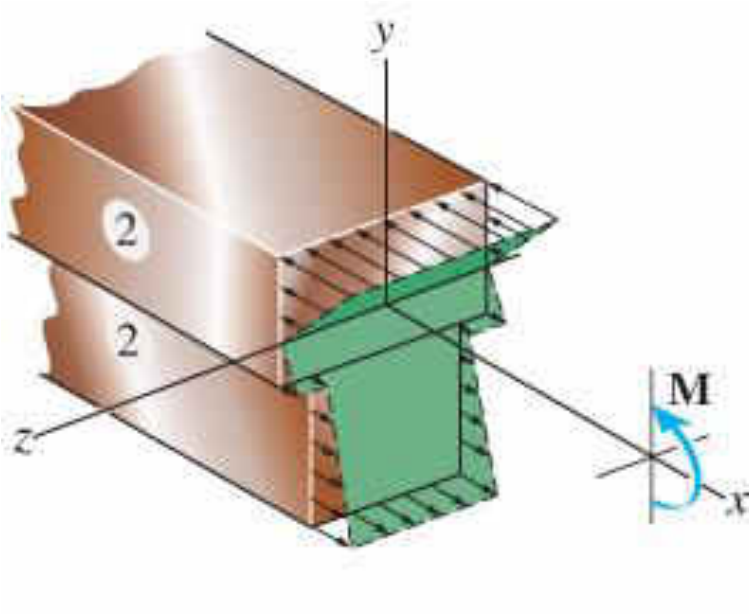


Beam transformed to material (2)
(e)

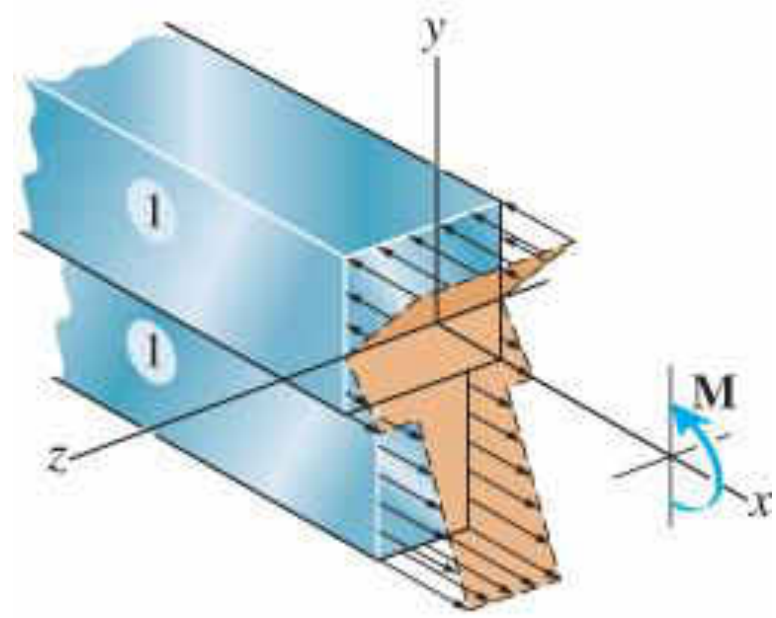


Beam transformed to material (1)
(f)

- Once “transformed”, the normal-stress distribution over the transformed x-section will be linear as shown below.



Bending-stress variation for beam transformed to material 2
(g)



Bending-stress variation for beam transformed to material 1
(h)

- For “transformed” material, stress on transformed section has to be multiplied by transformation factor n (or n')

$$\sigma = n\sigma'$$

Equation 6-21

IMPORTANT

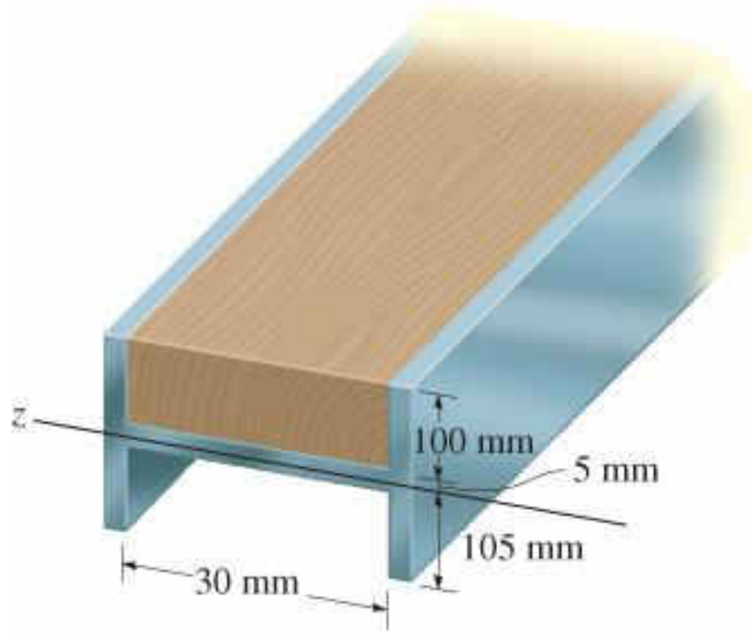
- Composite beams made from different materials to efficiently carry a load
- Application of flexure formula requires material to be homogeneous, and x-section of beam must be transformed into a single material

IMPORTANT

- Transformation factor is a ratio of the moduli of different materials that make up the beam. It converts dimensions of x-section of composite beam into a beam of single material.
- Stress in transformed section must be multiplied by transformation factor to obtain stress in actual beam

Composite beam as shown. If allowable normal stress for steel is $(\sigma_{allow})_{st}$ = 168 MPa and for wood is $(\sigma_{allow})_{w}$ = 21 MPa, determine maximum bending moment beam can support, with and without reinforcement.

E_{st} = 200 GPa, E_w = 12 GPa, Moment of inertia of steel beam is $I_z = 7.93 \times 10^6$ mm^4, x-sectional area is $A = 5493.75\ mm^2$



(a)

Without board

Neutral axis coincides with the *z* axis. Direct application of flexure formula to steel beam yields

$$(\sigma_{\text{allow}})_{\text{st}} = \frac{Mc}{I_z}$$

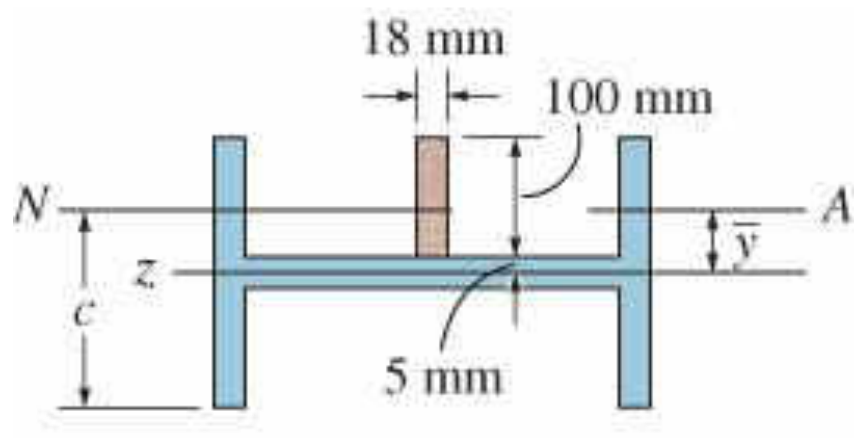
$$\vdots$$

$$M = 12.688\ \text{kN}\cdot\text{m}$$

With board

Easier to transform wood to equivalent amount of steel. Thus, $n = E_w/E_{st}$.

$$b_{st} = nb_w = \frac{12(10^3)\,\text{MPa}}{200(10^3)\,\text{MPa}}(300\,\text{mm}) = 18\,\text{mm}$$



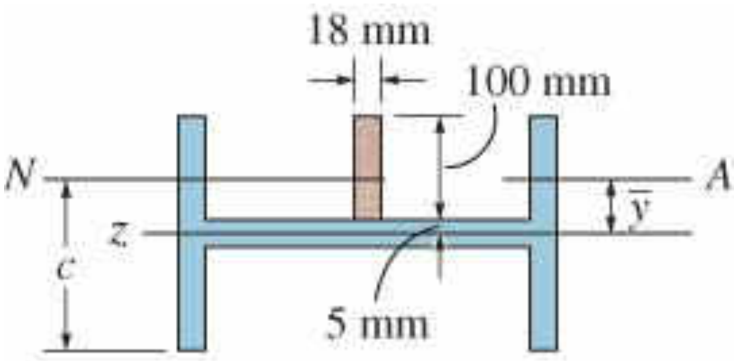
(b)

With board

Neutral axis is at $\bar{y} = \dfrac{\Sigma \tilde{y} A}{\Sigma A} = \ldots = 13.57\ \text{mm}$

Moment of inertia about neutral axis is

$$I = \ldots = 13.53(10^6)\ \text{mm}^4$$



(b)

With board

Maximum normal stress in steel will occur at bottom of the beam. Here

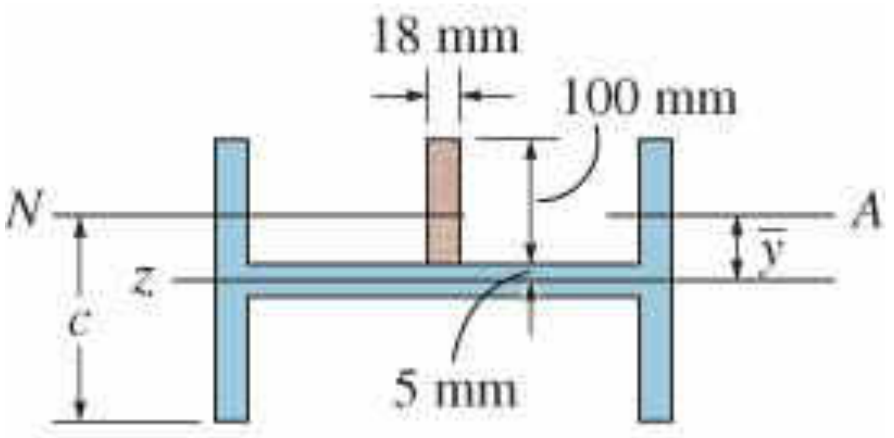
$$c = 105 \text{ mm} + 13.57 \text{ mm} = 118.57 \text{ mm}.$$

Therefore,

$$(\sigma_{\text{allow}})_{\text{st}} = \frac{Mc}{I}$$

$$\vdots$$

$$M = 19.17 \text{ kN}\cdot\text{m}$$



(b)

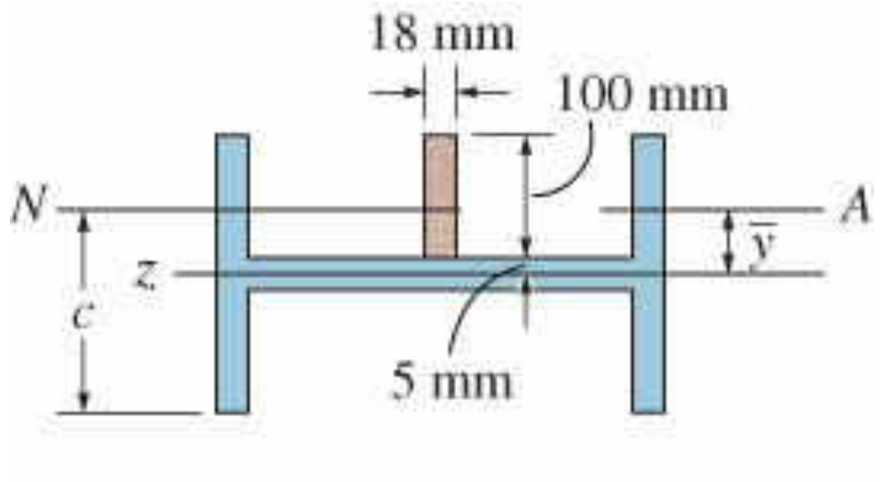
With board

Maximum normal stress in wood occur at top of the beam. Here $c' = 105 \text{ mm} - 13.57 \text{ mm} = 91.43 \text{ mm}$. Since $\sigma_w = n\sigma_{st}$, maximum moment based on allowable stress for wood is

$$(\sigma_{allow})_w = n \frac{M'c'}{I}$$

$$\vdots$$

$$M' = 51.79 \text{ kN}\cdot\text{m}$$



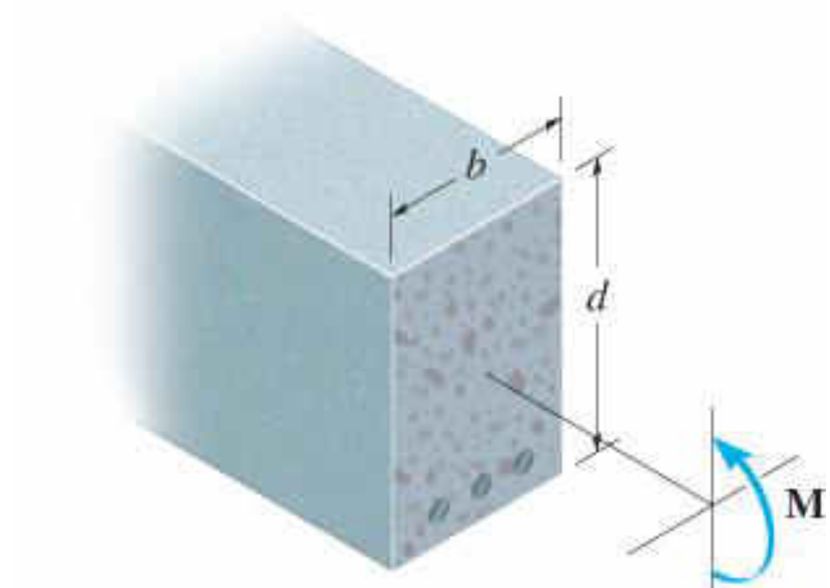
(b)

With board

By comparison, maximum moment limited by allowable steel in the steel. Thus, M = 19.17 kN·m.

Note also that by using board as reinforcement, one provides an additional 51% moment capacity for the beam

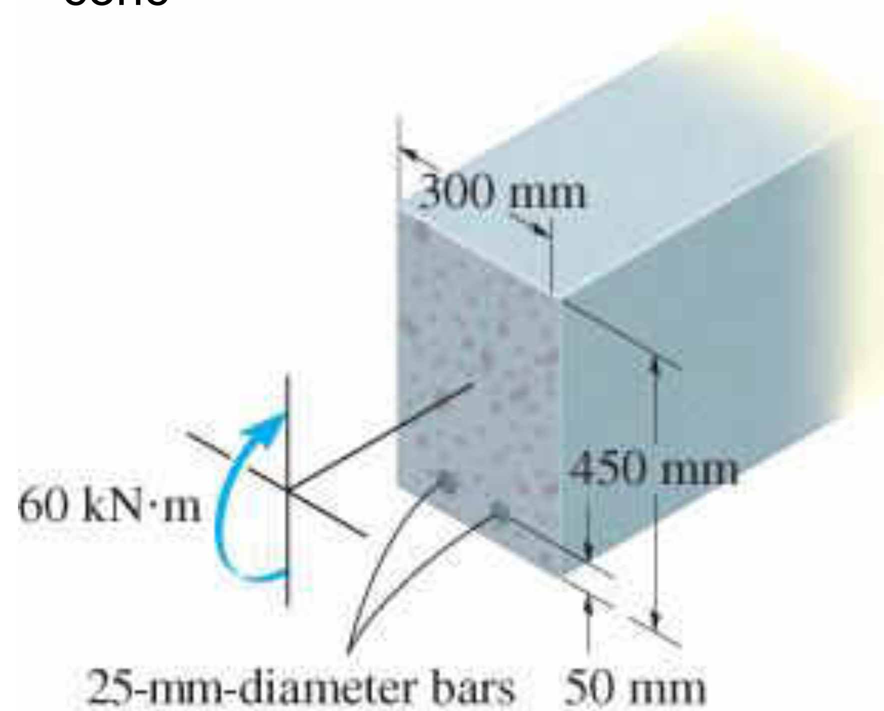
- Steel reinforcing rods is placed in concrete to resist tension cracking
- The rods are placed farthest away from beam's neutral axis. However, they also need concrete coverage to prevent corrosion or loss of strength in case of fire



(a)

If reinforced concrete beam is subjected to bending moment of M = 60 kN·m, determine the normal stress in each of the steel reinforcing rods and maximum normal stress in the concrete.

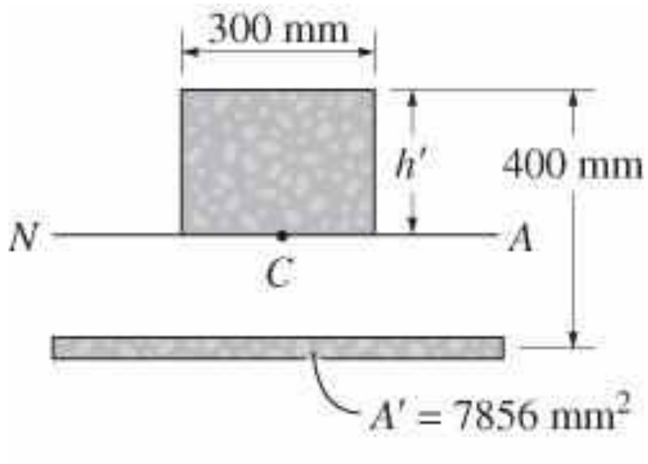
Take E_{st} = 200 GPa and E_{conc} = 25 GPa.



Section properties

Total area of steel, $A_{st} = 2[\pi(12.5\text{ mm})^2] = 982\text{ mm}^2$ will be transformed into an equivalent area of concrete.

$$A' = nA_{st} = \ldots = 7856\,\text{mm}^2$$



(b)

Centroid must lie on the neutral axis, thus $\Sigma\, \bar{y}A = 0$

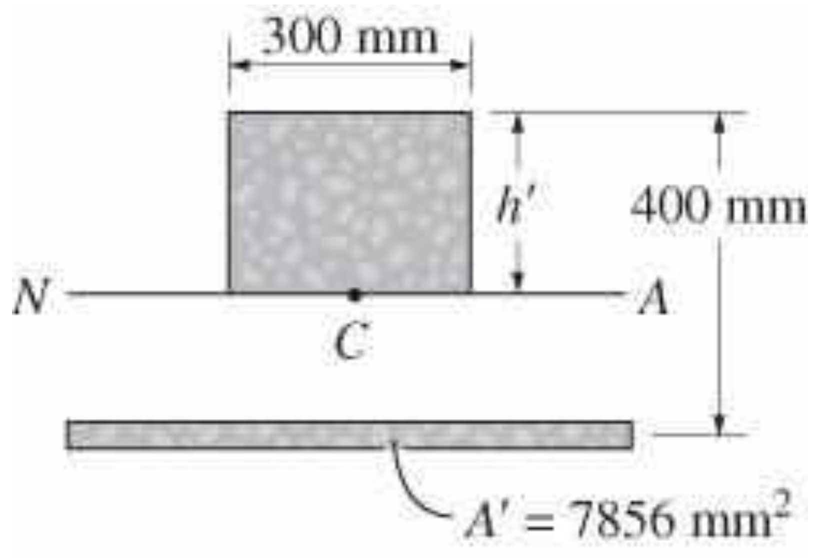
$$(h')^2 + 52.37h' - 20949.33 = 0$$

Solving for positive root, $h' = 120.90\,\text{mm}$

Section properties

Using computed value of *h'*, moment of inertia of transformed section about neutral axis is

$$I = \ldots = 788.67 \times 10^6 \text{ mm}^4$$

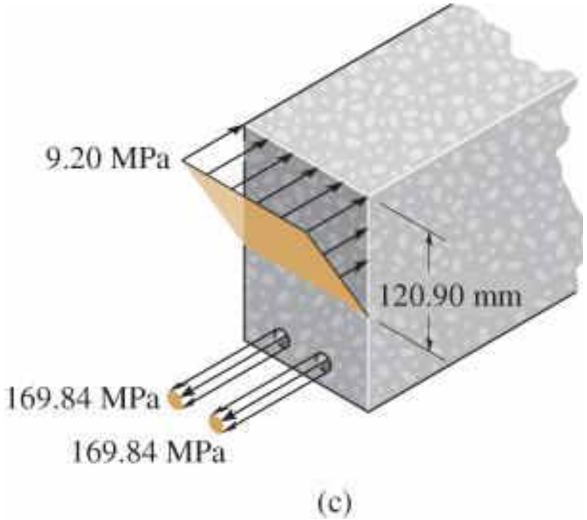


(b)

Normal stress

Apply flexure formula to transformed section, maximum normal stress in concrete is

$$(\sigma_{conc})_{max} = \ldots = 9.20\ \text{MPa}$$



Normal stress resisted by "concrete" strip, is

$$\sigma'_{conc} = \ldots = 21.23\ \text{MPa}$$

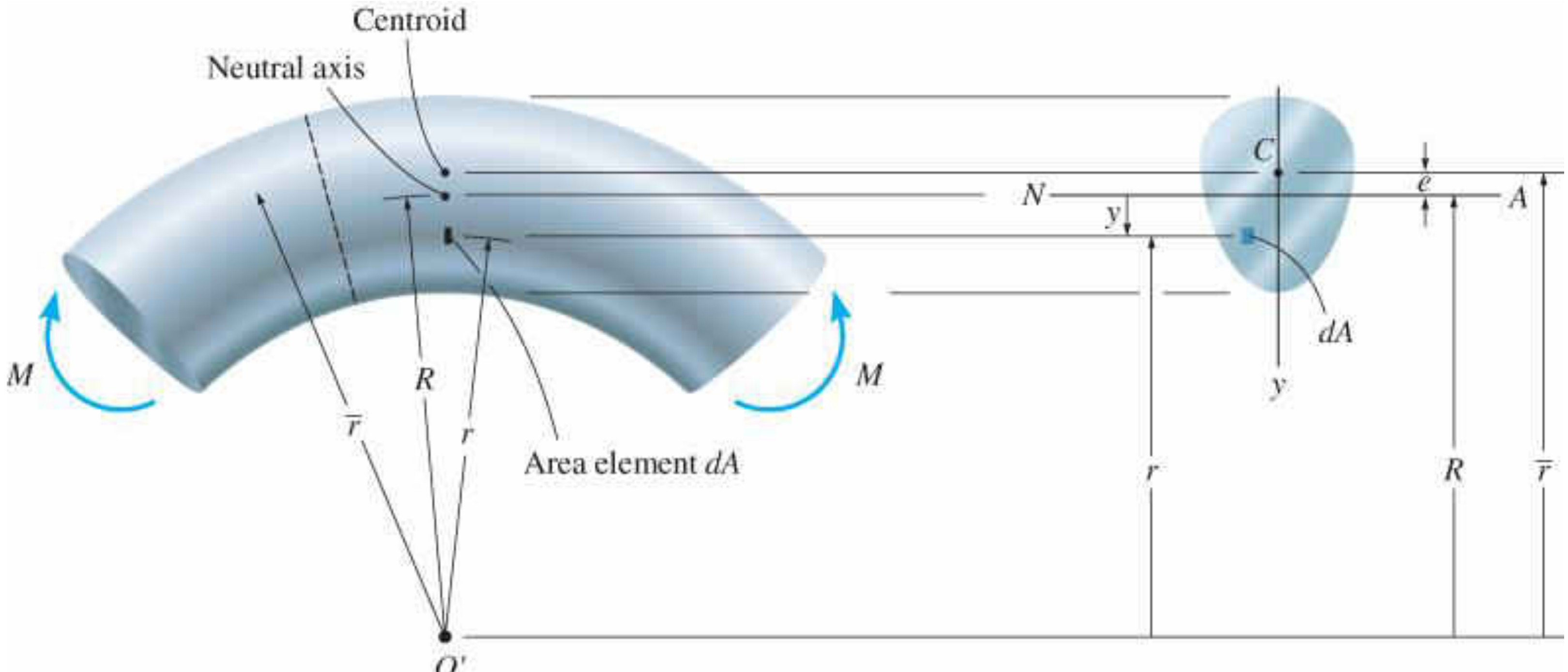
Normal stress in each of the rods is

$$\sigma'_{st} = n\sigma'_{conc} = \ldots = 169.84\ \text{MPa}$$

- Flexure formula only applies to members that are straight as normal strain varies linearly from the neutral axis
- Thus another equation needs to be formulated for curved beam, i.e., a member that has a curved axis and is subjected to bending

- Assumptions for analysis:
  1. X-sectional area is constant and has an axis of symmetry that is perpendicular to direction of applied moment **M**
  2. Material is homogeneous and isotropic and behaves in linear-elastic manner under loading
  3. X-sections of member *remain plane* after moment applied and distortion of x-section within its own will be neglected

- By first principles:



(a)

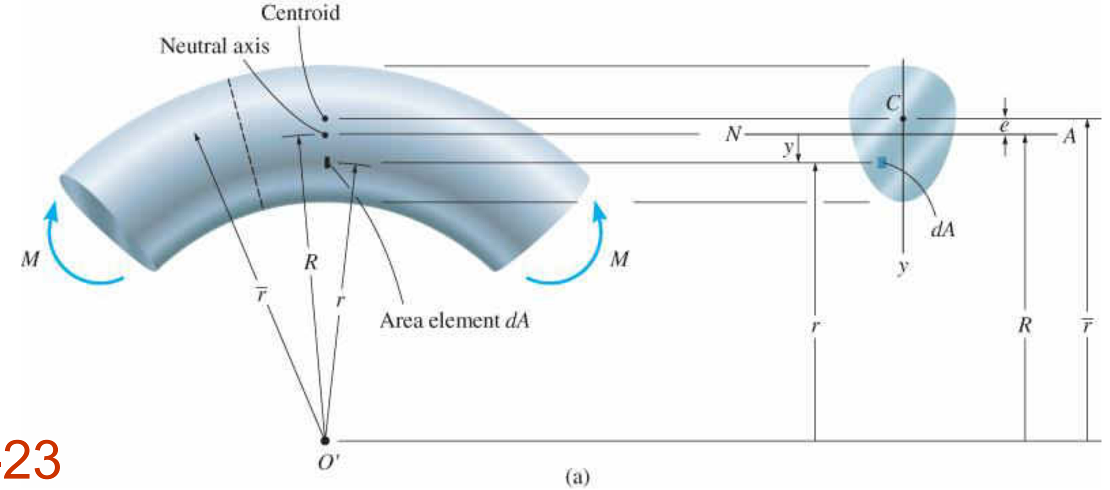
$$\sigma = Ek\left(\frac{R-r}{r}\right)$$

Equation 6-22

Location of neutral axis:

$$R = \frac{A}{\int_A \frac{dA}{r}}$$

Equation 6-23



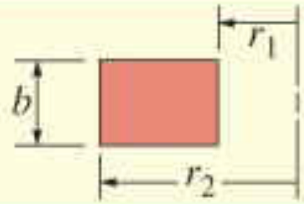
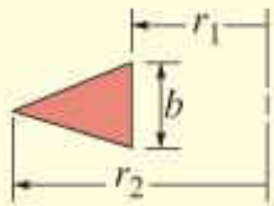
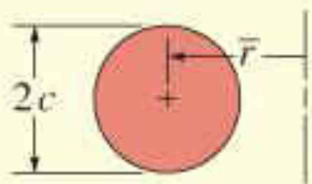
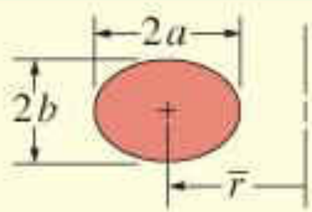
R = location of neutral axis, specified from center of curvature O' of member

A = x-sectional area of the member

R = arbitrary position of the area element dA on x-section specified from center of curvature P' of member

Common x-sections to use in integral in Eqn 6-23

$$R = \frac{A}{\int_A \frac{dA}{r}}$$

Shape	Area	$\int_A \frac{dA}{r}$
	$b(r_2 - r_1)$	$b \ln \frac{r_2}{r_1}$
	$\frac{b}{2}(r_2 - r_1)$	$\frac{b\ r_2}{(r_2 - r_1)}\left(\ln \frac{r_2}{r_1}\right) - b$
	πc^2	$2\pi\left(\bar{r} - \sqrt{\bar{r}^2 - c^2}\right)$
	πab	$\frac{2\pi b}{a}\left(\bar{r} - \sqrt{\bar{r}^2 - a^2}\right)$

Normal stress in curved beam:

$$\sigma = \frac{M(R-r)}{Ar(\underline{r}-R)}$$

Equation 6-24

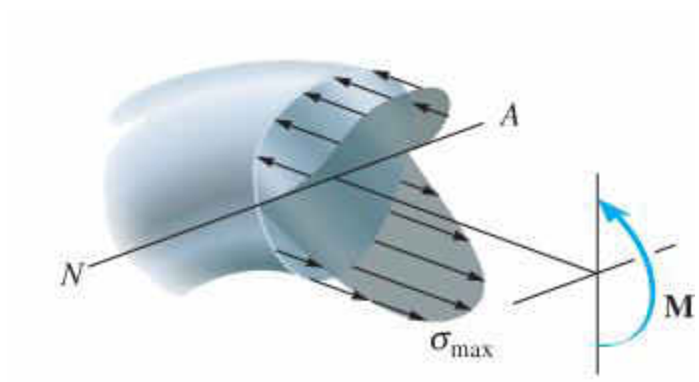
$$\sigma = \frac{My}{Ae(R-y)}$$

Equation 6-25

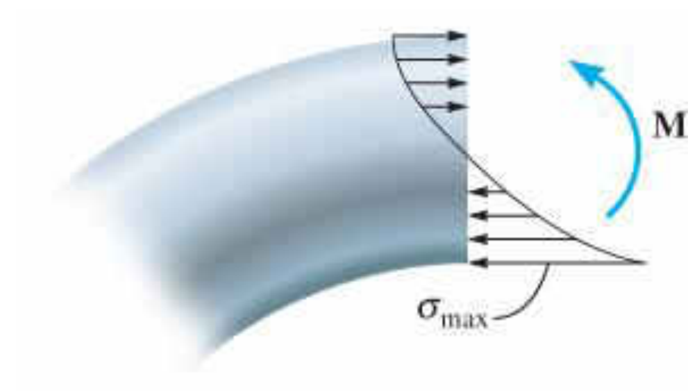
- The above equations represent 2 forms of the curved-beam formula, used to determine the normal-stress distribution in a member

Normal stress in curved beam:

- The stress distribution is as shown, hyperbolic, and is sometimes called circumferential stress
- Radial stress will also be created as a result
- If radius of curvature is greater than 5 times the depth of member, flexure formula can be used to determine the stress instead



(d)



Bending stress variation
(profile view)

(c)

IMPORTANT

- Curved beam formula used to determine circumferential stress in a beam when radius of curvature is less than five times the depth of the beam
- Due to beam curvature, normal strain in beam does not vary linearly with depth as in the case of straight beam. Thus, neutral axis does not pass through centroid of section
- Ignore radial stress component of bending, if x-section is a solid section and not made from thin plates

Procedure for analysis

Section properties

- Determine x-sectional area A and location of centroid $\bar{r}$, measured from centre of curvature
- Compute location of neutral axis, R using Eqn 6-23 or Table 6-2. If x-sectional area consists of n "composite" parts, compute $\int dA/r$ for each part.
- From Eqn 6-23, for entire section,
- In all cases, $R < \bar{r}$

$$R = \frac{A}{\int_A \frac{dA}{r}}$$

Procedure for analysis

Normal stress

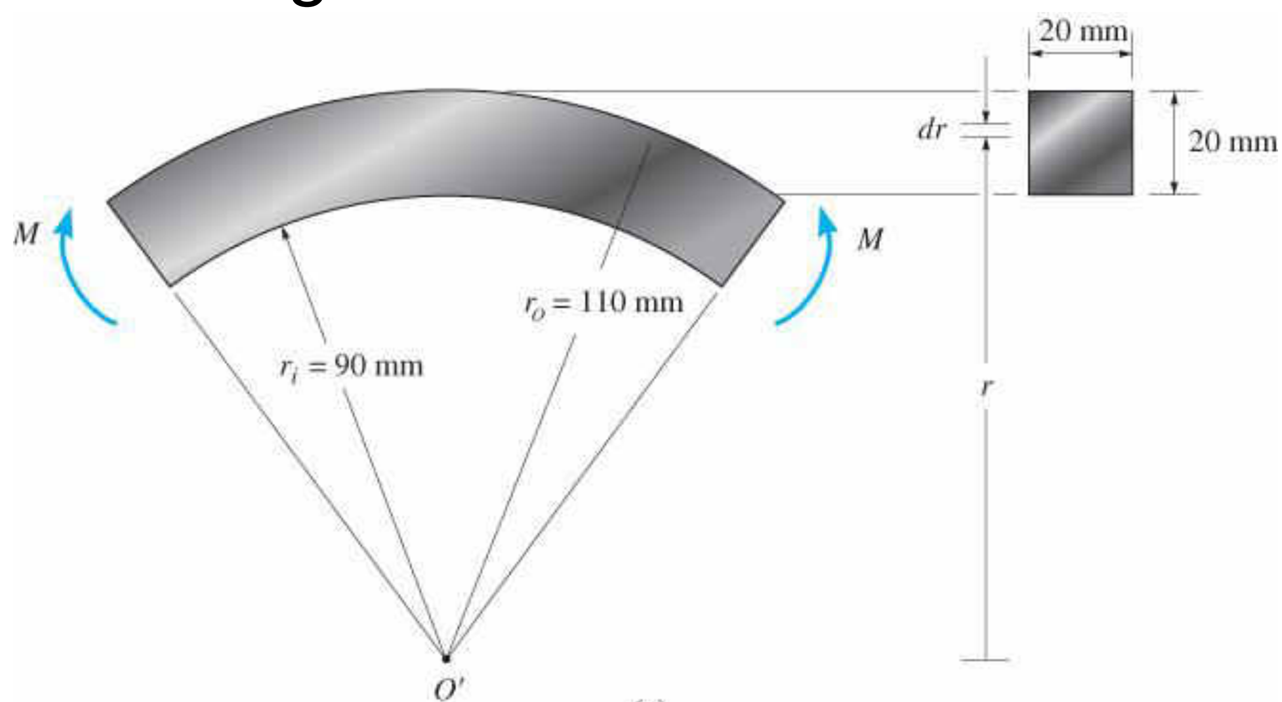
- Normal stress located at a pt r away from the centre of curvature is determined Eqn 6-24. If distance y to pt is measured from neutral axis, then compute $e = r - R$ and use Eqn 6-25
- Since $r - R$ generally produces a very small number, it is best to calculate r and R with sufficient capacity so that subtraction leads to e with at least 3 significant figures

Procedure for analysis

Normal stress

- Positive stress indicates tensile stress, negative means compressive stress
- Stress distribution over entire x-section can be graphed, or a volume element of material can be isolated and used to represent stress acting at the pt on x-section where it has been calculated

Steel bar with rectangular x-section is shaped into a circular arc. Allowable normal stress is σ_{allow} = 140 MPa. Determine maximum bending moment **M** that can be applied to the bar. What would this moment be if the bar was straight?



(a)

Internal moment

Since **M** tends to increase bar's radius of curvature, it is positive.

Section properties

Location of neutral axis is determined using Eqn 6-23.

$$\int_A \frac{dA}{r} = \int_{90\text{ mm}}^{110\text{ mm}} \frac{(20\text{ mm})\,dr}{r} = 4.0134\text{ mm}$$

Similar result can also be obtained from Table 6-2.

Section properties

We do not know if normal stress reaches its maximum at the top or bottom of the bar, so both cases must be compute separately.

Since normal stress at bar top is $\sigma = -140$ MPa

$$\sigma = \frac{M(R - r_o)}{A r_o(\bar{r} - R)}$$

$$\vdots$$

$$M = 0.199\,\mathrm{kN \cdot m}$$

Section properties

Likewise, at bottom of bar, σ = +140 MPa

$$\sigma = \frac{M(R - r_i)}{A r_i (\bar{r} - R)}$$

$$\vdots$$

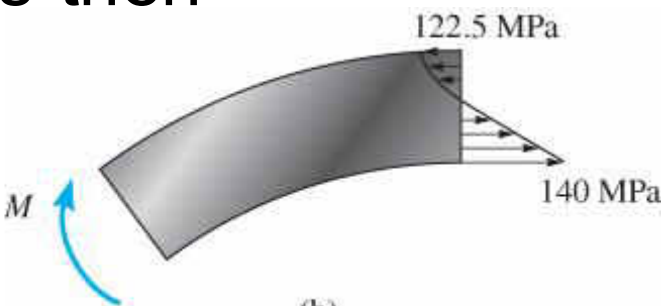
$$M = 0.174\,\mathrm{kN \cdot m}$$

By comparison, maximum that can be applied is $0.174\,\mathrm{kN \cdot m}$, so maximum normal stress occurs at bottom of the bar.

Section properties

Compressive stress at top of bar is then

$$\sigma = 122.5\ \mathrm{N/mm^2}$$



(b)

By comparison, maximum that can be applied is $0.174\ \mathrm{kN \cdot m}$, so maximum normal stress occurs at bottom of the bar.

If bar was straight?

$$\sigma = Mc/I$$

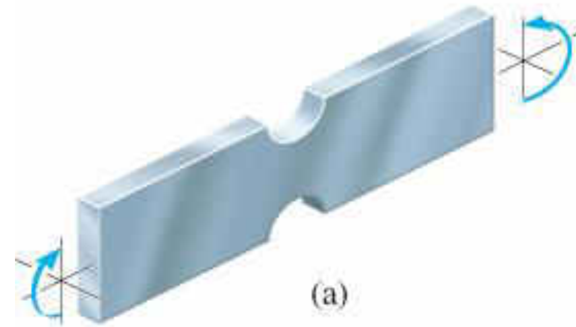
$$\vdots$$

$$M = 0.187\,\mathrm{kN\cdot m}$$

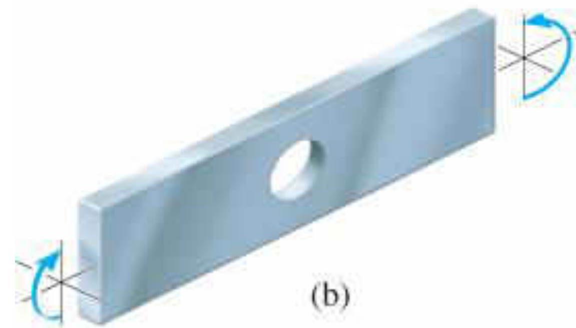
This represents an error of about 7% from the more exact value determined above.

- Flexure formula can only be used to determine stress distribution within regions of a member where x-sectional area is constant or tapers slightly
- If x-section suddenly changes, normal-stress and strain distributions become nonlinear and they can only be obtained via experiment or mathematical analysis using the theory of elasticity

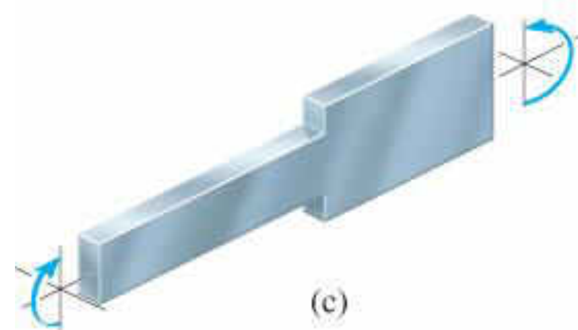
- Common discontinuities include members having notches on their surfaces, holes for passage of fasteners or abrupt changes in outer dimensions of member’s x-section
- The *maximum* normal stress at the discontinuities occur at the *smallest* x-sectional area



(a)



(b)



(c)

- For design, we only need to know the maximum normal stress developed at these sections, not the actual stress distribution
- Thus, the maximum normal stress due to bending can be obtained using the stress-concentration factor *K*

$$\sigma = K \frac{Mc}{I}$$

Equation 6-26

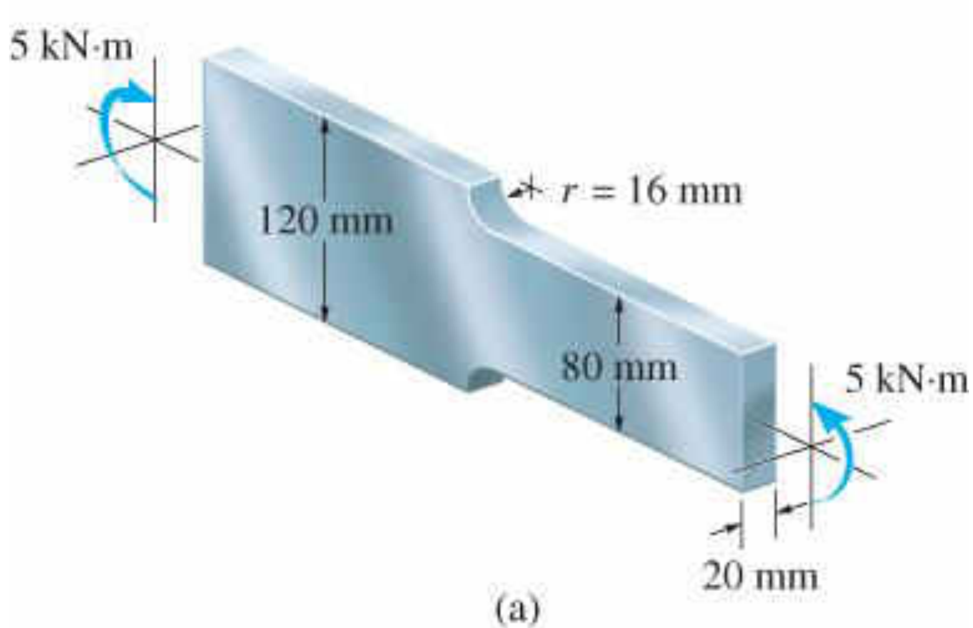
IMPORTANT

- Stress concentrations in members subjected to bending occur at pts of x-sectional change, such as notches and holes, because here the stress and strain become nonlinear.
- The more severe the change, the larger the stress distribution
- For design/analysis, not necessary to know the exact stress distribution around x-sectional change
- The maximum normal stress occurs at the smallest x-sectional area

IMPORTANT

- The maximum normal stress can be obtained using stress concentration factor K, which is determined through experiment and is a function of the geometry of the member
- If material is brittle or subjected to fatigue loading, stress concentrations in the member need to be considered in design

Transition in x-sectional area of steel bar is achieved using shoulder fillets as shown. If bar is subjected to a bending moment of 5kN·m, determine the maximum normal stress developed in the steel. σ_Y = 500 MPa.

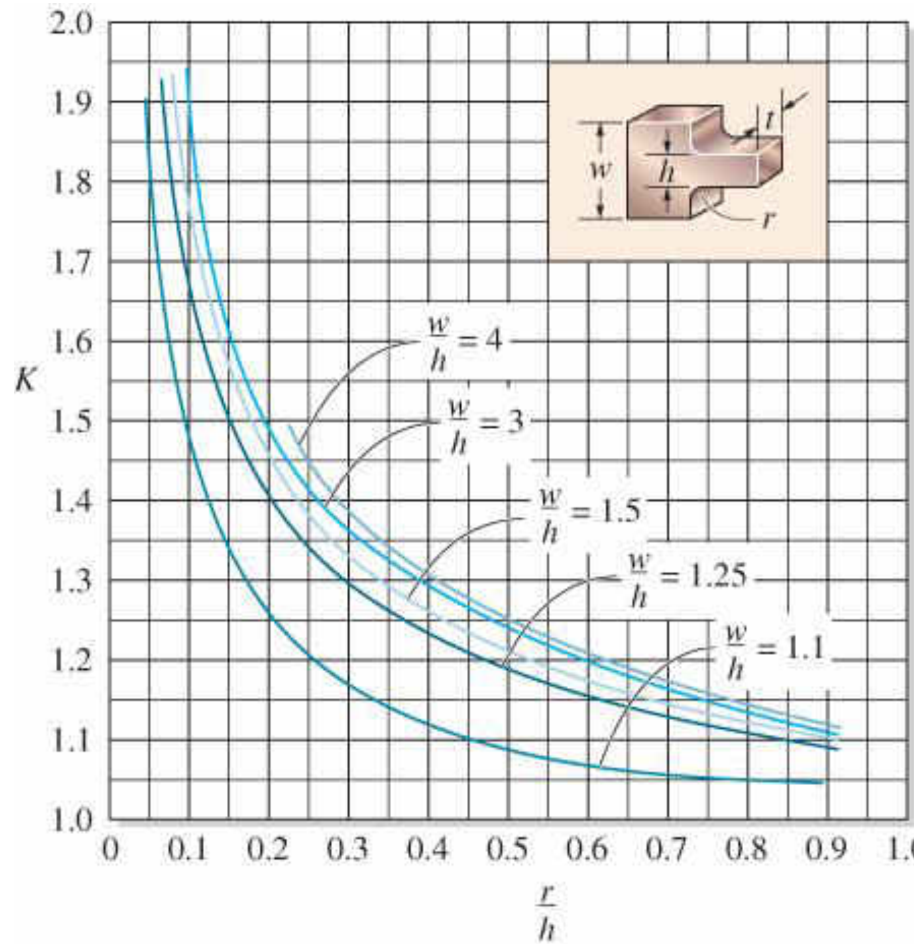


Moment creates largest stress in bar at base of fillet. Stress concentration factor can be determined from the graph.

$$r/h = ... = 0.2$$

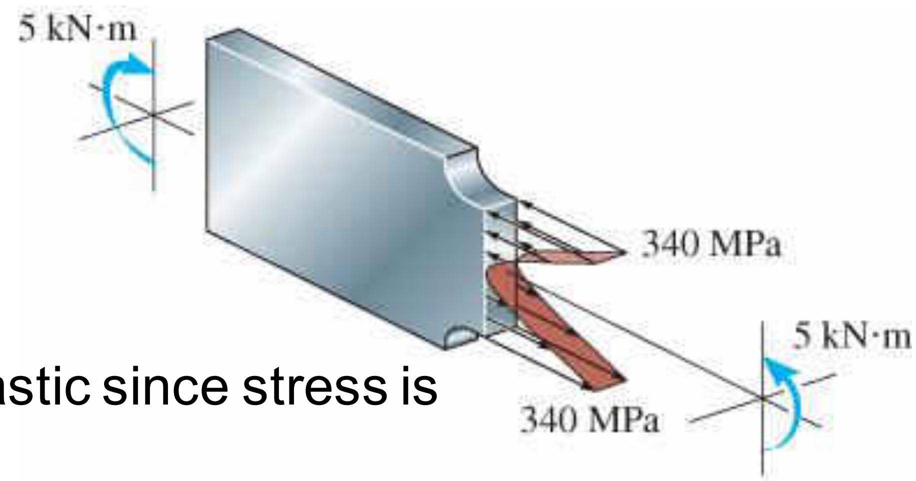
$$w/h = ... = 1.5$$

From above values, we get
K = 1.45



Applying Eqn 6-26:

$$\sigma = K\frac{Mc}{I} = \ldots = 340\,\text{MPa}$$

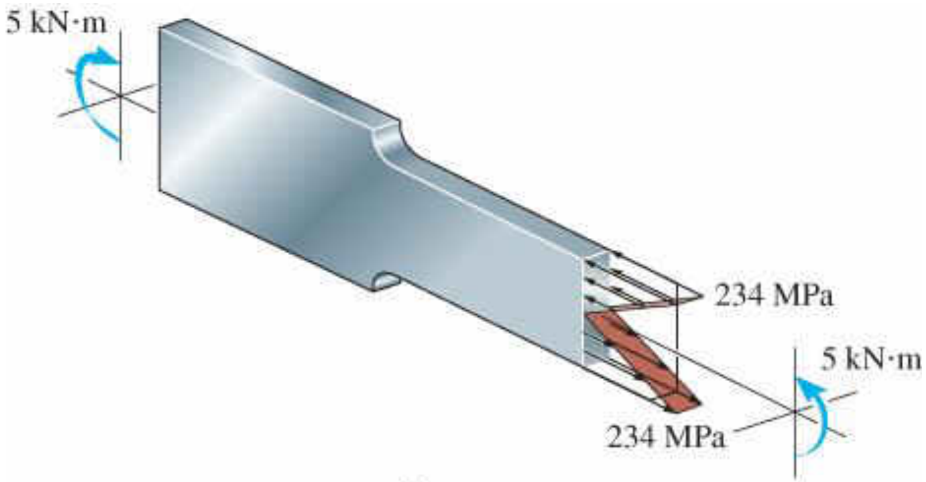


(b)

Result indicates that steel remains elastic since stress is below yield stress.

Normal stress distribution is nonlinear.

However, by Saint-Venant’s principle, the localized stresses smooth out and become linear when one moves at a distance of 80 mm or more to right of transition.



(c)

Thus, the flexure formula gives $\sigma_{\max} = 234\ \mathrm{MPa}$.

Note that choice of a larger-radius fillet will significantly reduce $\sigma_{\max}$, since as r increase, K will decrease.

Linear normal-strain distribution

- Based on geometric considerations, it was shown in section 6.3 that normal strains that develop in the material always vary linearly from zero at neutral axis of x-section to a maximum at the farthest pt from the neutral axis

Resultant force equals zero

- Since there is only a resultant internal moment acting on the x-section, resultant force caused by stress distribution must be equal to zero.

$$F_R = \Sigma F_x; \qquad \int_A \sigma \, dA = 0$$

Equation 6-27

Resultant moment

- Resultant moment at section must be equivalent to moment caused by the stress distribution about the neutral axis

$$(M_R)_z = \Sigma M_z; \qquad M = \int_A y\,(\sigma\, dA)$$

Equation 6-28

Maximum elastic moment

- Since each of the forces acts through the centroid of the volume of its associated triangular stress block, we have

$$M_Y = (1/6)(bh^2 \sigma_Y)$$

Equation 6-29

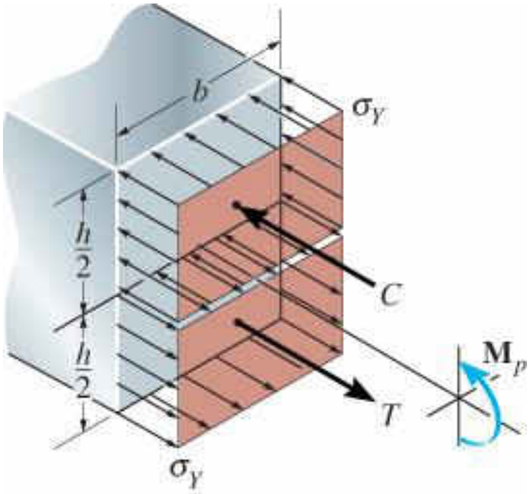
Plastic moment

- From first principles, we derive the plastic moment

$$M_P = (3/2)\, M_Y$$

Equation 6-32

- Its value is unique only for the rectangular section shown below, since analysis depends on geometry of the x-section



Plastic moment

(f)

Plastic moment

- Beams used in steel buildings are sometimes designed to resist a plastic moment. The codes usually list a design property called the shape factor:

$$k = M_P/M_Y$$ Equation 6-33

- The *k*-value specifies the additional moment capacity a beam can support beyond its maximum elastic moment.

Ultimate moment

- Trial-and-error procedure is used to solve such a problem:
  1. For given moment M, assume location of neutral axis and slope of “linear” strain distribution
  2. Graphically establish the stress distribution on member’s x-section using σ-ε curve to plot values of stress and strain.

Ultimate moment

- Trial-and-error procedure is used to solve such a problem:

3. Determine the volumes enclosed by tensile and compressive stress “blocks”. They need to be equal, otherwise, adjust the location neutral axis till they are equal
4. Once tensile force = compressive force, compute their corresponding moments. Moment arms are measured from the neutral axis to the centroids of the volumes defined by the stress distributions

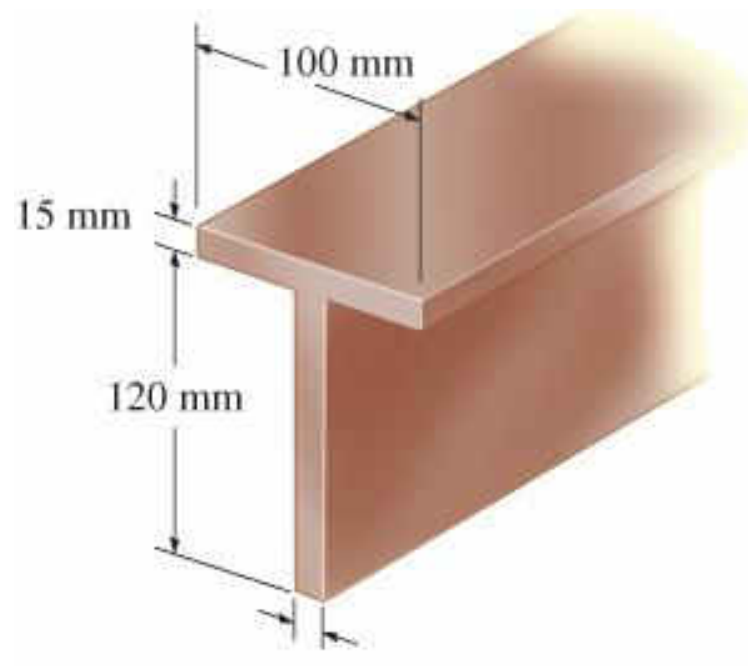
IMPORTANT

- Normal strain distribution over x-section of a beam is based only on geometric considerations and has been found to always remain linear, regardless of load applied
- Normal stress distribution must be determined from the material behavior, or stress/strain diagram once the strain is established
- Location of neutral axis is determined from the condition that resultant force on the x-section is zero

IMPORTANT

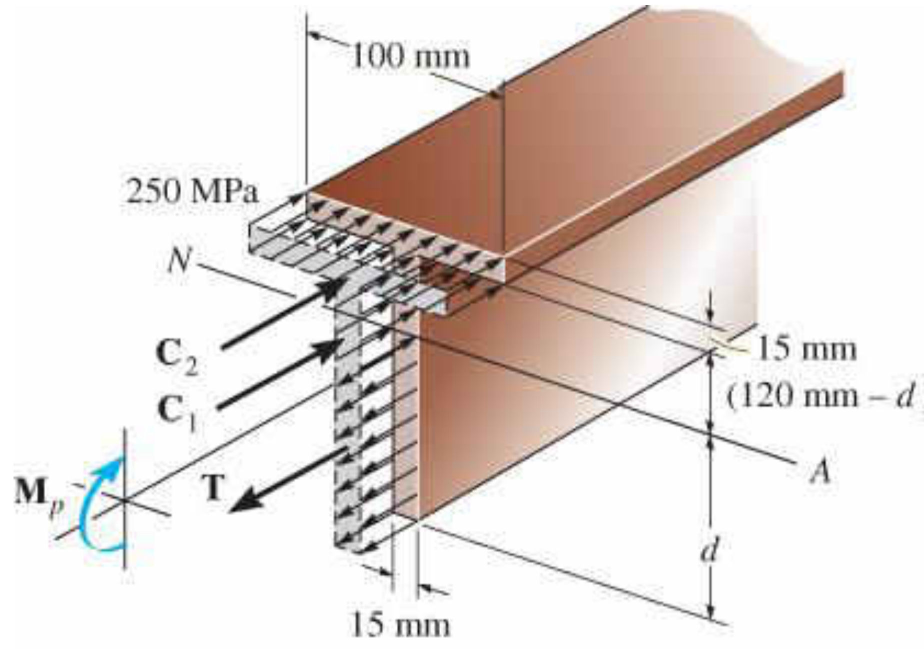
- Resultant internal moment on x-section must be equal to moment of stress distribution about the neutral axis
- Perfectly plastic behavior assumes normal stress distribution is constant over the x-section, and beam continue to bend, with no increase in moment. This moment is called the plastic moment.

T-beam has dimensions as shown. If it is made of an elastic perfectly plastic material having tensile and compressive yield stress of $\sigma_Y = 250$ MPa, determine the plastic moment that can be resisted by the beam.



(a)

“Plastic” stress distribution acting over beam’s x-sectional area is shown. The x-section is not symmetric with respect to horizontal axis, thus neutral axis will not pass through centroid of x-section.



(b)

To determine location of neutral axis, we require the stress distribution to produce a zero resultant force on the x-section.

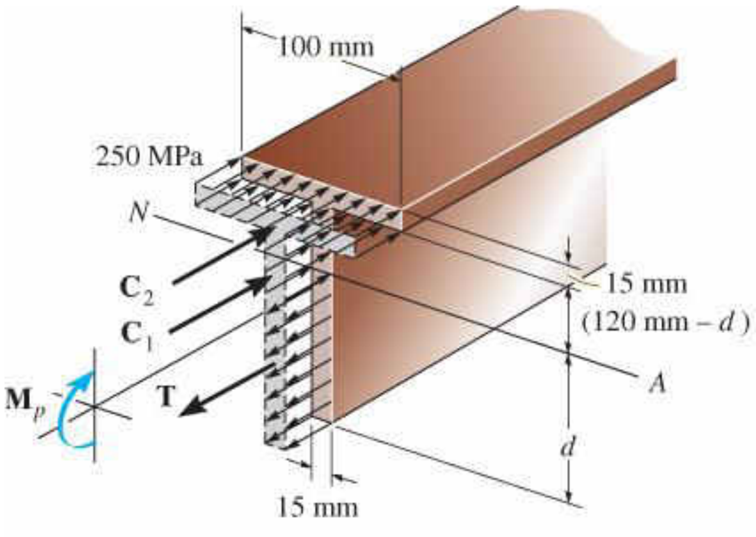
Assuming $d \leq 120\,\text{mm}$, we have

$\int_A \sigma\, dA = 0;$ $\qquad\qquad T - C_1 - C_2 = 0$

$$\vdots$$

$$d = 0.110\,\text{m}$$

$$< 0.120\,\text{m (OK!)}$$



(b)

Using this result, forces acting on each segment:

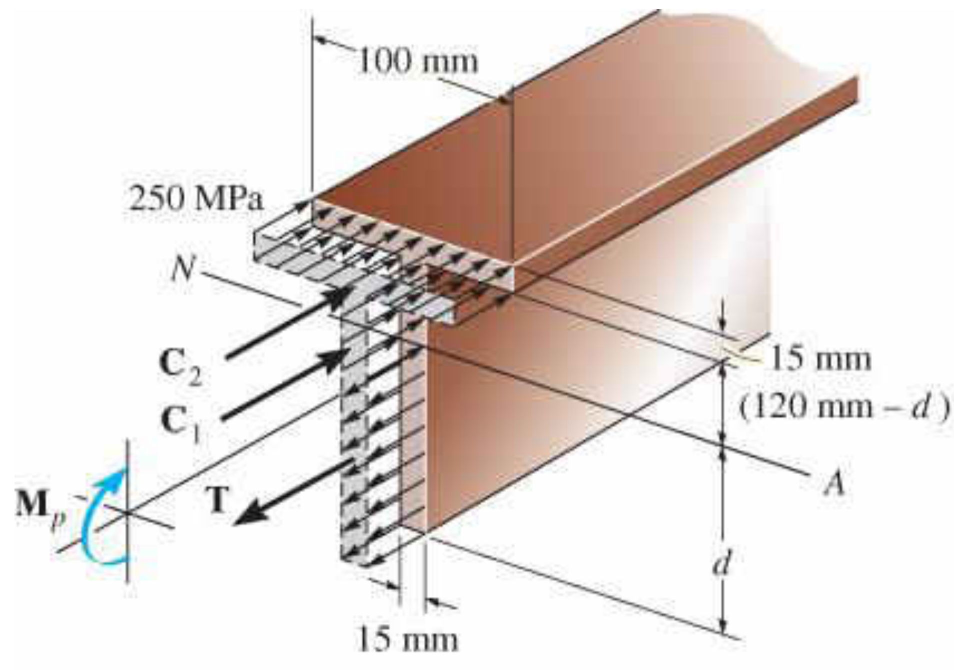
$$T = \ldots = 412.5\,\text{kN}$$

$$C_1 = \ldots = 37.5\,\text{kN}$$

$$C_2 = \ldots = 375\,\text{kN}$$

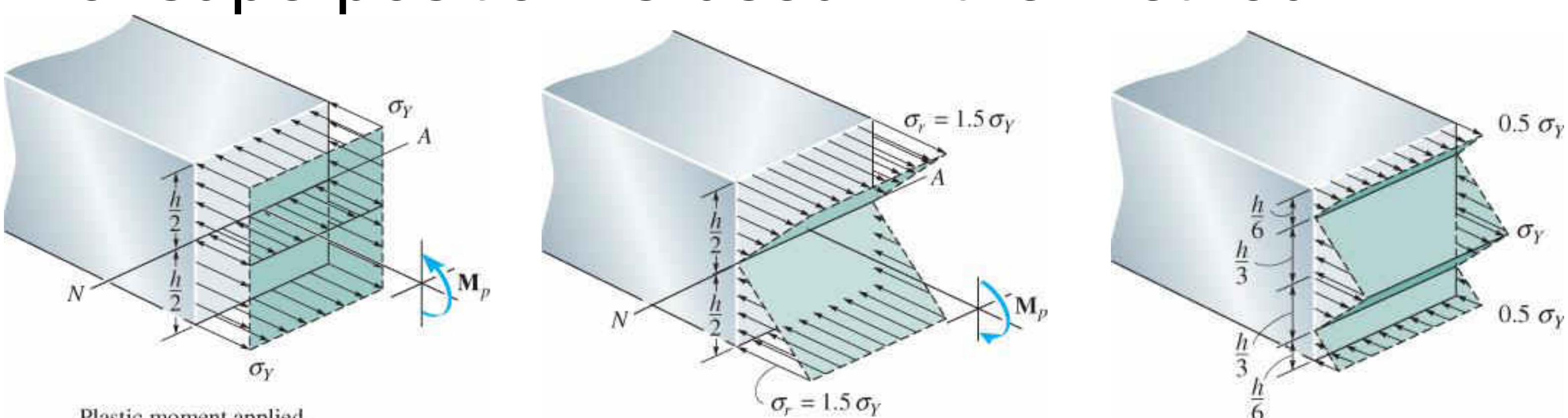
Hence, resulting plastic moment about the neutral axis is

$$M_P = \ldots = 29.4\,\text{kN}\cdot\text{m}$$



(b)

- Like the case for torsion, residual stress is important when considering fatigue and other types of mechanical behavior
- A method is developed here to determine residual stress for a member subjected to bending
- Plastic moment, modulus of rupture and principles of superposition is used in the method



Plastic moment applied causing plastic strain

(c)

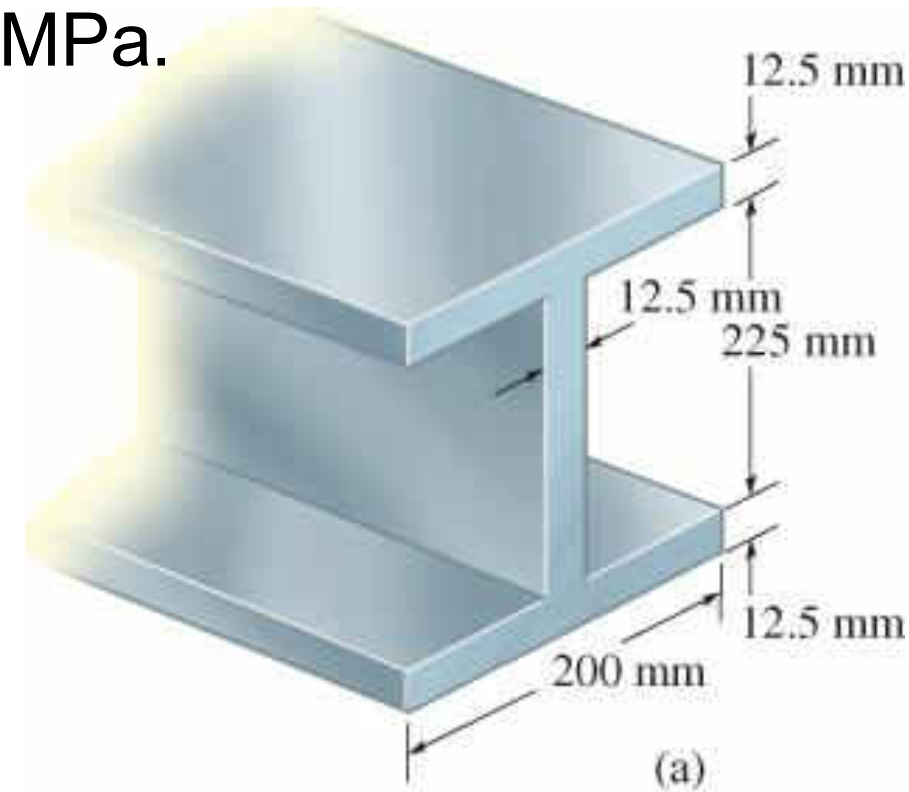
Plastic moment reversal causing elastic strain

(d)

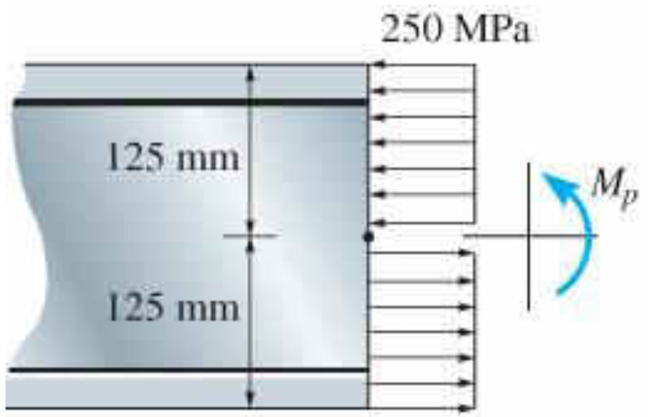
Residual stress distribution in beam

(e)

Steel wide-flange beam shown is subjected to a fully plastic moment of M_P. If moment is removed, determine the residual-stress distribution in the beam. Material is elastic perfectly plastic and has a yield stress of σ_Y = 250 MPa.

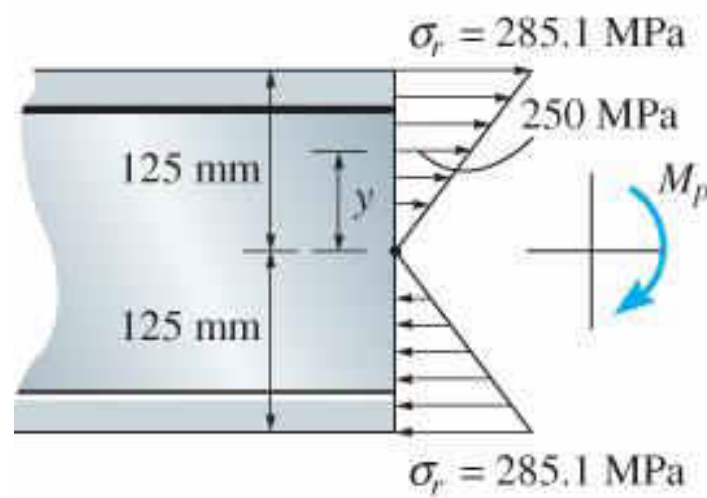


Normal stress distribution caused by M_P is shown in (b). When M_P is removed, material responds elastically. The assumed elastic stress distribution is shown in (c). Modulus of rupture σ_r, is calculated from the flexure formula.



Plastic moment applied
(profile view)

(b)

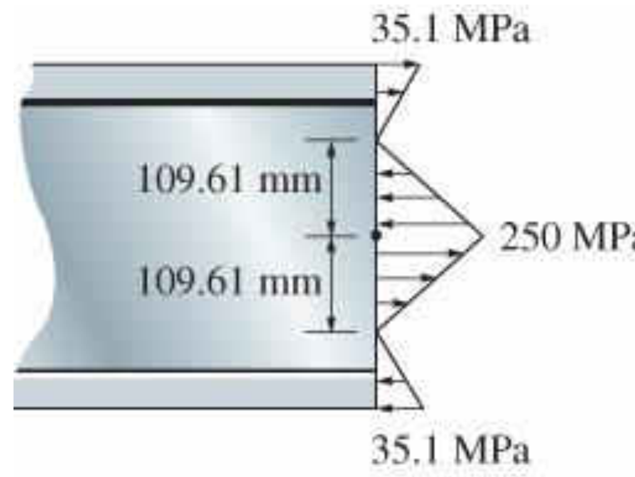


Plastic moment reversed
(profile view)

(c)

Using M_P = 188 kN·m and I = 82.44 × 10⁶ mm⁴.

$$\sigma_{\max} = \frac{Mc}{I} \quad \ldots \quad \sigma_{\text{allow}} = 285.1\,\text{MPa}$$



Residual stress distribution

(d)

As expected, $\sigma_r < 2\sigma_Y$.

Superposition of stresses gives residual-stress distribution as shown. Note that point of zero normal stress was determined by proportion,

$$\frac{281.51\,\text{MPa}}{125\,\text{mm}} = \frac{2501\,\text{MPa}}{y}$$

$$y = 109.61\,\text{mm}$$

- Shear and moment diagrams are graphical representations of internal shear and moment within a beam.
- They can be constructed by sectioning the beam an arbitrary distance x from the left end, finding **V** and **M** as functions of x, then plotting the results
- Another method to plot the diagrams is to realize that at each pt, the slope of the shear diagram is a negative of the distributed loading, $w = dV/dx$; and slope of moment diagram is the shear, $V = dM/dx$.

- Also, the (-ve) area under the loading diagram represents the change in shear, $\Delta V = -\int w\, dx$.
- The area under the shear diagram represents the change in moment, $\Delta M = \int V\, dx$. Note that values of shear and moment at any pt can be obtained using the method of sections
- A bending moment tends to produce a linear variation of normal strain within a beam.
- Provided that material is homogeneous, Hooke's law applies, and moment applied does not cause yielding, then equilibrium is used to relate the internal moment in the beam to its stress distribution

- That results in the flexure formula, $\sigma = Mc/I$, where I and c are determined from the neutral axis that passes through the centroid of the x-section
- If x-section of beam is not symmetric about an axis that is perpendicular to neutral axis, then unsymmetrical bending occurs
- Maximum stress can be determined from formulas, or problem can be solved by considering the superposition of bending about two separate axes

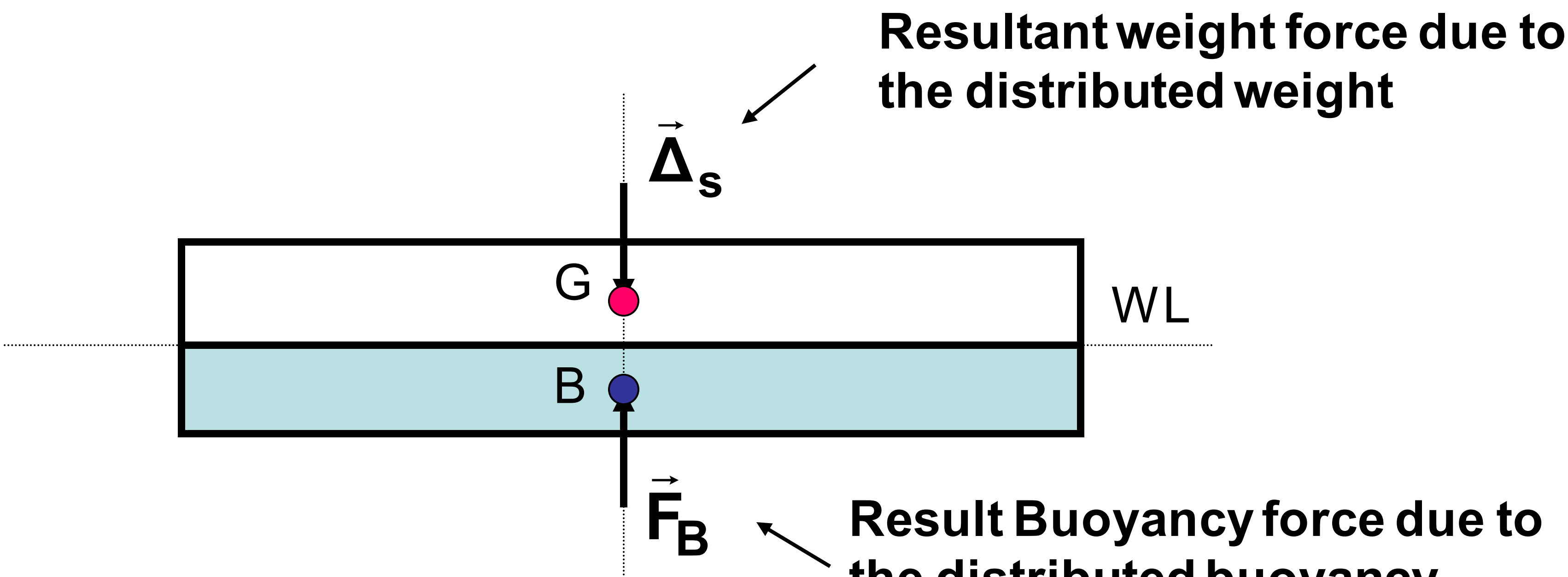
- Beams made from composite materials can be “transformed” so their x-section is assumed to be made from a single material
- A transformation factor, $n = E_1/E_2$ is used to do this.
- Once the “transformation” is done, the stress in the beam can be determined in the usual manner of the flexure formula
- Curved beams deform such that normal strain does not vary linearly from the neutral axis

- Provided that material of the curved beam is homogeneous, linear elastic and x-section has axis of symmetry, then curved beam formula can be used to determine the bending stress, $\sigma = My/[Ae(R - y)]$
- Stress concentrations occur in members having a sudden change in their x-section, such as holes and notches.
- The maximum bending stresses in such locations is determined using a stress concentration factor K, that is found empirically, $\sigma = K\sigma_{avg}$.

- If bending moment causes material to exceed its elastic limit, then normal strain remains linear, however stress distribution varies in accordance with shear strain diagram and balance of force and moment equilibrium. Thus plastic and ultimate moments can be determined.
- If an applied plastic or ultimate moment is released, it will cause the material to respond elastically, thereby inducing residual stresses in the beam

Structural Load

Distributed Forces ; weight & buoyancy



< Floating Body in Static Equilibrium>

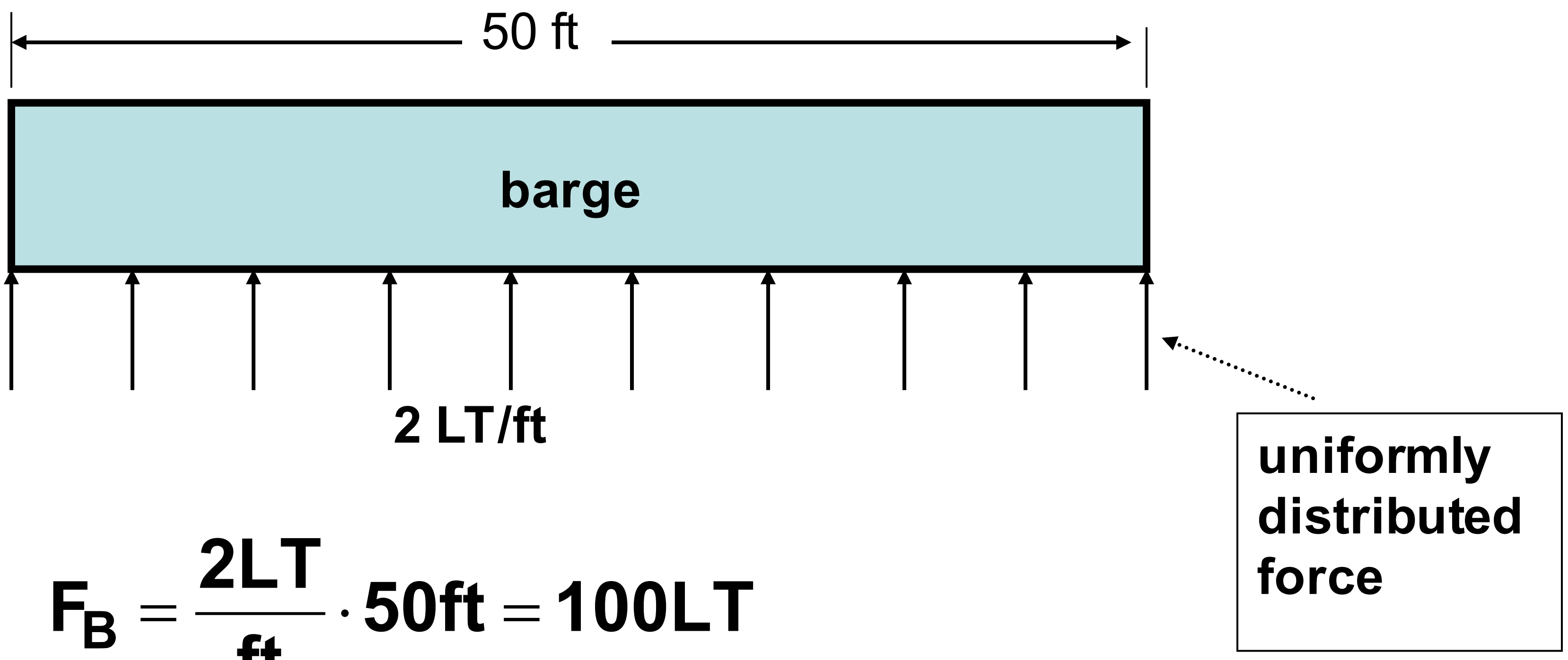
-Two forces are equal in magnitude.
-The centroid of the forces are vertically in line.

Bending stresses

Distributed Forces

Distributed Buoyancy

- **Buoyant forces can be considered as a *distributed force*.**

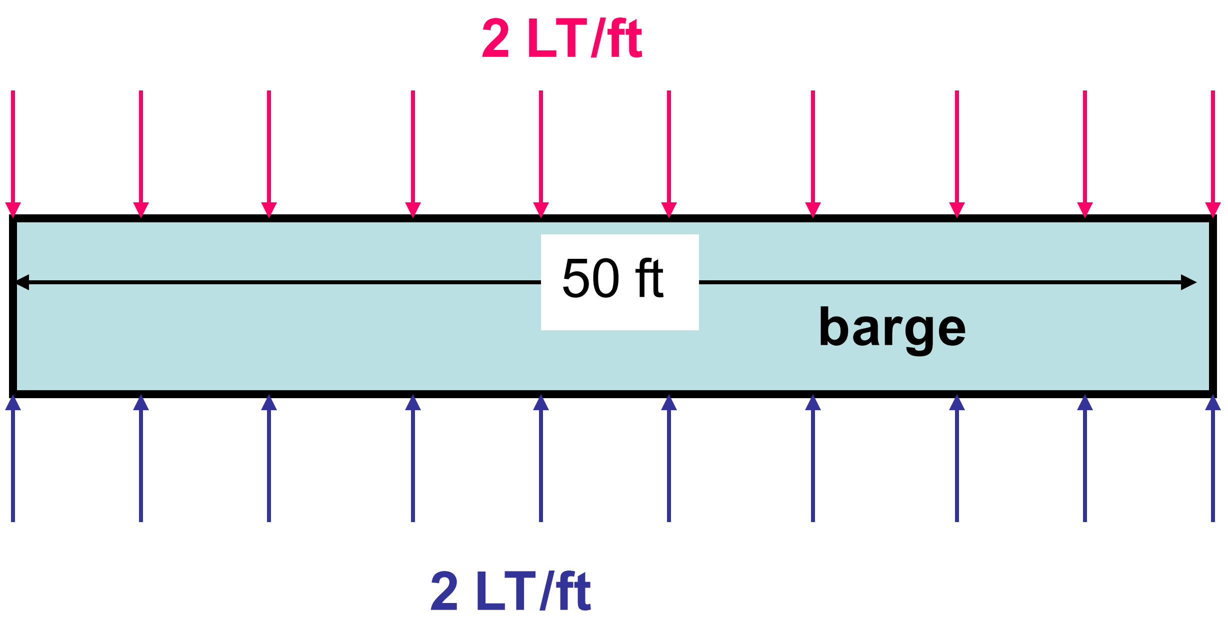


$$F_B = \frac{2LT}{ft} \cdot 50ft = 100LT$$

Distributed Forces

Distributed Weight

-**Weight of ship can be presented as a *distributed force*.**
- **Case I : *Uniformly distributed weight***

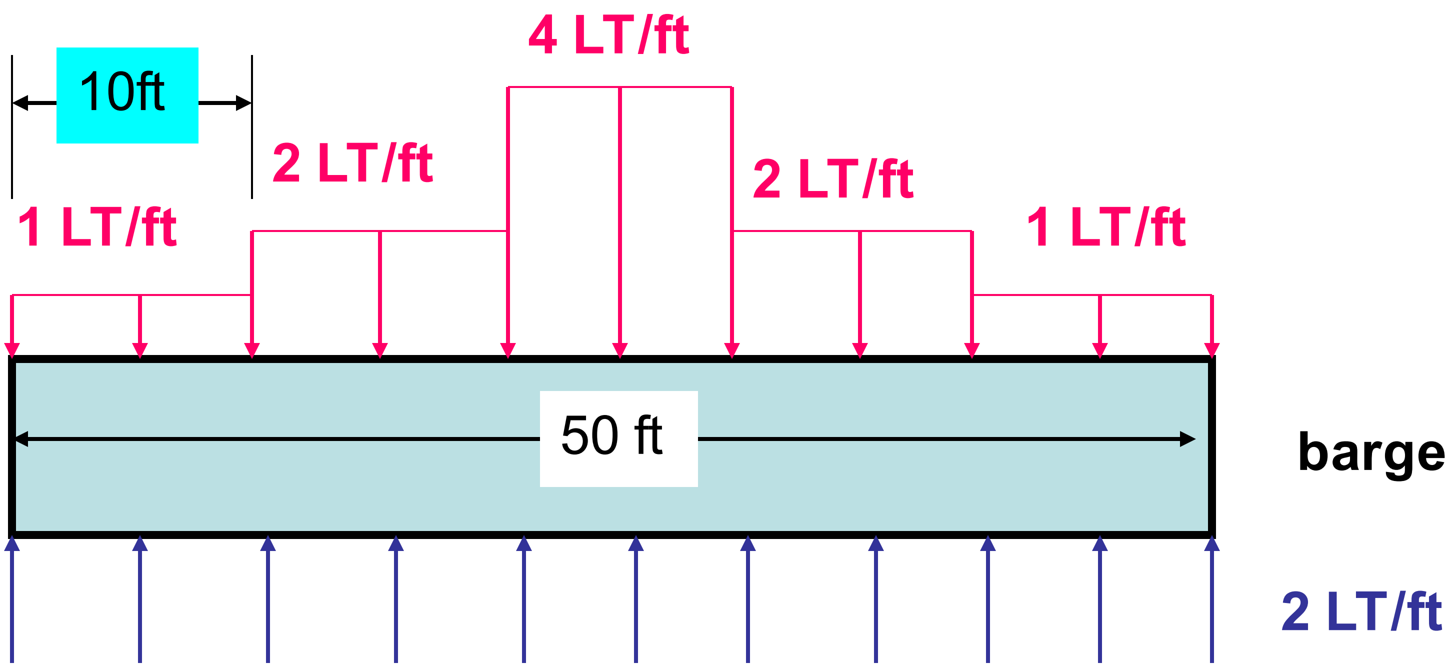


$$\Delta_s = \frac{2LT}{ft} \cdot 50ft = 100LT = F_B$$

Distributed Forces

Distributed Weight

- Case II : *Non-uniformly distributed weight*



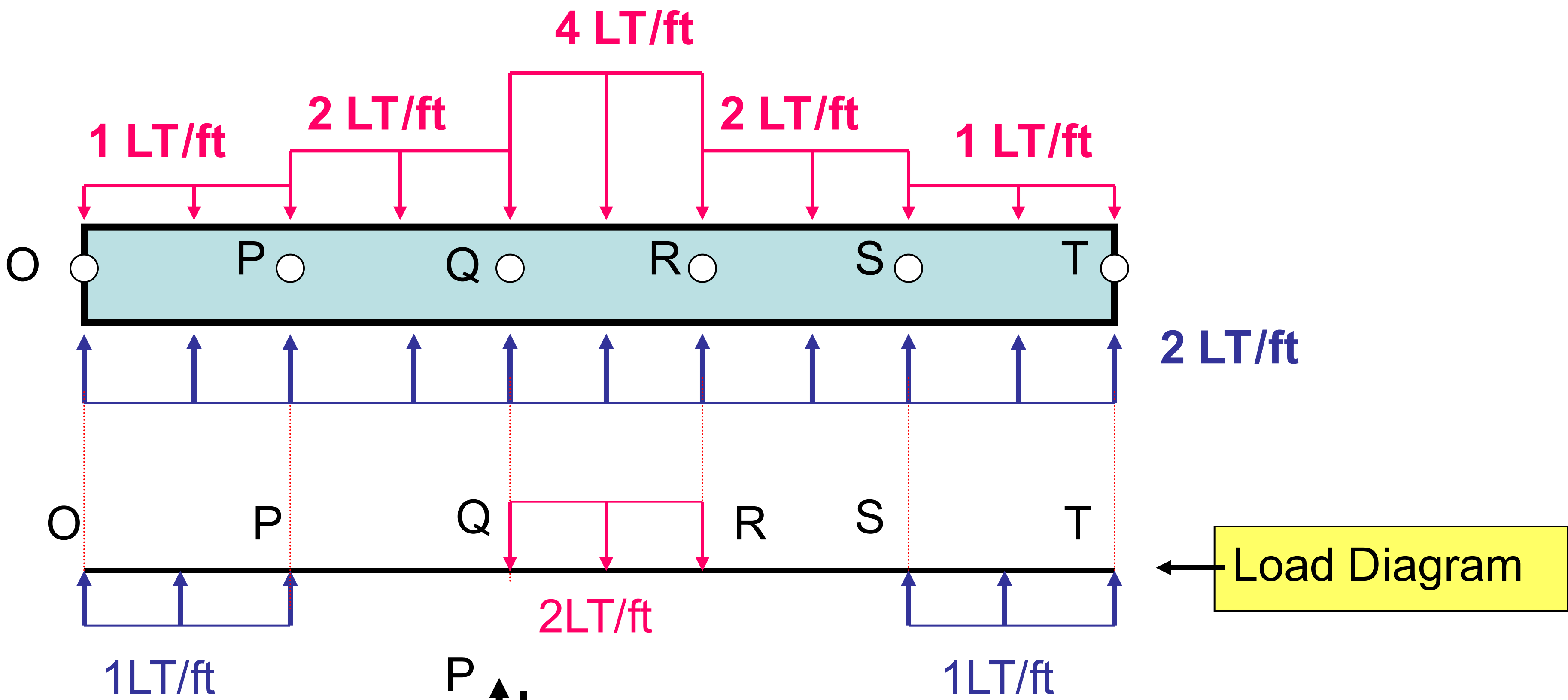
$$\Delta_s = \frac{1LT}{ft}\cdot 10ft + \frac{2LT}{ft}\cdot 10ft + \frac{4LT}{ft}\cdot 10ft + \frac{2LT}{ft}\cdot 10ft + \frac{1LT}{ft}\cdot 10ft = 100LT = F_B$$

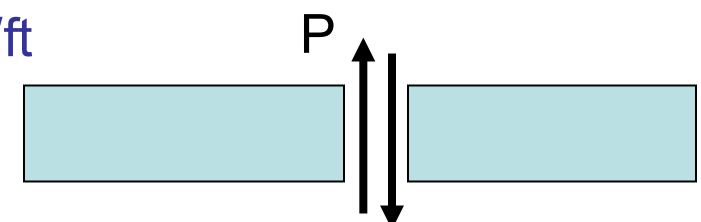
$w_{FB} = F_B/L$ (distributed load = F_B/length) $w_{FB} = \frac{100LT}{50ft} = 2$ LT/ft

Shear Stress

Shear stress present at points P, Q, R, S & T due to unbalanced forces at top and bottom.

Load diagram can be drawn by summing up the distributed force vertically.

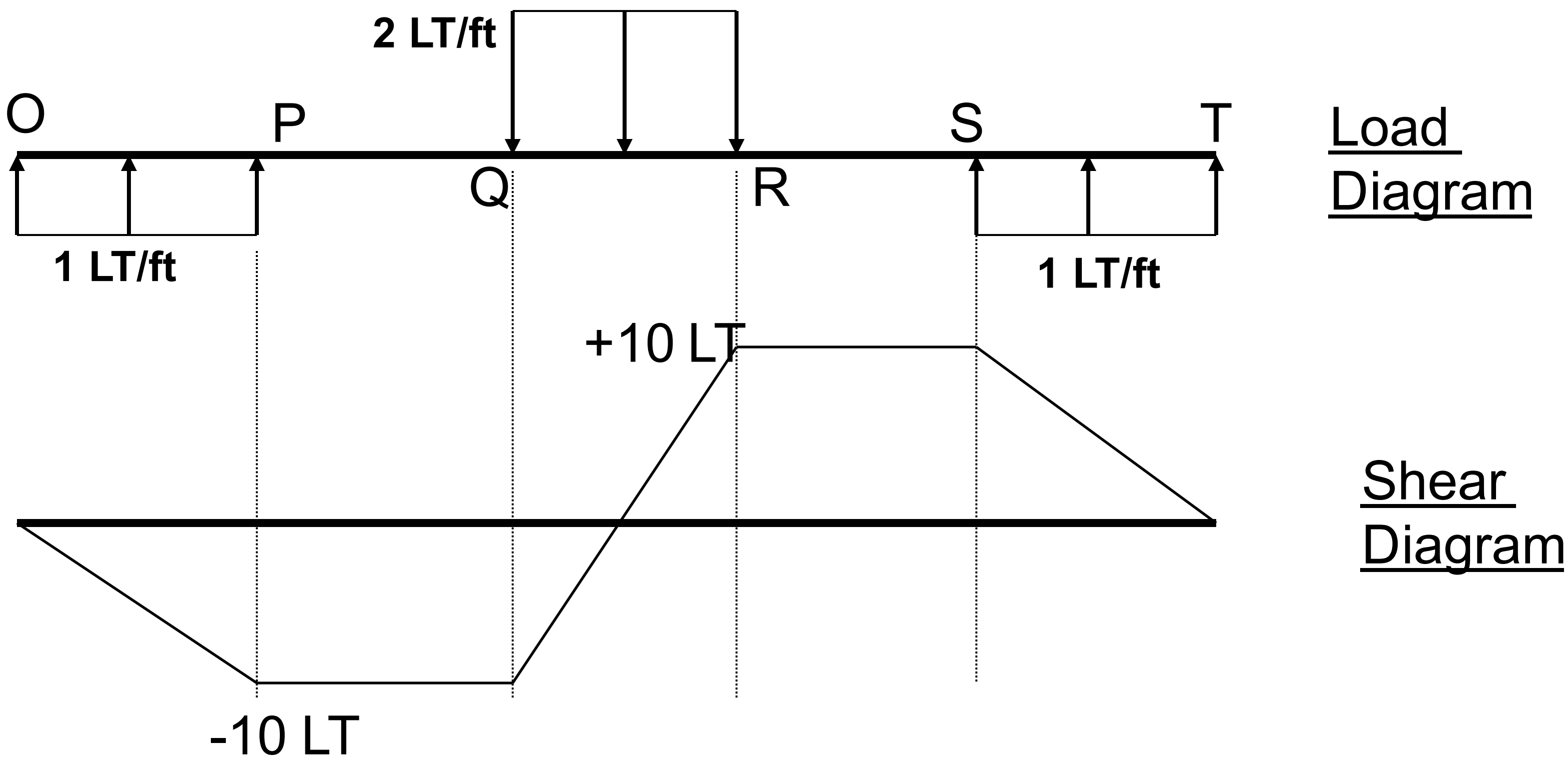




Shear Force at point P

Shear Stress

Maximum shear stresses occur where the load diagram crosses the x-axis (or equals 0).



Shear Stress

How to Reduce Shear Stress of ship

To change the underwater hull shape so that buoyancy distribution matches that of weight distribution.

- The step like shape is very inefficient with regard to the resistance.
- Since the loading condition changes every time, this method is not feasible.

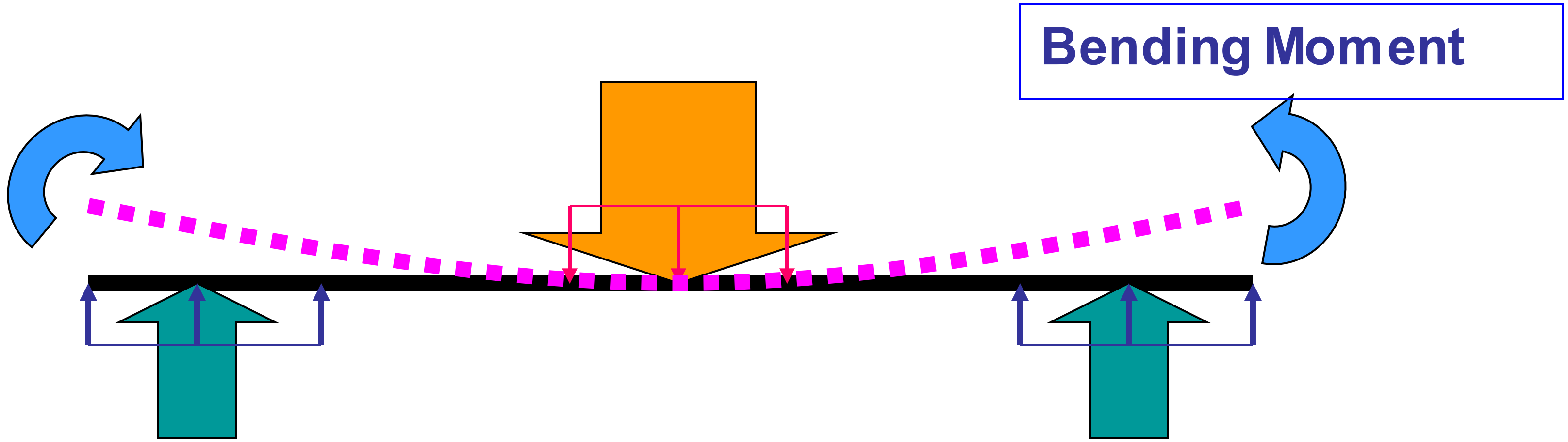
To concentrate the ship hull strength in an area where large shear stress exists . This can be done by

- *using higher strength material*
- *increasing the cross sectional area of the structure*.

Longitudinal Bending Stress

Longitudinal Bending Moment and Stress

Uneven load distribution will produce a longitudinal *Bending Moment.*



- **Buoyant force concentrates at bow and stern.**
- **Weight concentrates at middle of ship.**

The longitudinal bending moment will create a significant stress in the structure called *bending stress*.

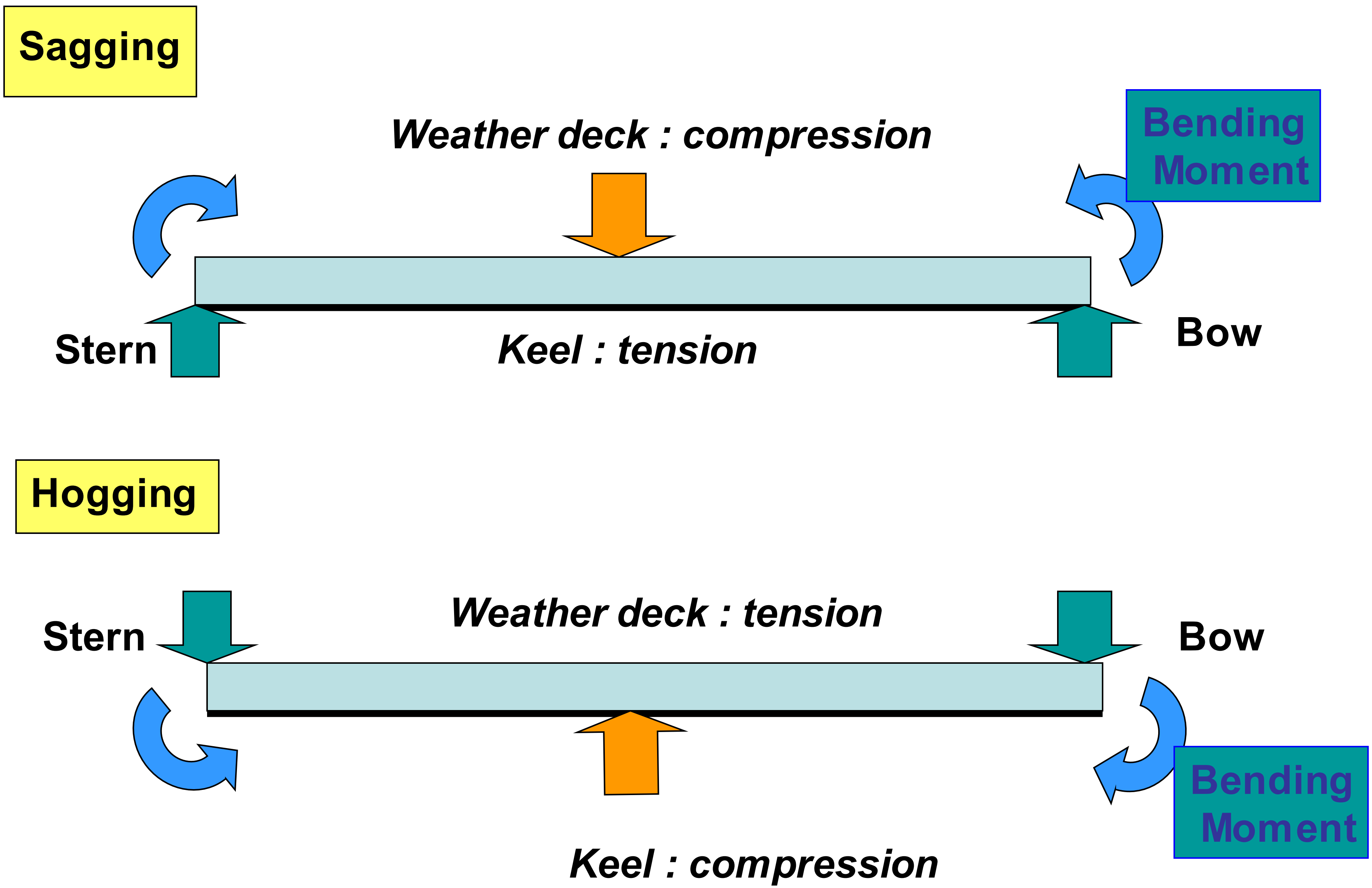
Longitudinal Bending Stress

A ship has similar bending moments, but the buoyancy and many loads are distributed over the entire hull instead of just one point.

The upward force is buoyancy and the downward forces are weights.

Most weight and buoyancy is concentrated in the middle of a ship, where the volume is greatest.

Longitudinal Bending Stress



Longitudinal Bending Stress

The longitudinal bending moment creates a significant structural stress called the **bending stress**

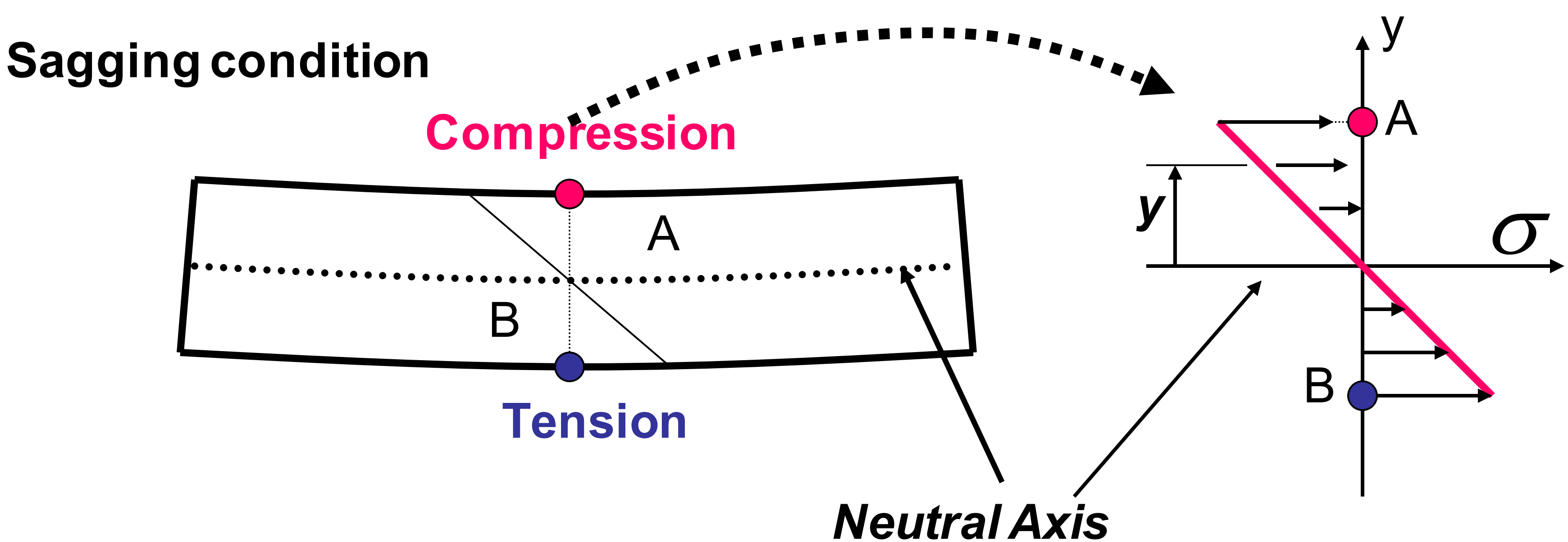
$$\boxed{\sigma = \frac{M\,y}{I}}$$

Where:

M = Bending Moment
I = 2^{nd} Moment of area of the cross section
y = Vertical distance from the neutral axis
σ = tensile (+) or compressive(-) stress

Longitudinal Bending Stress

Quantifying Bending Stress



Bending Stress :

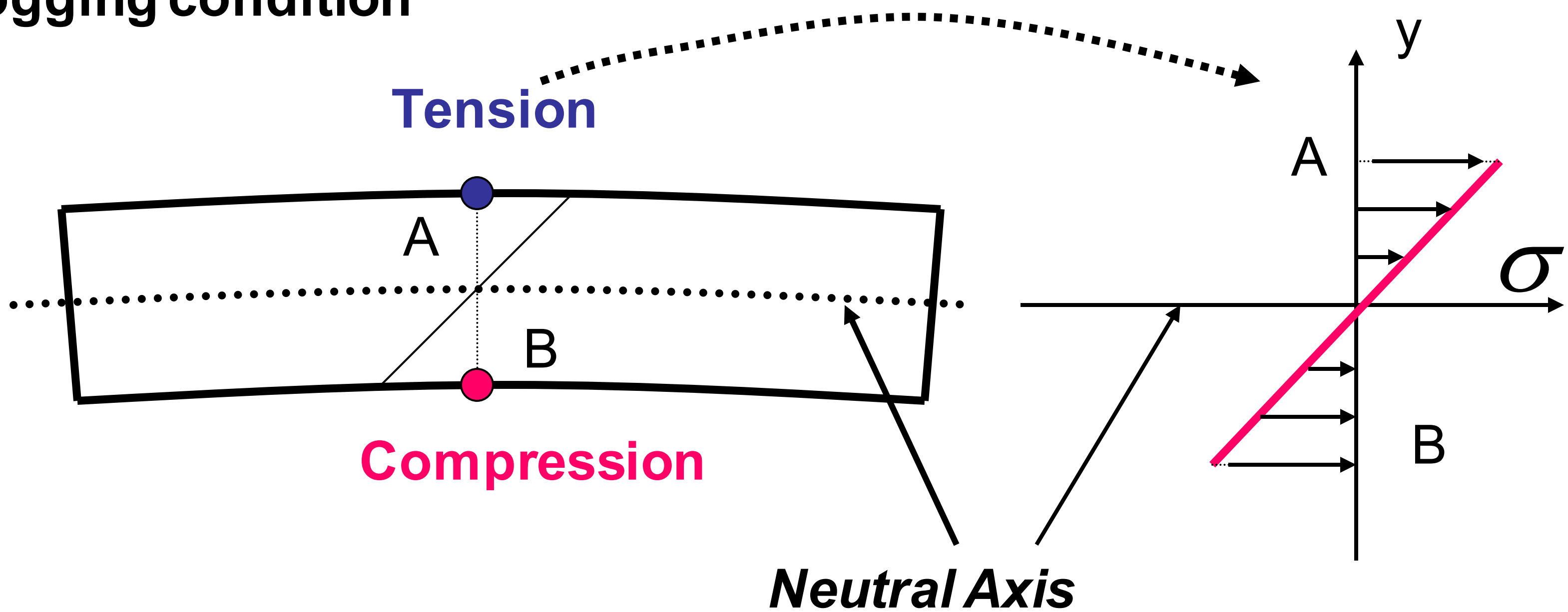
$$\sigma = \frac{M\,y}{I}$$

M **: Bending Moment**
I **: 2nd Moment of area of the cross section**
y **: Vertical distance from the neutral axis**
σ **: tensile (+) or compressive(-) stress**

Longitudinal Bending Stress

Quantifying Bending Stress

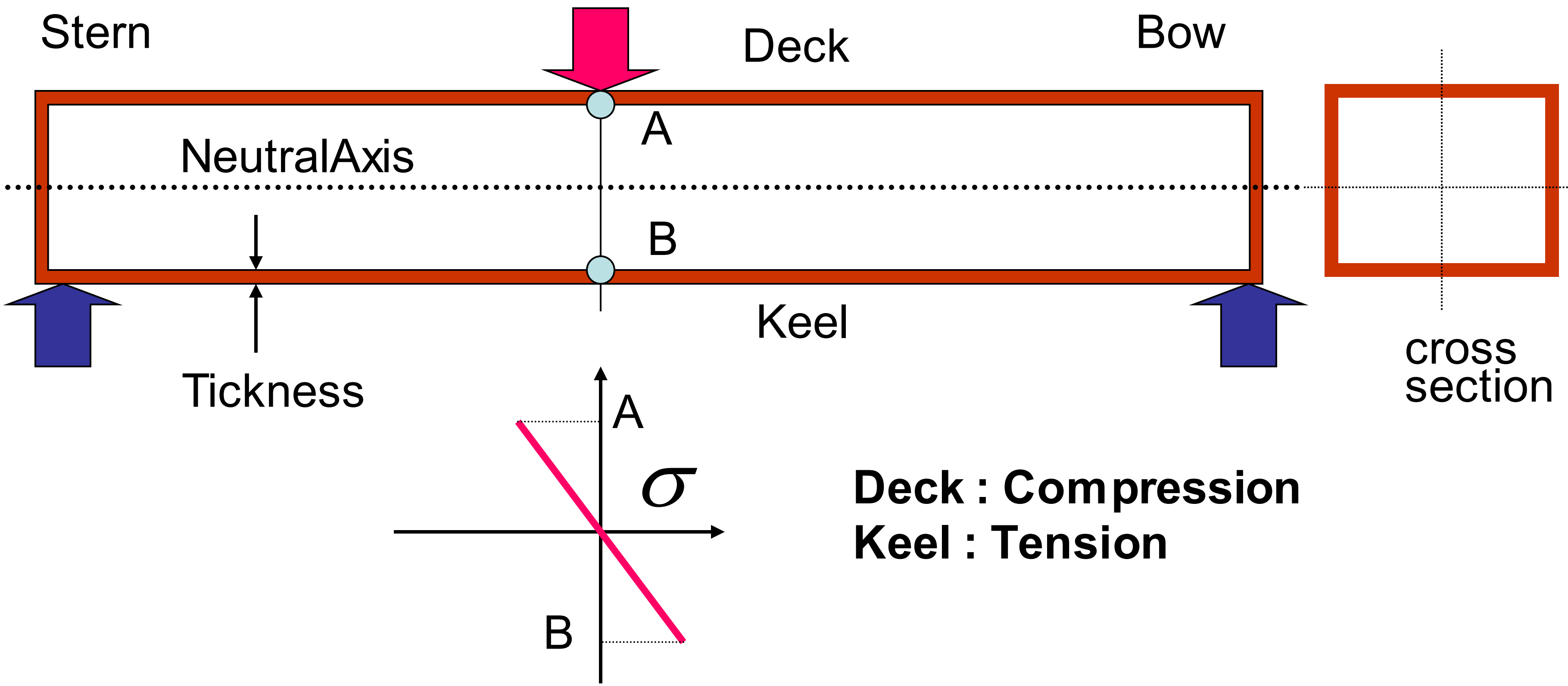
Hogging condition



Neutral Axis : geometric centroid of the cross section or transition between compression and tension

Longitudinal Bending Stress

Example :Bending Stress of Ship Hull

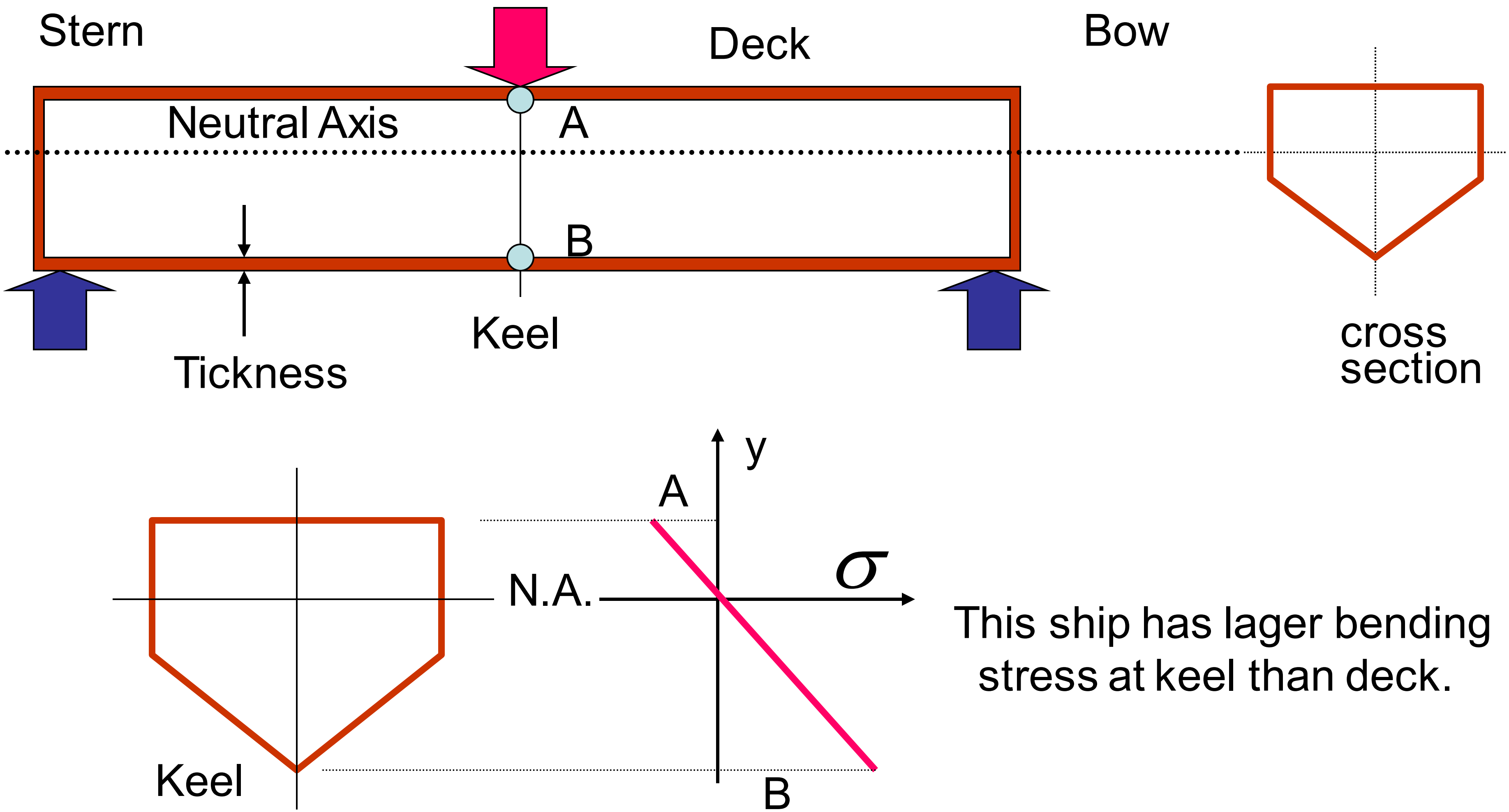


Deck : Compression
Keel : Tension

- ***Ship could be at sagging condition even in calm water .***
- ***Generally, bending moments are largest at the midship area.***

Longitudinal Bending Stress

Example :Bending Stress of Ship Hull



This ship has lager bending stress at keel than deck.

Longitudinal Bending Stress

Reducing the Effect of Bending stress

Bending moment are largest at amidship of a ship.

Ship will experience the greatest bending stress at the deck and keel.

The bending stress can be reduced by using:

- **<u>higher strength steel</u>**
- **<u>larger cross sectional area</u> of longitudinal structural elements**

Longitudinal Bending Stress

Hull Structure Interaction

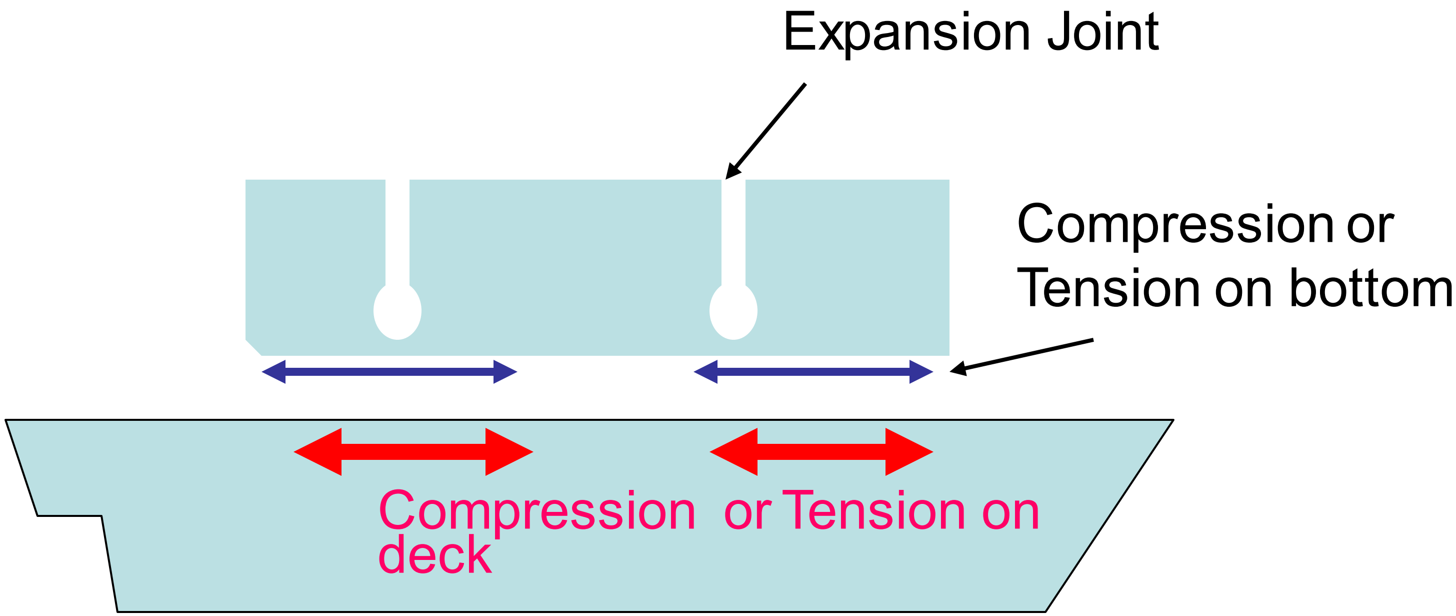
Bending stress at the superstructure is large because of its distance from the neutral axis.

In Sagging or Hogging condition, severe shear stresses between deck of hull and bottom of the superstructure will be created.

This shear stresses will cause crack in area of sharp corners where the hull and superstructure connect.

→ **This stress can be reduced by an *Expansion Joint***

Longitudinal Bending Stress



By using Expansion Joint, the super structure will be allowed to flex along with the hull.

Other Loads

Hydrostatic Loads

Loading associated with hydrostatic pressure

Hydrostatic Loads are considerable in *submarines*

Hydrostatic pressure :

$$P_{HydStatic} = \rho g h$$

Torsional Loads

Torsional Loads of hull are often insignificant

They can have effect on ships with large opening(s) in their weather deck. (e.g., research vessels)

Other Loads

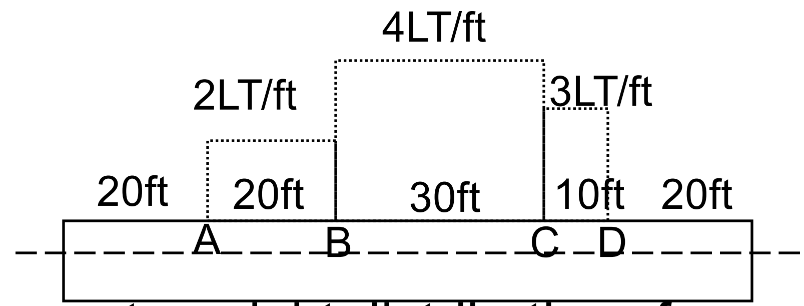
Weapon Loads

Loading due to explosion of weapons or shock impact, both in air and underwater

Naval Vessel should resist these forces

Naval vessel will often go through a series of *shock trials* during initial sea trials.

Example Problem



A 100ft long box shaped barge has an empty weight distribution of 2LT/ft. What is the total buoyant force floating the empty barge in calm water?

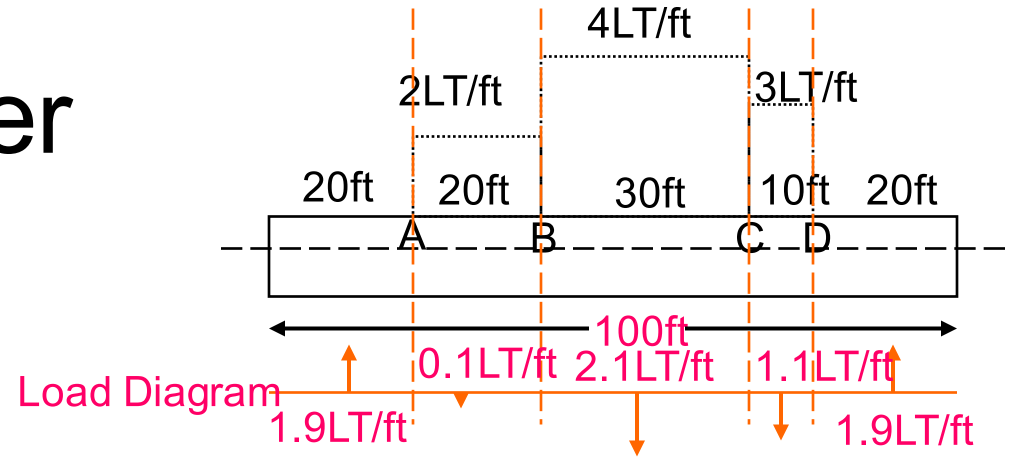
The barge is then loaded with the additional cargo weight distribution shown above. What is the buoyant force distribution in calm water for the loaded barge?

At which point, (A, B, C or D) is the barge under the greatest shear stress?

Is the barge in a hogging or sagging condition?

If a wave hits which peaks at the center of the barge and troughs at the ends, is the condition above mitigated or exacerbated?

Example Answer



$F_{B\ Total\ Empty}$=100ft×2LT/ft=200LT

$F_{B\ Total\ Loaded}$=200LT+20ft×2LT/ft+
30ft×4LT/ft+10ft×3LT/ft=390LT

$F_{B\ Dist'n}$=390LT/100ft=3.9LT/ft

Point A & D: Load Diagram Crosses X- Axis

Ends curling up - Sagging
(Mitigated by providing additional support at center of barge)

Objectives :

1) To study some of the important parameters which affect column buckling, such as slenderness ratio (L/r) and least radius of gyration (r).

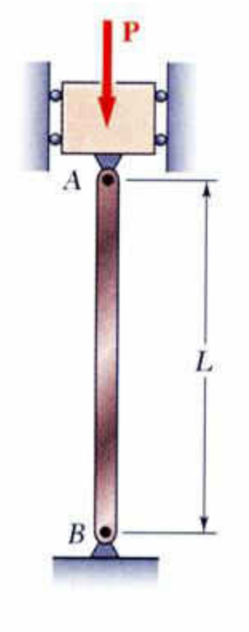
2) To determine the relationship between critical stress versus slenderness ratio of steel columns.

3) To confirm the validity of Euler's analysis of P_{cr} ,the critical buckling load and the relationship of P_{cr} to column slenderness.

Compression Members :

- Columns: structural members which undergo compressive stress.

- Column Design :
  1) Column Strength : Its ability to support loads without experiencing excessive stress.
  2) Column Stability : Its ability to support loads without experiencing sudden change in its configuration.

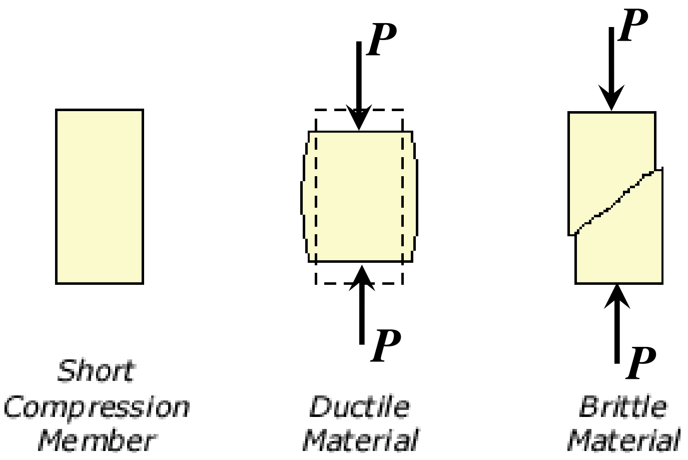


- Column Failure :
  1) Material Failure
  2) Buckling

$$\boxed{\sigma = \frac{P}{A}}$$

Material Failure:

■ The failure of a *short column* resulting from the compression axial force looks like:

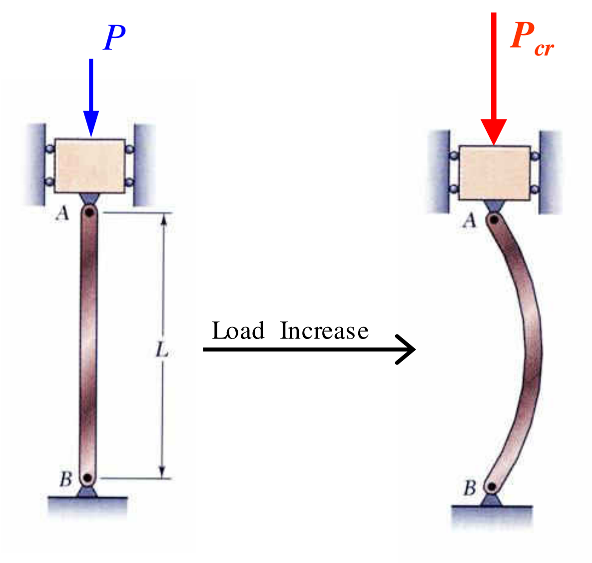


$$\sigma = \frac{P}{A} \begin{cases} \sigma \leq \sigma_{\text{ultimate}} \longrightarrow \textit{Safe} \\ \sigma > \sigma_{\text{ultimate}} \longrightarrow \textit{Failure} \end{cases}$$

Buckling :

■ However, when a compression member becomes longer, the role of the *geometry* and *stiffness* (Young's Modulus) becomes more and more important.

■ For a *long (slender) column*, buckling occurs way before the normal stress reaches the strength of the column material.



$$\boxed{\sigma_{cr} = \frac{P_{cr}}{A}}$$

σ_{cr} : Buckling Stress

P_{cr} : Buckling Load

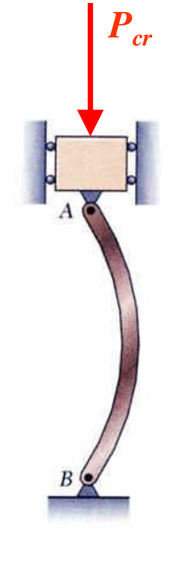
$\sigma < \sigma_{cr}$: ***Safe***

$\sigma \geq \sigma_{cr}$: ***Buckling***

Buckling Stress:

■ What does buckling stress (σ_{cr}) depend on ?

- Modulus of Elasticity (***E***)
- Length of the column (***L***)
- Dimensions of the cross section (***r*** for a rod)



■ The expression for critical stress is called ***Euler's*** (1707-1783) formula. For a both end pinned column we have:

$$\boxed{\sigma_{cr} = \frac{\pi^2 E}{(L_e / r_g)^2}}$$

Euler's Formula:

■ An important parameter in Euler's formula is:

$$\lambda = \frac{L_e}{r_g}$$

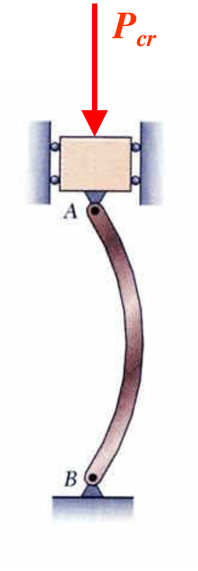
is called <u>*slenderness ratio*</u>.

$$\sigma_{cr} = \frac{\pi^2 E}{(L_e / r_g)^2}$$

L_e : effective length $\longrightarrow$ Column length; Support condition

r_g : radius of gyration

$$r_g = \sqrt{\frac{I}{A}}$$



An Example:

■ For example for a rod with both ends pinned we have :

- Both ends pinned :

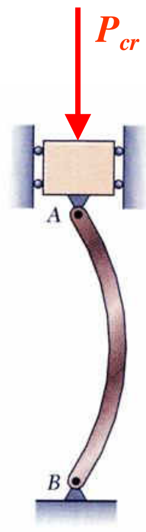
$$L_e = L$$

- Radius of gyration :

$$r_g = \sqrt{\frac{I}{A}} = \sqrt{\frac{\pi d^4/64}{\pi d^2/4}} = \frac{d}{4}$$

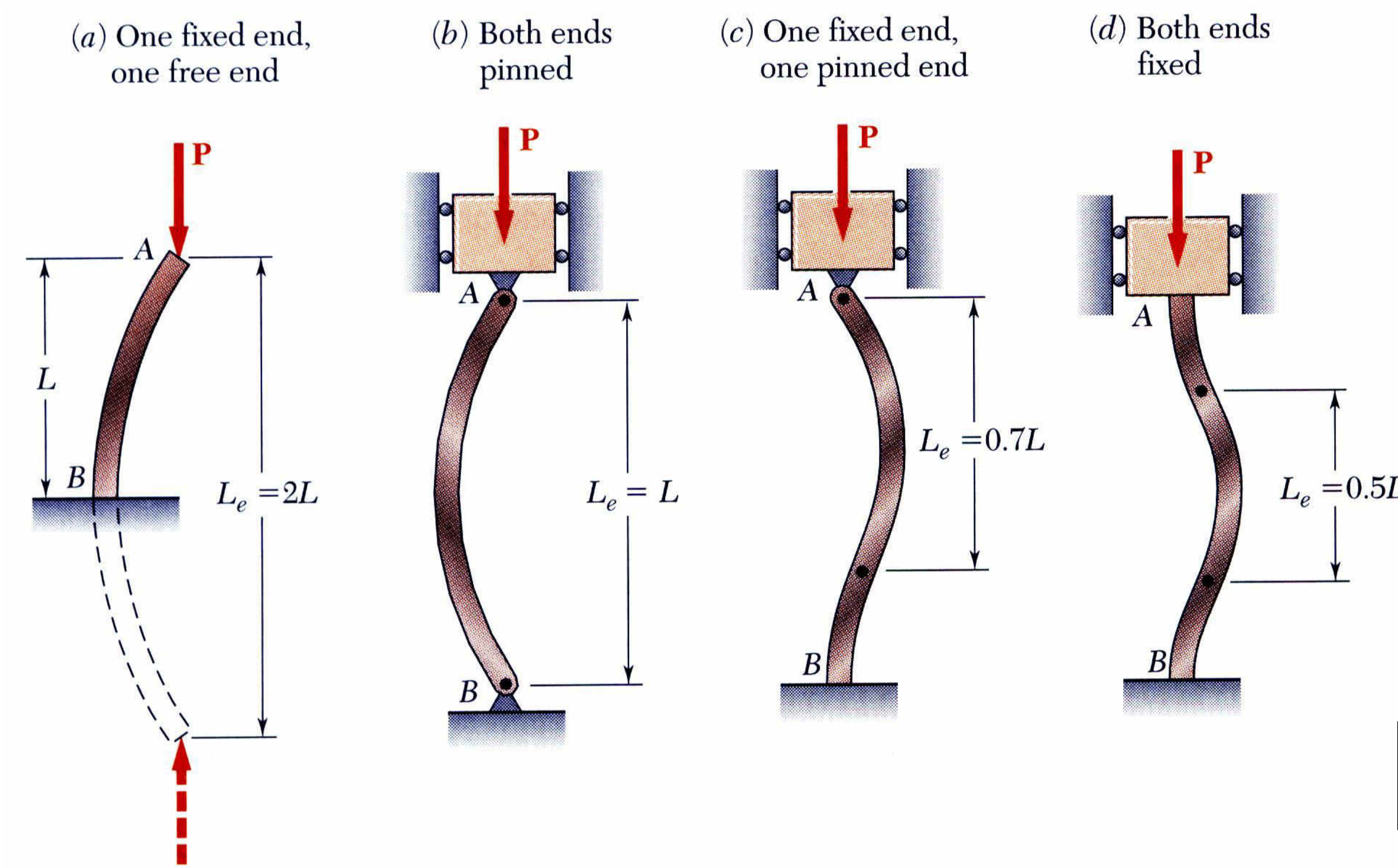
- And the slenderness ratio is :

$$\lambda = \frac{4L}{d}$$



Effective Length:

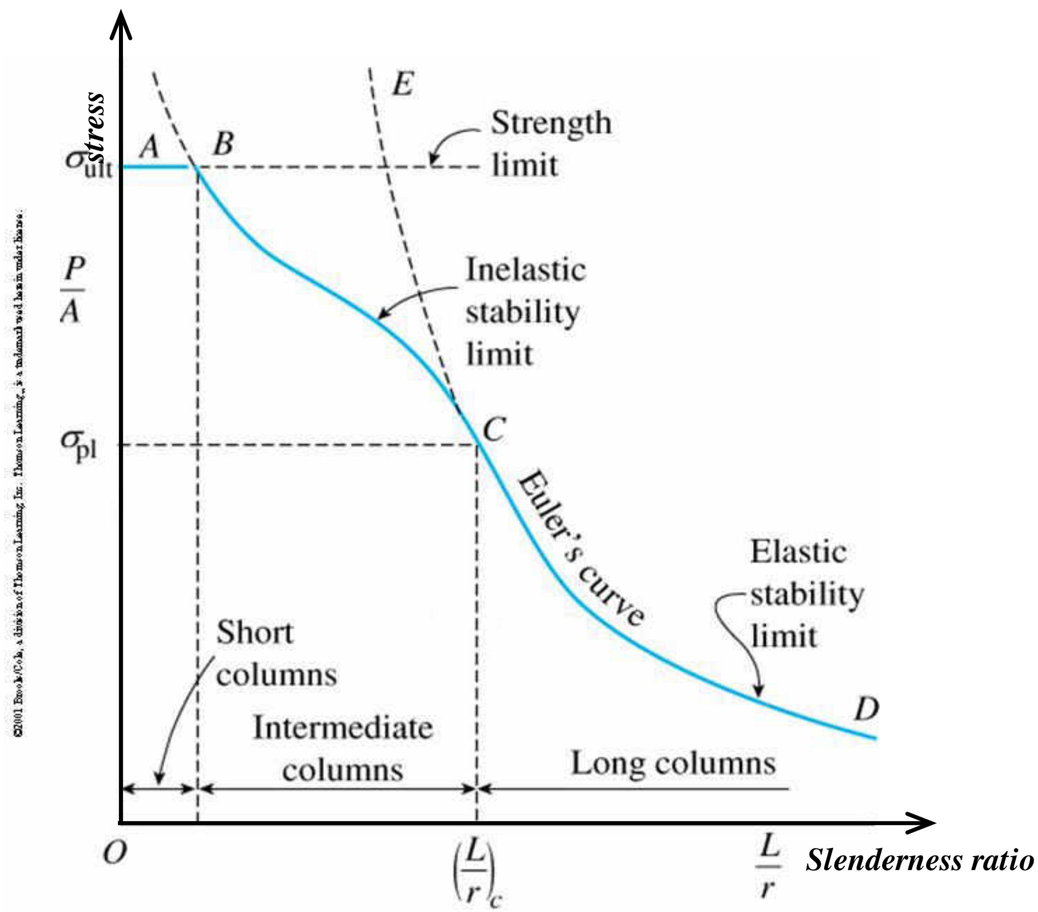
■ How to figure out L_e based on the supports conditions ?



$$\sigma_{cr} = \frac{\pi^2 E}{(L_e / r_g)^2}$$

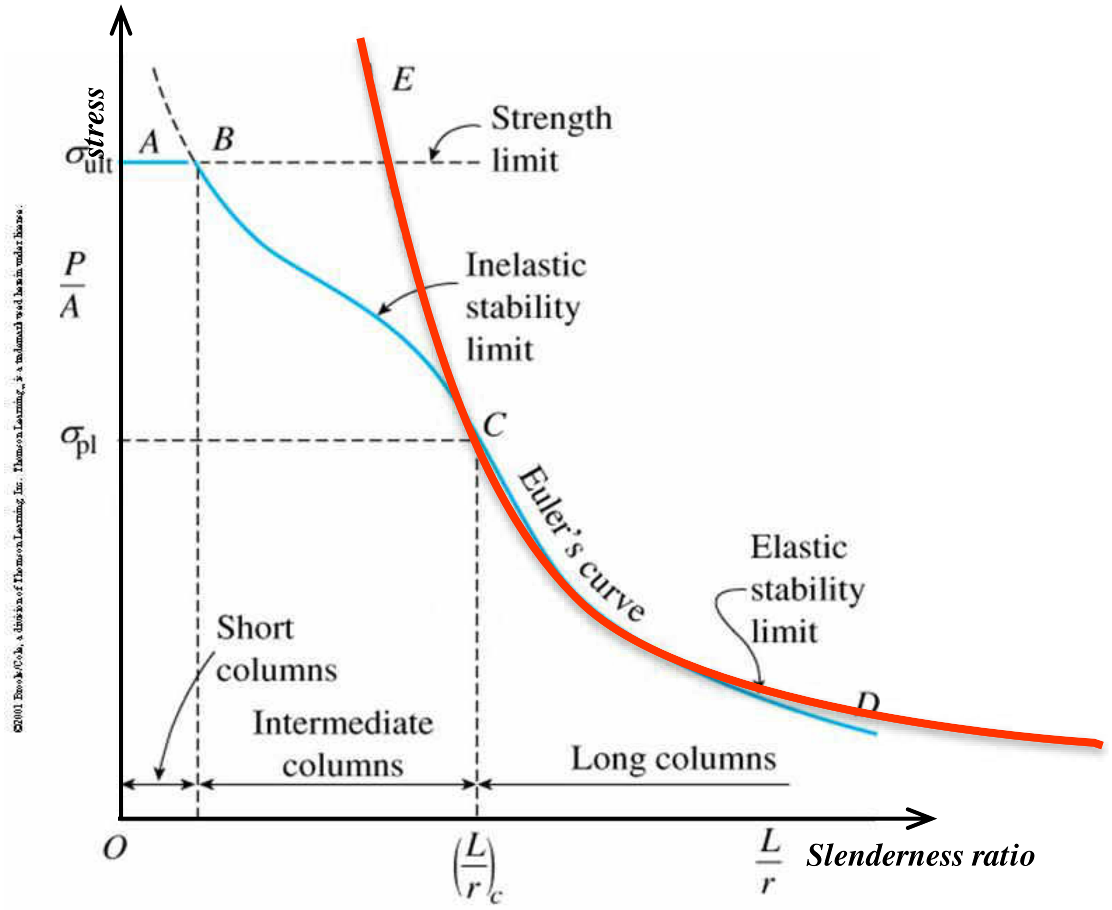
Buckling Curve:

■ If the critical stress (σ_{cr}) vs. slenderness ratio is plotted :



Buckling Curve:

■ If the critical stress (σ_{cr}) vs. slenderness ratio is plotted :



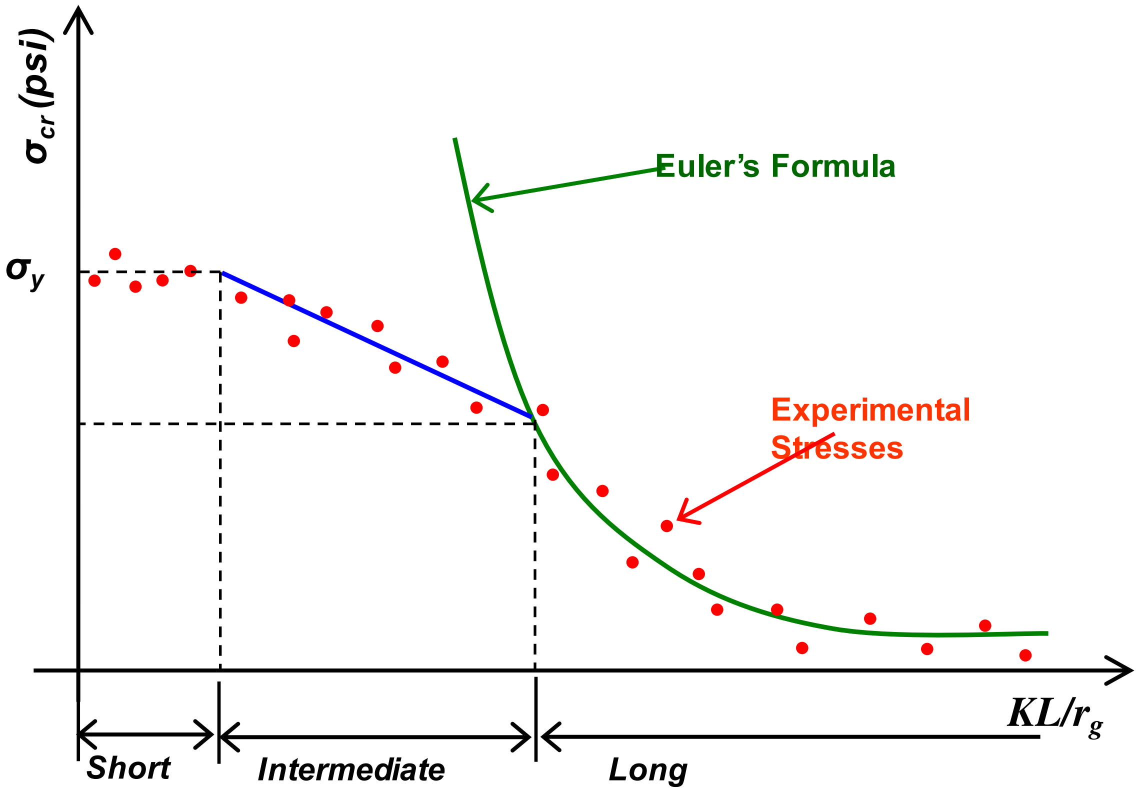
Experiment Procedure:

- For this experiment several steel columns with different lengths and cross section diameters are put under compression and the buckling loads are measured. Then the corresponding critical stresses are compared with those calculated from Euler's formula.

Sample Number	Length *(in)*	Diameter *(in)*	***A*** *(in^2)*	***r_g*** *(in)*	Expt. P_{cr} *(lbs)*	***L/r_g***	Expt. σ_{cr} *(psi)*	Theo. σ_{cr} *(psi)*	*Error (%)*
1	*L1*	*d1*	*A1*	*d1/4*	*P1*		*P1/A1*		
2	*L2*	*d2*	*A2*	*d2/4*	*P2*		*P2/A2*		
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
n	*Ln*	*dn*	*An*	*dn/4*	*Pn*		*Pn/An*		

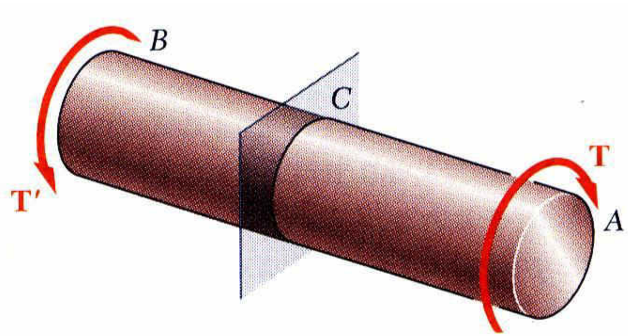
Experiment Procedure:

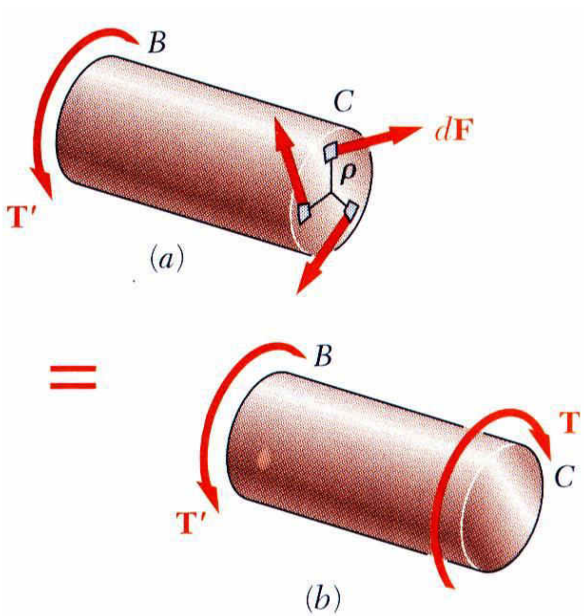
■ Finally, experimental and theoretical critical stresses are plotted vs. slenderness ratio :



Torsion

Net Torque Due to Internal Stresses





- Resultant of internal shearing stresses is an internal torque, equal and opposite to applied torque,

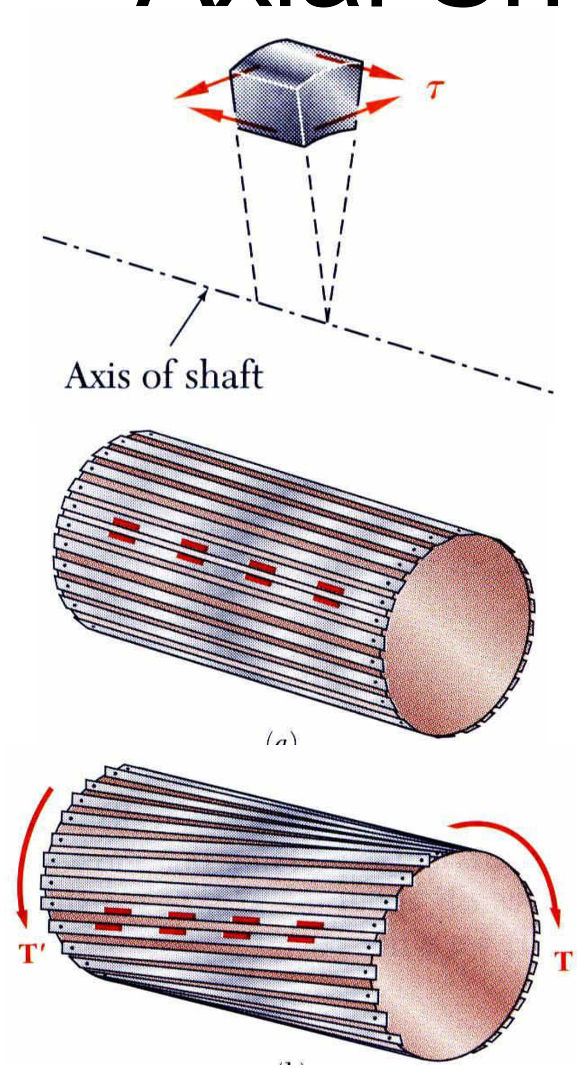
$$T = \int \rho \, dF = \int \rho (\tau \, dA)$$

- Although net torque due to shearing stresses is known, the distribution of the stresses is not

- Distribution of shearing stresses is statically indeterminate – must consider shaft deformations

- Unlike normal stress due to axial loads, the distribution of shearing stresses due to torsional loads cannot be assumed uniform.

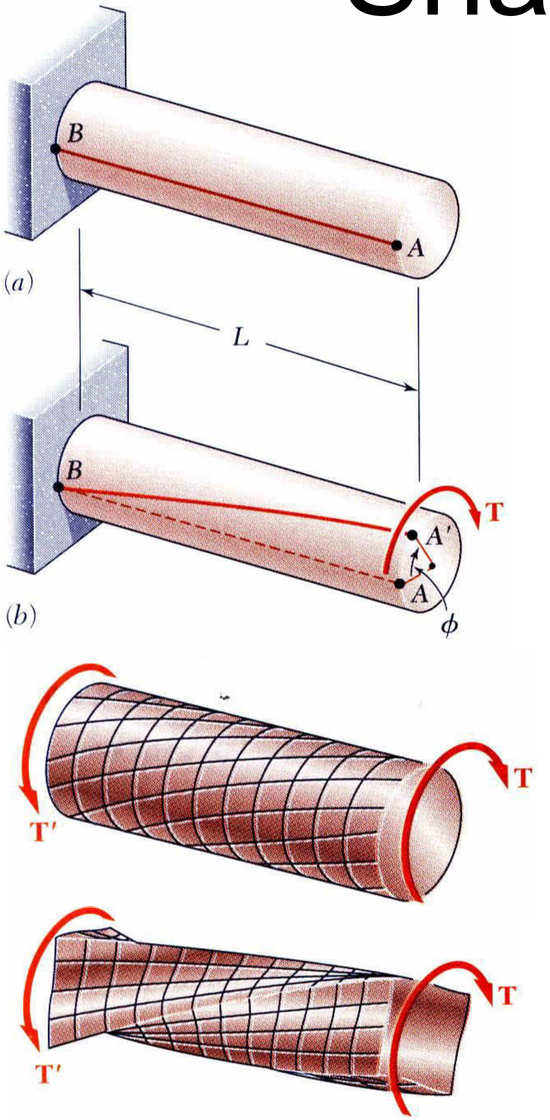
Axial Shear Components



- Torque applied to shaft produces shearing stresses on planes perpendicular to shaft axis.
- Moment equilibrium requires existence of equal shear stresses on planes containing the shaft axis, i.e., "axial shear stresses".
- Existence of axial shear stresses is demonstrated by considering a shaft made up of axial slats.

  Slats slide with respect to each other when equal and opposite torques applied to shaft ends.

Shaft Deformations



- Will see that angle of twist of shaft is proportional to applied torque and to shaft length.

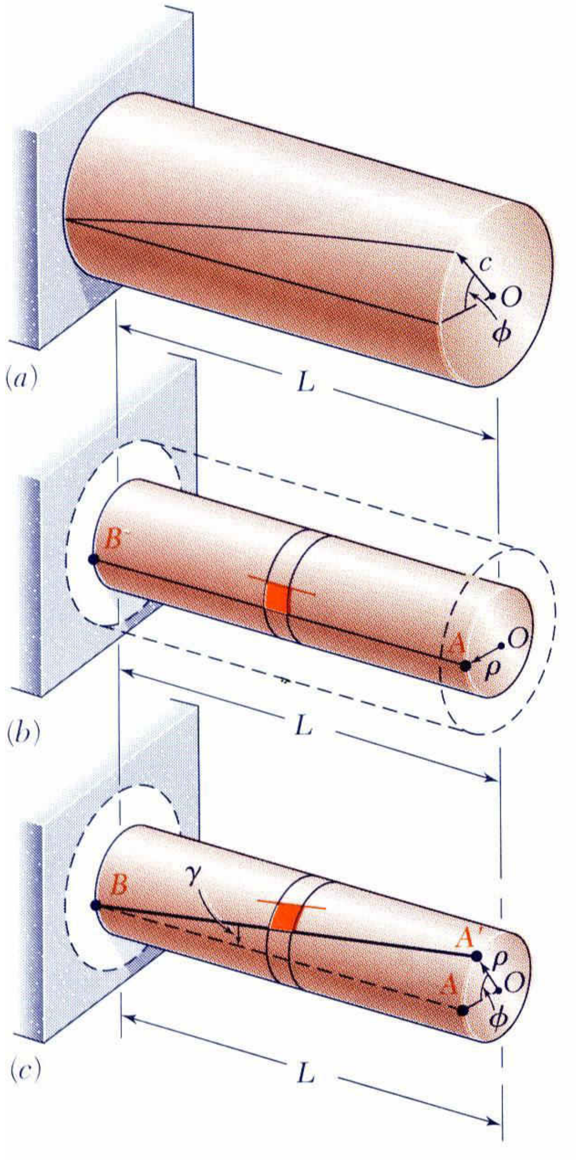
$$\phi \propto T$$

$$\phi \propto L$$

- When subjected to torsion, every cross-section of circular (solid or hollow) shaft remains plane and undistorted. This is due to axisymmetry of cross section.

- Cross-sections of noncircular (hence non-axisymmetric) shafts are distorted when subjected to torsion – since no axisymm.

Shearing Strain



- Consider interior section of shaft. When torsional load applied, a rectangular element on the interior cylinder deforms into rhombus.

- So shear strain equals angle between BA and BA'

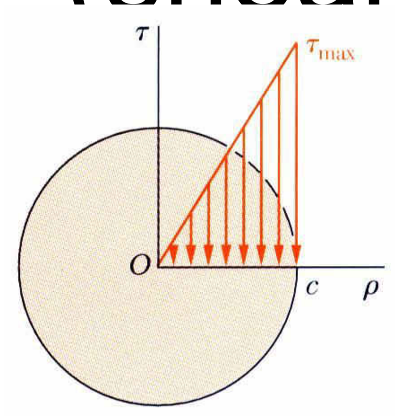
- Thus,

$$L\gamma = \rho\phi \quad \Rightarrow \quad \gamma = \frac{\rho\phi}{L}$$

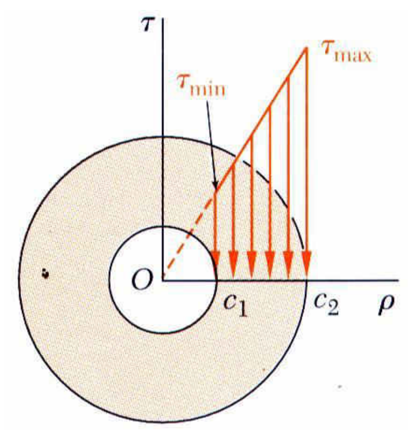
so shear strain proportional to twist and radius

$$\gamma_{\max} = \frac{c\phi}{L} \quad \text{and} \quad \gamma = \frac{\rho}{c}\gamma_{\max}$$

Torsion Formulae in elastic range (shear stress, angle of twist



$$J = \tfrac{1}{2}\pi c^4$$



$$J = \tfrac{1}{2}\pi\left(c_2^4 - c_1^4\right)$$

J= Polar moment of inertia

- Hooke's law,

$$\tau = G\gamma = G\frac{\rho\phi}{L} = G\frac{\rho}{c}\gamma_{\max} = \frac{\rho}{c}\tau_{\max}$$

So shearing stress also varies linearly with radial position in the section.

- Recall: sum of moments from internal stress distribution equals internal torque at the section,

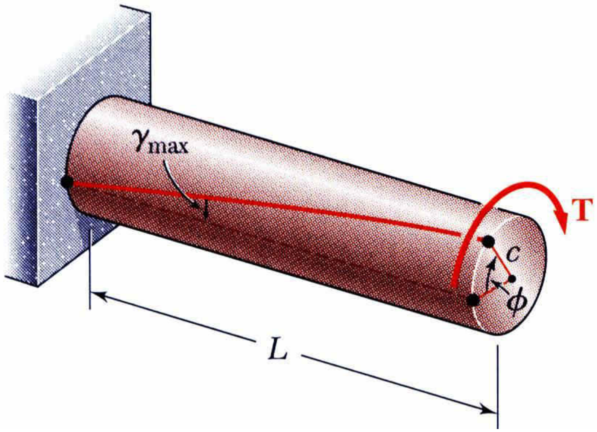
$$T = \int \rho\tau\, dA = \int \rho G\frac{\rho\phi}{L}dA = G\frac{\phi}{L}\int \rho^2\, dA$$

$$= GJ\frac{\phi}{L} = \tau\frac{J}{\rho}$$

- Thus, elastic torsion formulas are

$$\boxed{\tau = \frac{T\rho}{J} \quad ; \quad \phi = \frac{TL}{GJ}}$$

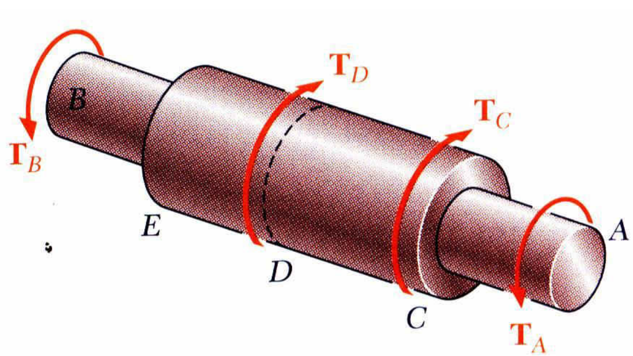
Torsion formulae in Elastic Range



$$\tau = \frac{T\rho}{J}, \quad \phi = \frac{TL}{GJ}$$

$$\tau_{\max} = \frac{Tc}{J}$$

$$\text{so } \frac{\phi}{L} = \alpha = \frac{T}{GJ}, \text{ a constant if } T \text{ constant}$$



- If torsional loading or shaft cross-section changes (discretely) along length, the angle of rotation is found as sum of segment rotations

$$\boxed{\phi = \sum_i \frac{T_i L_i}{J_i G_i}}$$

Comparison of Axial and Torsion formulae.

AE =Axial rigidity

GJ =Torsional rigidity

Axial Stiffness

$$k_A = \frac{AE}{L}$$

Torsional Stiffness

$$k_T = \frac{GJ}{L}$$

Axial Flexibilty: $f_A = k_A^{-1}$

Torsional Flexibilty: $f_T = k_T^{-1}$

Axial displacement

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i}$$

Torsional displacement

$$\phi = \sum_i \frac{T_i L_i}{J_i G_i}$$

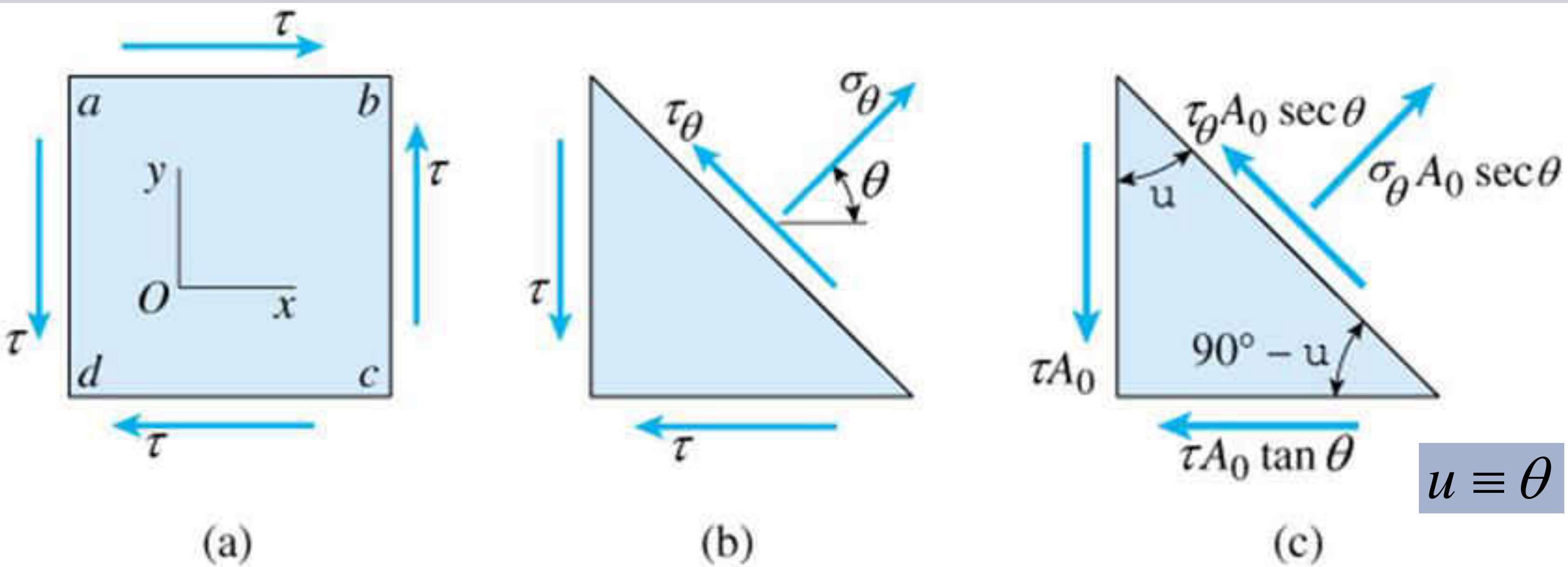
Axial stress

$$\sigma = \frac{P}{A}$$

Torsional stress

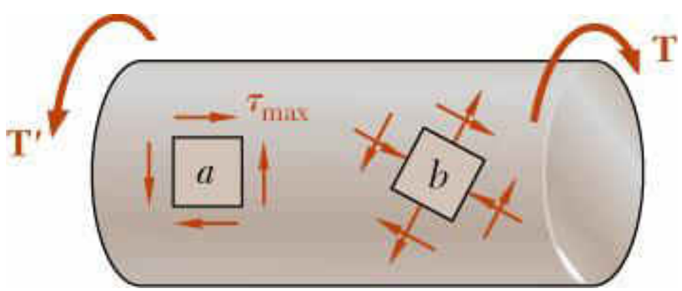
$$\tau = \frac{T\rho}{J}$$

Stressed on Inclined Plane



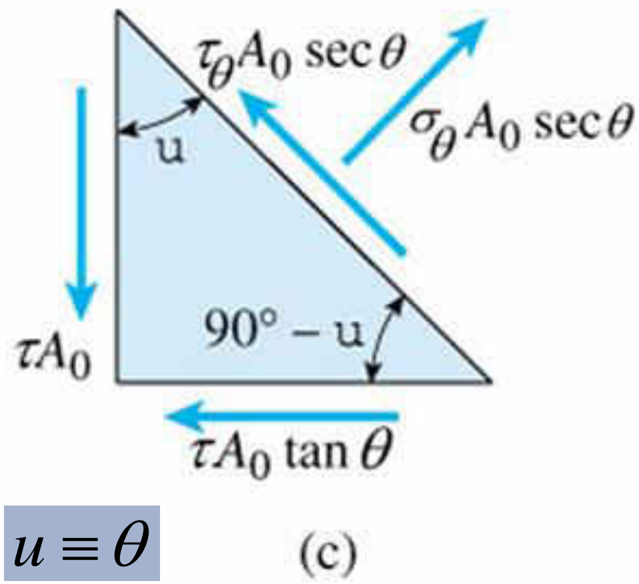
$u \equiv \theta$

(a) element in pure shear generated due to applied torque,
(b) stresses acting on inclined plane of a triangular stress element,
(c) forces acting on the triangular stress element (FBD).



Sign convention for stresses on inclined plane (Normal stresses tensile positive, shear stresses producing counterclockwise rotation positive.)

Stressed on Inclined Plane



$u \equiv \theta$

Equilibrium normal to plane,

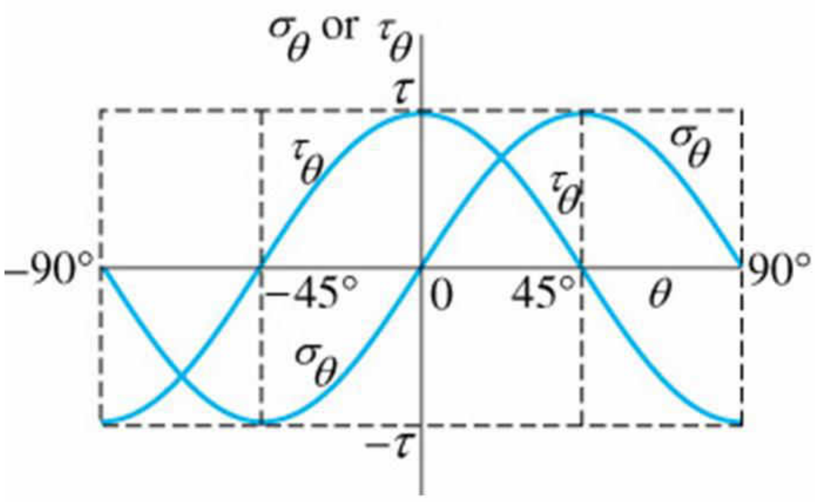
$$\sigma_\theta A_o \sec\theta = \tau A_o \sin\theta + \tau A_o \tan\theta \cos\theta \quad (1)$$

$$\boxed{\sigma_\theta = \tau \sin 2\theta}$$

Equilibrium along plane,

$$\tau_\theta A_o \sec\theta = \tau A_o \cos\theta - \tau A_o \tan\theta \sin\theta \quad (2)$$

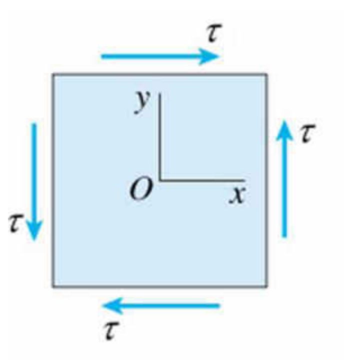
$$\boxed{\tau_\theta = \tau \cos 2\theta}$$



Graph of σ_θ and τ_θ versus θ.

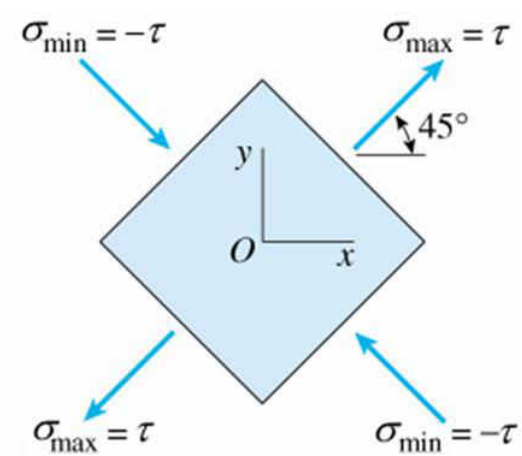
$$\tau_\theta = \tau \cos 2\theta$$

Maximum/Minimum shear stress occurs at $\theta = 0^o$ or 90^o plane $\tau_{max} = +\tau; \tau_{min} = -\tau$

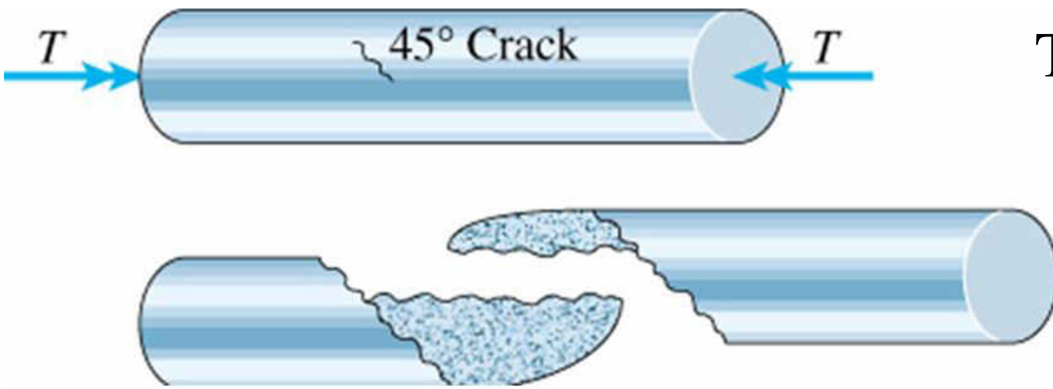


$$\sigma_\theta = \tau \sin 2\theta$$

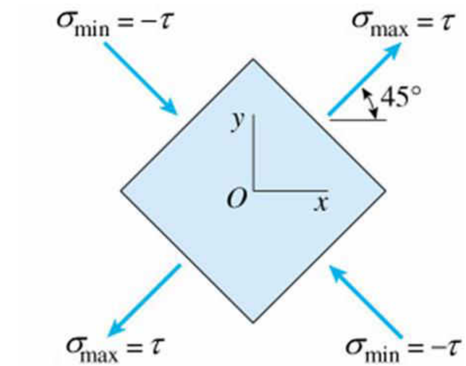
Maximum/minimum normal stress occurs at θ=+45 or -45^o plane $\sigma_{max} = +\tau; \sigma_{min} = -\tau$

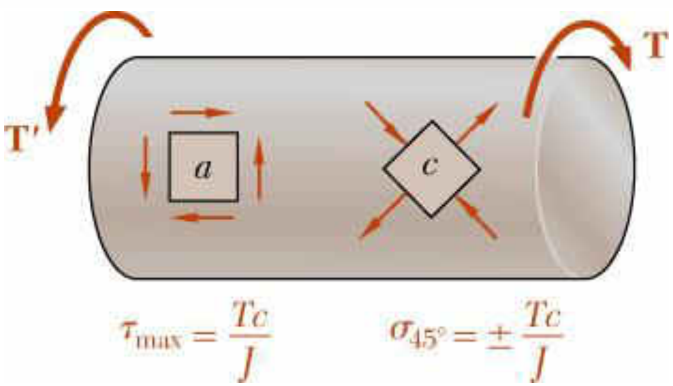


Failure of Brittle material



Try on a piece of chalk!

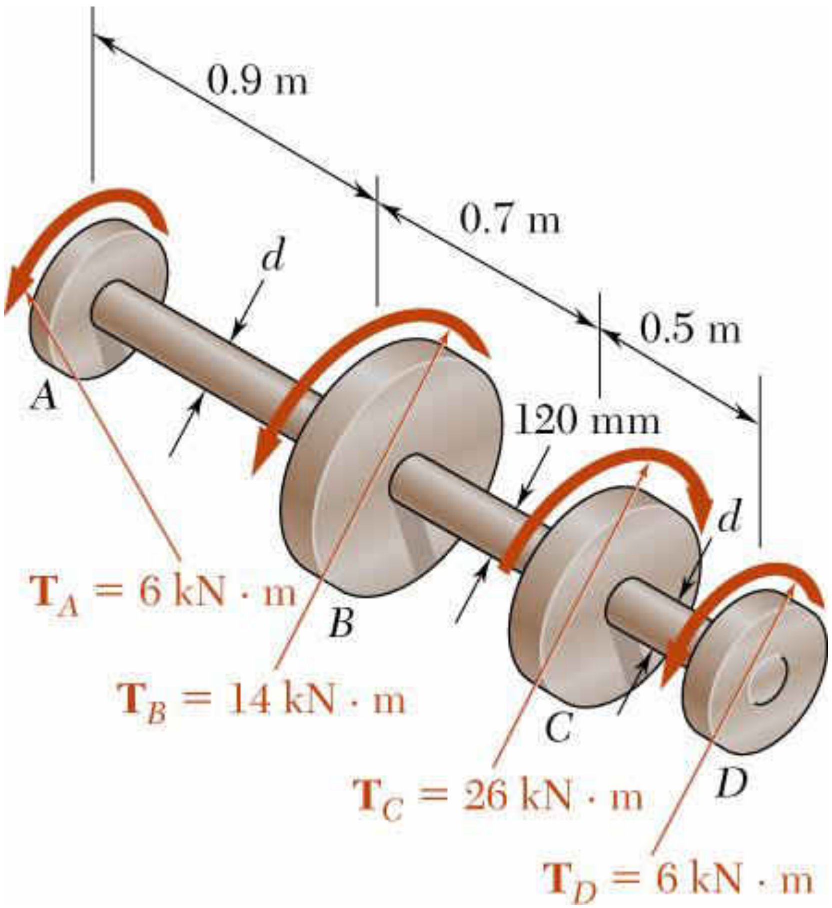




Reason: Brittle materials are weak in tension and maximum normal stress (tensile) plane in this case is 45°

Remember: Ductile materials are weak in shear and brittle materials are weak in tension. Thus, for ductile material failure occurs on maximum shear stress plane, and for brittle material failure occurs on maximum normal (tensile) stress plane.

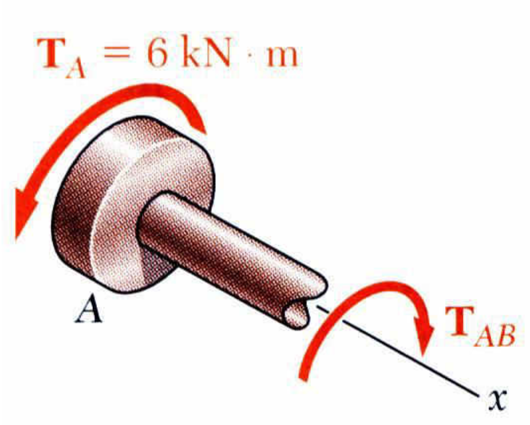
Example 3.1

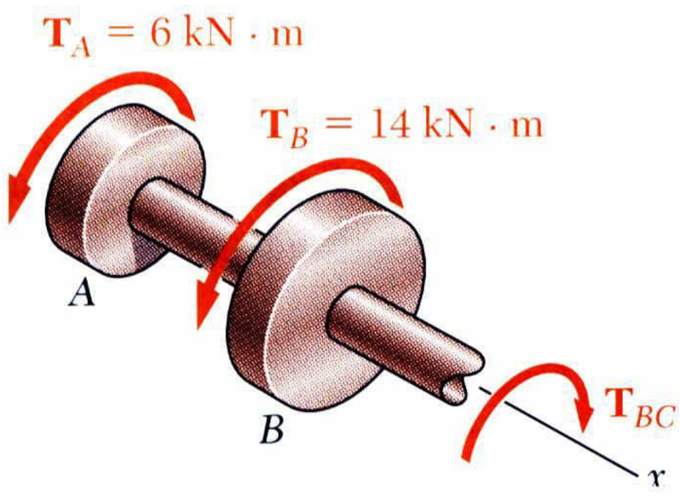


Shaft *BC* hollow, inner dia. 90 mm, outer dia. 120mm. Shafts *AB* and *CD* solid, dia. *d*. For loading shown, find (*a*) min. and max. shearing stress in *BC*, (*b*) required dia. *d* of *AB* and *CD* if allowable shearing stress in them is 65 MPa.

Example 3.1

- Cut sections, use equilibrium to find internal torque.



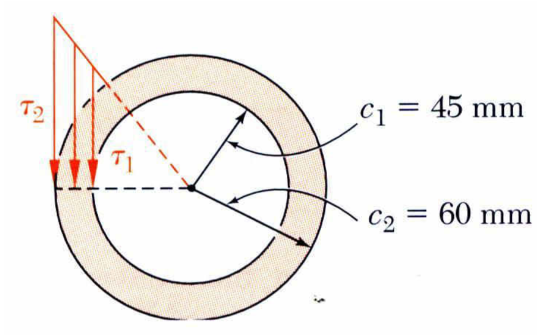


$$\sum M_x = 0 = (6\,\text{kN}\cdot\text{m}) - T_{AB}$$
$$T_{AB} = 6\,\text{kN}\cdot\text{m} = T_{CD}$$

$$\sum M_x = 0 = (6\,\text{kN}\cdot\text{m}) + (14\,\text{kN}\cdot\text{m}) - T_{BC}$$
$$T_{BC} = 20\,\text{kN}\cdot\text{m}$$

Example 3.1

- Apply elastic torsion formulae to find min. and max. stress in *BC*



$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}\left[(0.060)^4 - (0.045)^4\right]$$

$$= 13.92\times10^{-6}\,\mathrm{m}^4$$

$$\tau_{max} = \tau_2 = \frac{T_{BC}c_2}{J} = \frac{(20\,\mathrm{kN\cdot m})(0.060\,\mathrm{m})}{13.92\times10^{-6}\,\mathrm{m}^4}$$

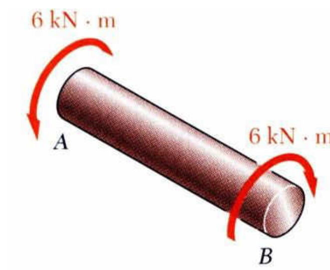
$$= 86.2\,\mathrm{MPa}$$

$$\frac{\tau_{min}}{\tau_{max}} = \frac{c_1}{c_2} \qquad \frac{\tau_{min}}{86.2\,\mathrm{MPa}} = \frac{45\,\mathrm{mm}}{60\,\mathrm{mm}}$$

$$\tau_{min} = 64.7\,\mathrm{MPa}$$

$$\boxed{\begin{aligned}\tau_{max} &= 86.2\,\mathrm{MPa}\\ \tau_{min} &= 64.7\,\mathrm{MPa}\end{aligned}}$$

- Given allowable shearing stress and applied torque, find required dia. of *AB* and *CD*

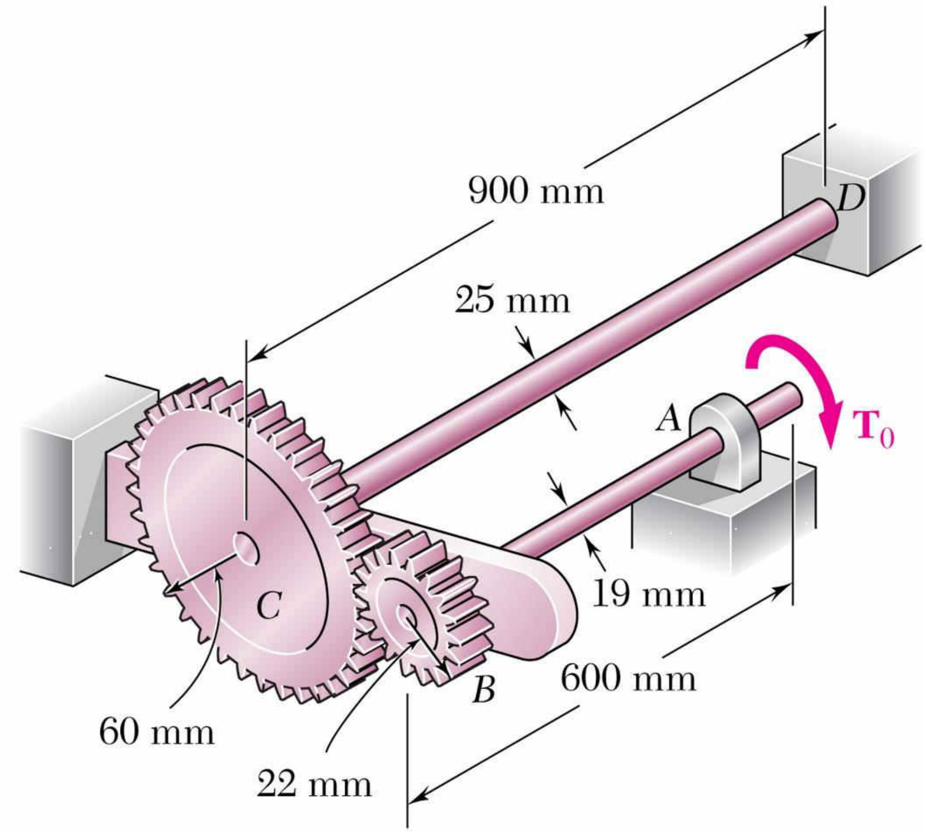


$$\tau_{max} = \frac{Tc}{J} = \frac{Tc}{\frac{\pi}{2}c^4} \qquad 65MPa = \frac{6\,\mathrm{kN\cdot m}}{\frac{\pi}{2}c^3}$$

$$c = 38.9\times10^{-3}\,\mathrm{m}$$

$$\boxed{d = 2c = 77.8\,\mathrm{mm}}$$

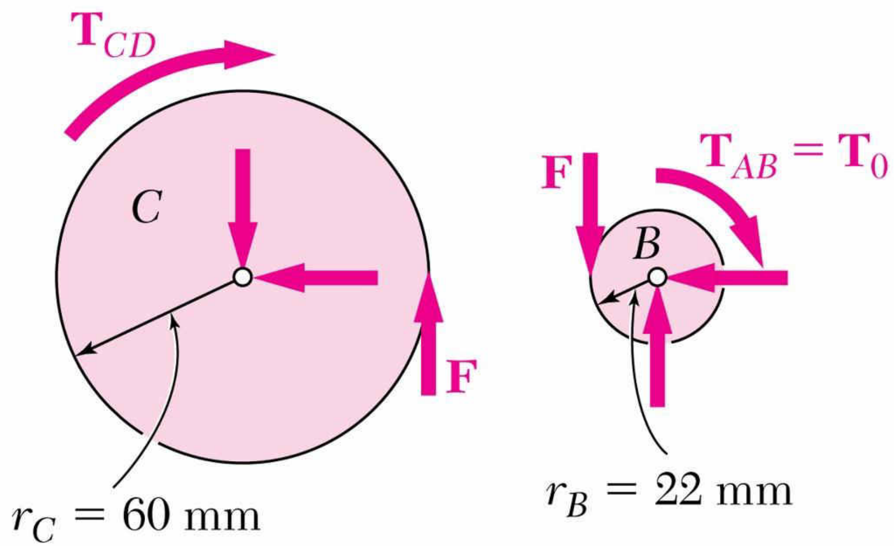
Example 3.2



Two solid steel shafts connected by gears. For each shaft $G = 77$ GPa and allowable shearing stress 55 Mpa. Find (*a*) largest torque T_0 that can be applied to end of AB, (*b*) corresponding angle through which end A rotates.

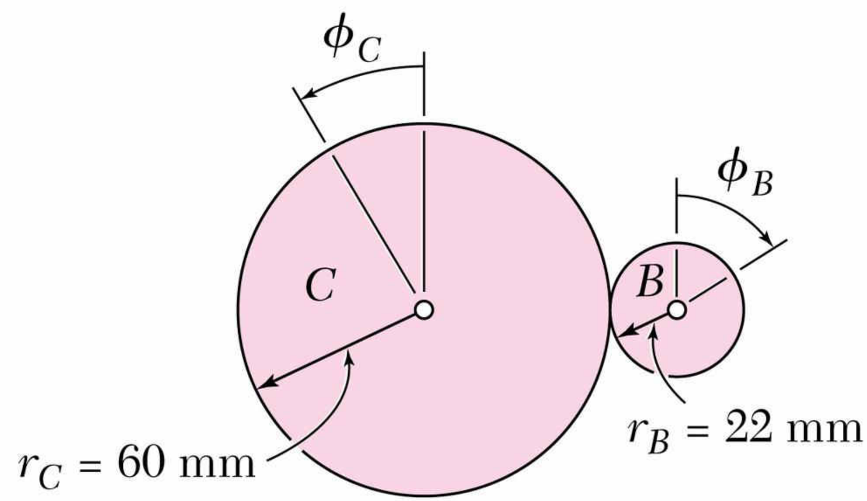
Example 3.2

- Equilibrium



$$\sum M_B = 0 = F(22\,\text{mm}) - T_0$$
$$\sum M_C = 0 = F(60\,\text{mm}) - T_{CD}$$
$$T_{CD} = 2.73 T_0$$

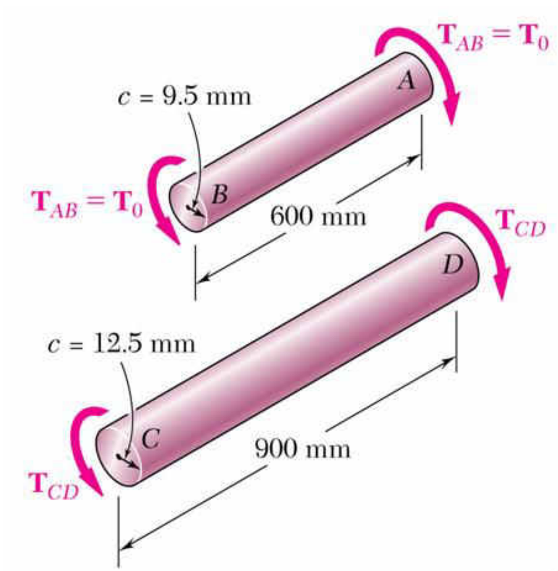
- Kinematic constraint of no slipping between gears (to relate rotations)

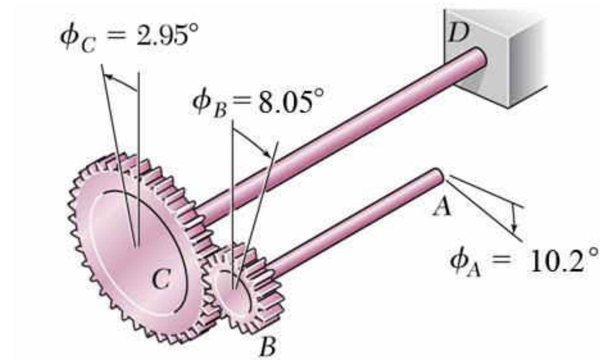


$$r_B \phi_B = r_C \phi_C$$
$$\phi_B = \frac{r_C}{r_B}\phi_C = \frac{60\,\text{mm}}{20\,\text{mm}}\phi_C$$
$$\phi_B = 2.73\,\phi_C$$

Example 3.2

- Find T_0 for max allowable torque on each shaft – choose smallest
- Find the corresponding angle of twist for each shaft and the net angular rotation of end A





$$\tau_{\max} = \frac{T_{AB}c}{J_{AB}} \qquad 55\times10^{6}\,\text{Pa} = \frac{T_0\left(9.5\times10^{-3}\,\text{m}\right)}{\frac{\pi}{2}\left(9.5\times10^{-3}\,\text{m}\right)^4}$$

$$T_0 = 74.1\,\text{Nm}$$

$$\tau_{\max} = \frac{T_{CD}c}{J_{CD}} \qquad 55\times10^{6}\,\text{Pa} = \frac{2.8T_0\left(12.5\times10^{-3}\,\text{m}\right)}{\frac{\pi}{2}\left(12.5\times10^{-3}\,\text{m}\right)^4}$$

$$T_0 = 61.8\,\text{Nm}$$

$$\boxed{T_0 = 61.8\,\text{Nm}}$$

$$\phi_{A/B} = \frac{T_{AB}L}{J_{AB}G} = \frac{(61.8\,\text{Nm})(0.6\,\text{m})}{\frac{\pi}{2}(0.0095\,\text{m})^4\left(77\times10^9\,\text{Pa}\right)}$$

$$= 0.0376\,\text{rad} = 2.15^\circ$$

$$\phi_{C/D} = \frac{T_{CD}L}{J_{CD}G} = \frac{2.73(61.8\,\text{Nm})(0.6\,\text{m})}{\frac{\pi}{2}(0.0125\,\text{m})^4\left(77\times10^9\,\text{psi}\right)}$$

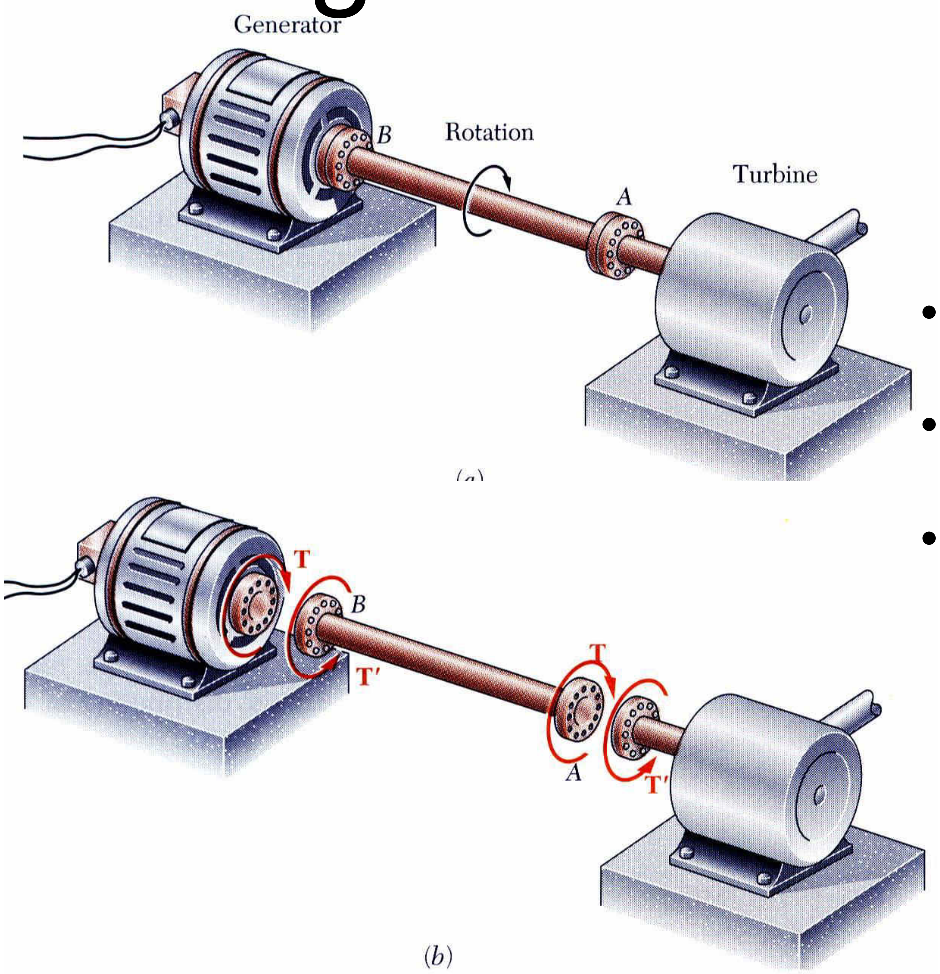
$$= 0.0514\,\text{rad} = 2.95^\circ$$

$$\phi_B = 2.73\phi_C = 2.73\left(2.95^\circ\right) = 8.05^\circ$$

$$\phi_A = \phi_B + \phi_{A/B} = 8.05^\circ + 2.15^\circ$$

$$\boxed{\phi_A = 10.2^\circ}$$

Design of Transmission Shafts



- Turbine exerts torque *T* on shaft
- Shaft transmits torque to generator
- Generator applies equal and opposite torque *T*' on shaft.

Design of Transmission Shafts

- Transmission shaft performance specifications are:
  - power
  - speed

- Designer must select shaft material and cross-section to meet performance specs. without exceeding allowable shearing stress.

- Determine torque applied to shaft at specified power and speed,

$$\mathbf{P} = \mathbf{T}\omega = 2\pi \mathbf{N}\mathbf{T}$$

$$\mathbf{T} = \frac{\mathbf{P}}{\omega} = \frac{\mathbf{P}}{2\pi \mathbf{N}}$$

P= Power (Watt)
T= torque (N-m)
ω= angular speed (rad/s)
N= revolution per sec

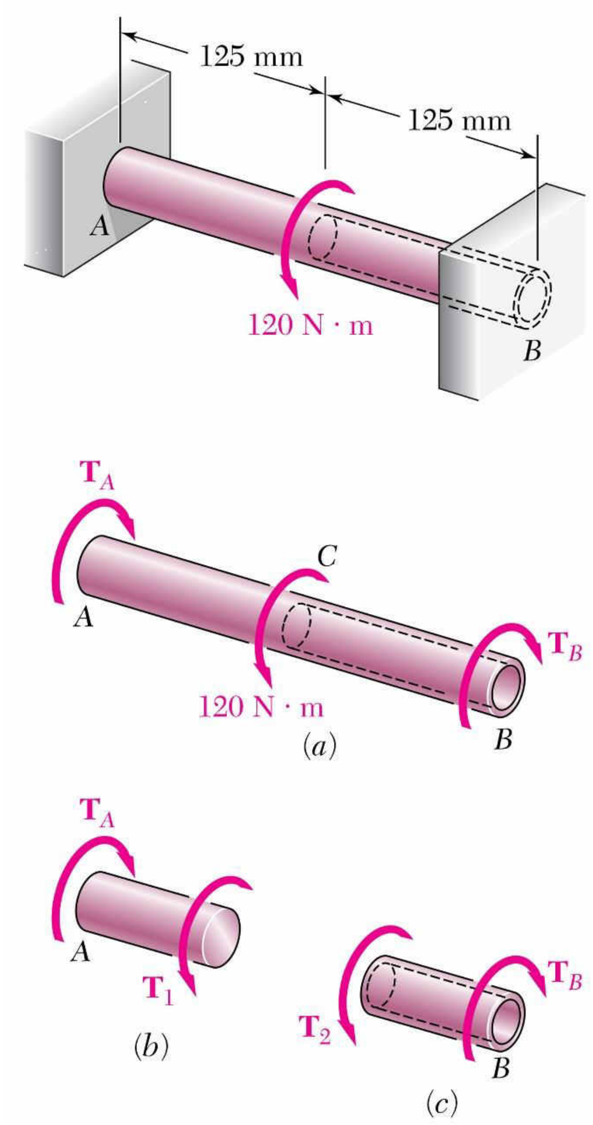
- Find cross-section so that max allowable shearing stress not exceeded,

$$\tau_{\max} = \frac{Tc}{J}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{T}{\tau_{\max}} \quad \text{(solid shafts)}$$

$$\frac{J}{c_2} = \frac{\pi}{2c_2}\left(c_2^4 - c_1^4\right) = \frac{T}{\tau_{\max}} \quad \text{(hollow shafts)}$$

Statically Indeterminate Shafts



- Given applied torque, find torque reactions at A and B.

- Equilibrium,

$$T_A + T_B = 120 \text{ N.m}$$

So problem is SID.

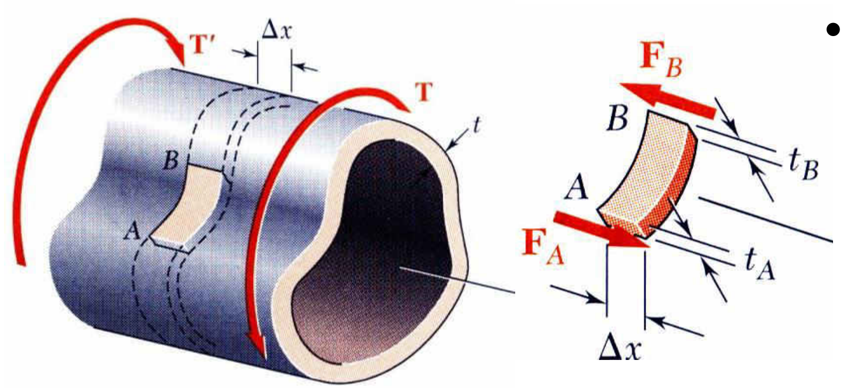
- Compatibility,

$$\phi = \phi_1 + \phi_2 = \frac{T_A L_1}{J_1 G} - \frac{T_B L_2}{J_2 G} = 0$$

- Solve equil. and compat,

$$T_A \left(1 + \frac{L_1 J_2}{L_2 J_1} \right) = 120 \,\text{N} \cdot \text{m}$$

Thin-Walled Hollow Shafts



- Since wall is thin, assume shear stress constant thru wall thickness. For AB, summing forces in x(shaft-axis)-direction,

$$\sum F_x = 0 = \tau_A\left(t_A \Delta x\right) - \tau_B\left(t_B \Delta x\right)$$

$$\tau_A t_A = \tau_B t_B = \tau t = q = \text{shear flow}$$

So shear flow constant and shear stress at section varies inversely with thickness

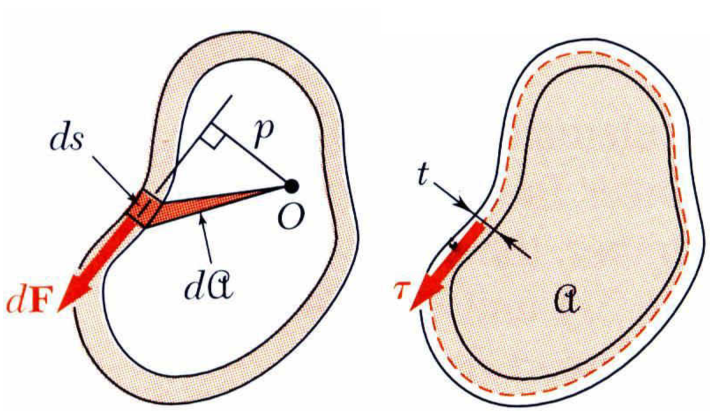
- Compute shaft torque from integral of the moments due to shear stress

$$dM_0 = p\,dF = p\left(\tau\, t\, ds\right) = q\left(p ds\right)$$

$$= q(r \sin\theta\, ds) = q(\mathbf{r} \times \mathbf{ds}) = 2q\, dA$$

$$T = \oint dM_0 = \oint 2q\, dA = 2qA$$

$$\tau = \frac{T}{2tA} \Rightarrow q = \frac{T}{2A}$$

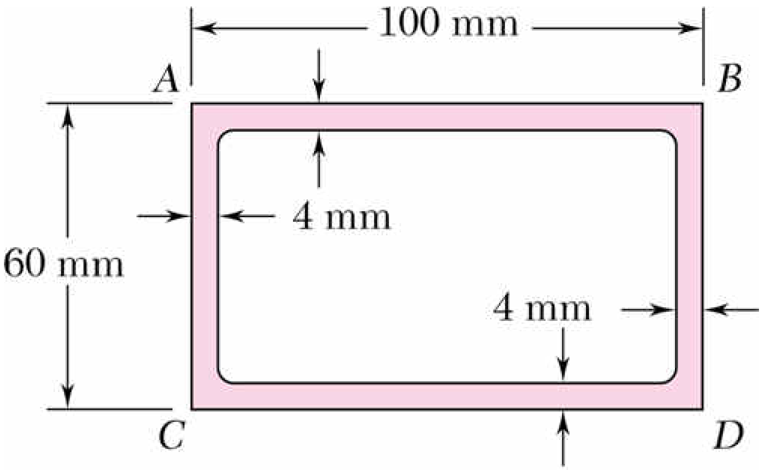


$$\boxed{\tau t = q = \text{constant}; \quad q = \frac{T}{2A}}$$

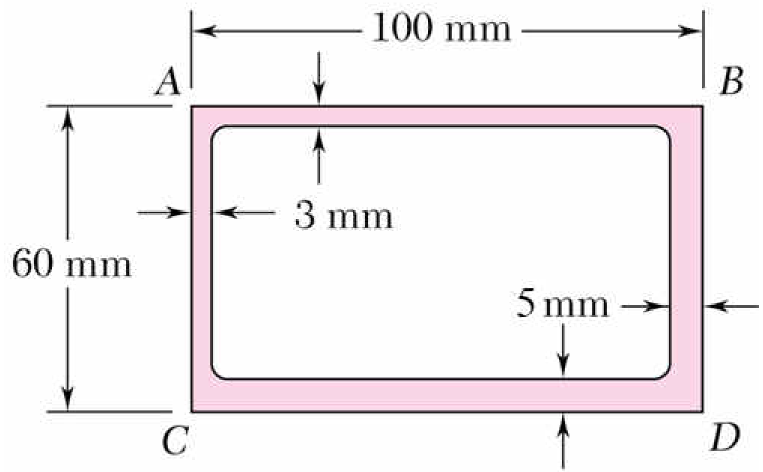
- Angle of twist (from Chapt 11)

$$\boxed{\phi = \frac{TL}{4A^2 G} \oint \frac{ds}{t}}$$

Example 3.3



(a)

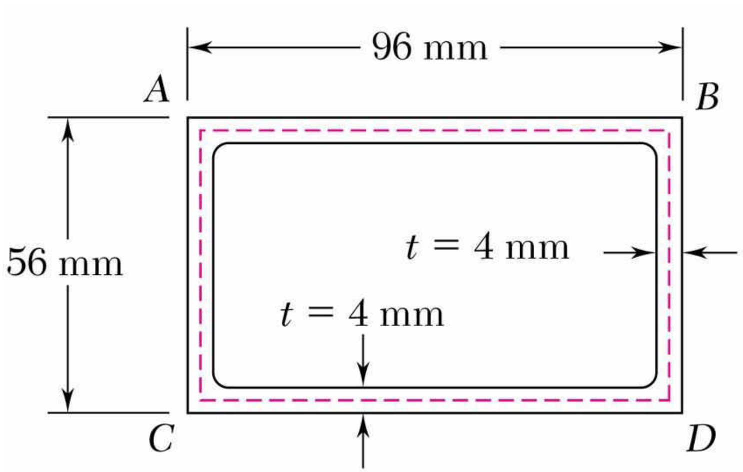


(b)

Extruded aluminum tubing with rectangular cross-section has torque loading 2.7 kNm. Find shearing stress in each of four walls considering (a) uniform wall thickness of 4 mm and (b) wall thicknesses of 3 mm on *AB* and *AC* and 5 mm on *CD* and *BD*.

Example 3.3

Find shear flow q.



$$A = (96\,\text{mm})(56\,\text{mm}) = 5376\,\text{mm}^2$$

$$q = \frac{T}{2A} = \frac{2700\,\text{Nm}}{2\left(5376\times10^{-6}\,\text{mm}^2\right)} = 251.12\times10^3\,\frac{\text{N}}{\text{m}}$$

Find corresponding shearing stress for each wall thickness.

With uniform wall thickness,

$$\tau = \frac{q}{t} = \frac{251.12\times10^3\,\text{N/m}}{0.004\,\text{m}}$$

$$\boxed{\tau = 62.8\,\text{MPa}}$$

With variable wall thickness

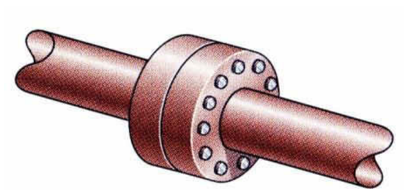
$$\tau_{AB} = \tau_{AC} = \frac{251.12\times10^3\,\text{N/m}}{0.003\,\text{m}}$$

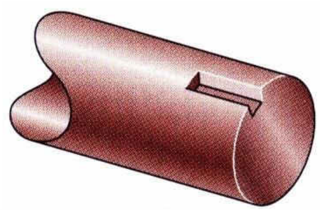
$$\boxed{\tau_{AB} = \tau_{BC} = 83.7\,\text{MPa}}$$

$$\tau_{BD} = \tau_{CD} = \frac{251.12\times10^3\,\text{N/m}}{0.005\,\text{m}}$$

$$\boxed{\tau_{BC} = \tau_{CD} = 50.2\,\text{MPa}}$$

Stress Concentrations





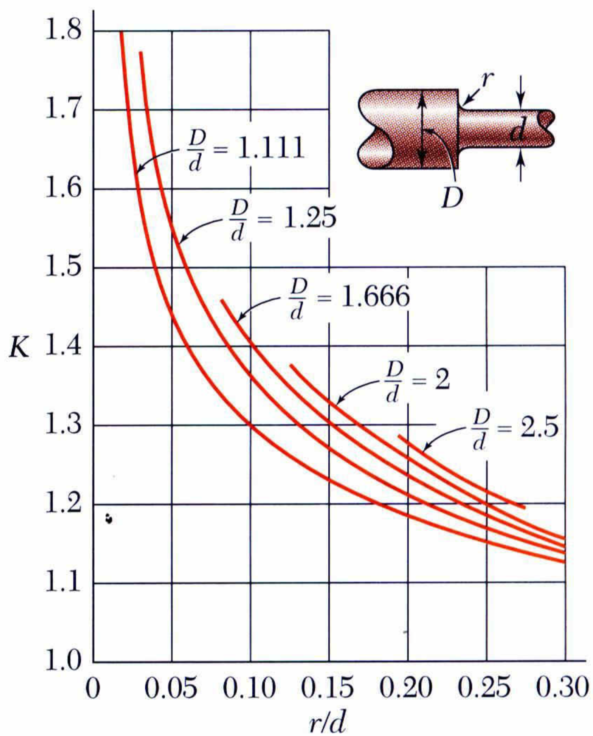


Fig. 3.32 Stress-concentration factors for fillets in circular shafts.†

- The derivation of the torsion formula,

$$\tau_{\max} = \frac{Tc}{J}$$

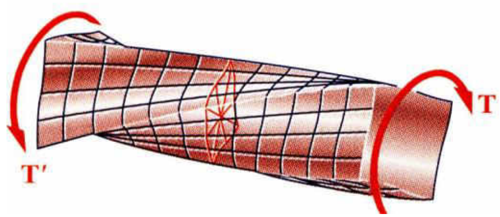
  assumed a circular shaft with uniform cross-section loaded through rigid end plates.

- The use of flange couplings, gears and pulleys attached to shafts by keys in keyways, and cross-section discontinuities can cause stress concentrations

- Experimental or numerically determined concentration factors are applied as

$$\tau_{\max} = K\frac{Tc}{J}$$

Torsion of Noncircular Members



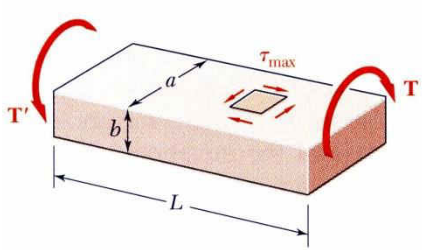
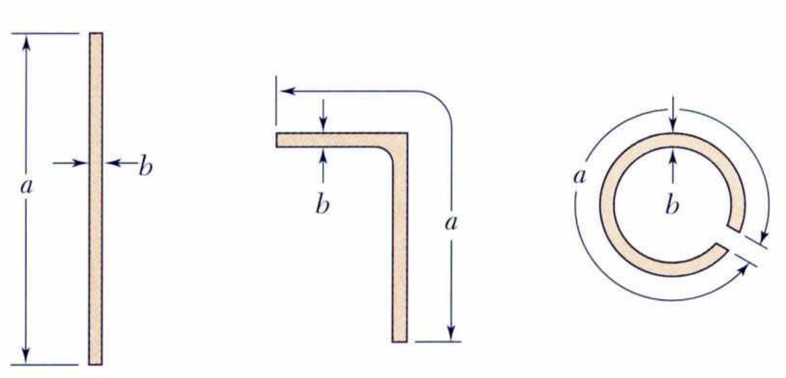


TABLE 3.1. Coefficients for Rectangular Bars in Torsion

a/b	c_1	c_2
1.0	0.208	0.1406
1.2	0.219	0.1661
1.5	0.231	0.1958
2.0	0.246	0.229
2.5	0.258	0.249
3.0	0.267	0.263
4.0	0.282	0.281
5.0	0.291	0.291
10.0	0.312	0.312
∞	0.333	0.333

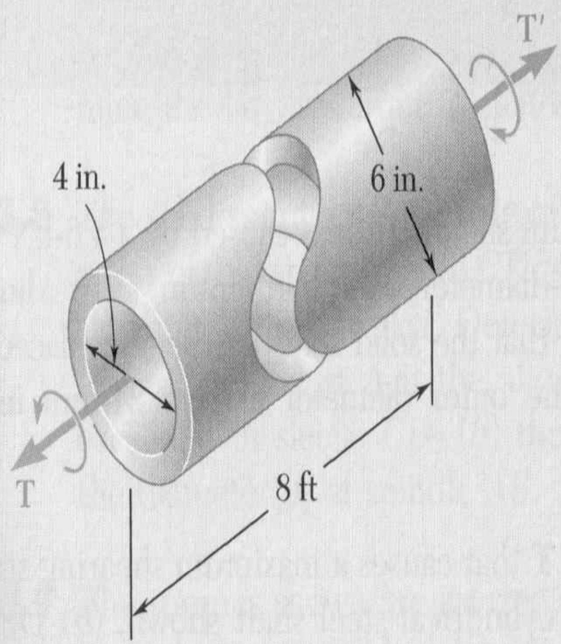


- Previous torsion formulas are valid for axisymmetric or circular shafts
- Planar cross-sections of noncircular shafts do not remain planar and stress and strain distribution do not vary linearly
- For uniform rectangular cross-sections,

$$\tau_{max} = \frac{T}{c_1 ab^2} \qquad \phi = \frac{TL}{c_2 ab^3 G}$$

- At large values of a/b, the maximum shear stress and angle of twist for other open sections are the same as a rectangular bar.

Example 3.4



The preliminary design of a large shaft connecting a motor to a generator calls for the use of a hollow shaft with inner and outer diameters of 4 in. and 6 in., respectively. Knowing that the allowable shearing stress is 12 ksi, determine the maximum torque that can be transmitted (*a*) by the shaft as designed, (*b*) by a solid shaft of the same weight, (*c*) by a hollow shaft of the same weight and of 8-in. outer diameter.

Example 3.4

Ex 3-4

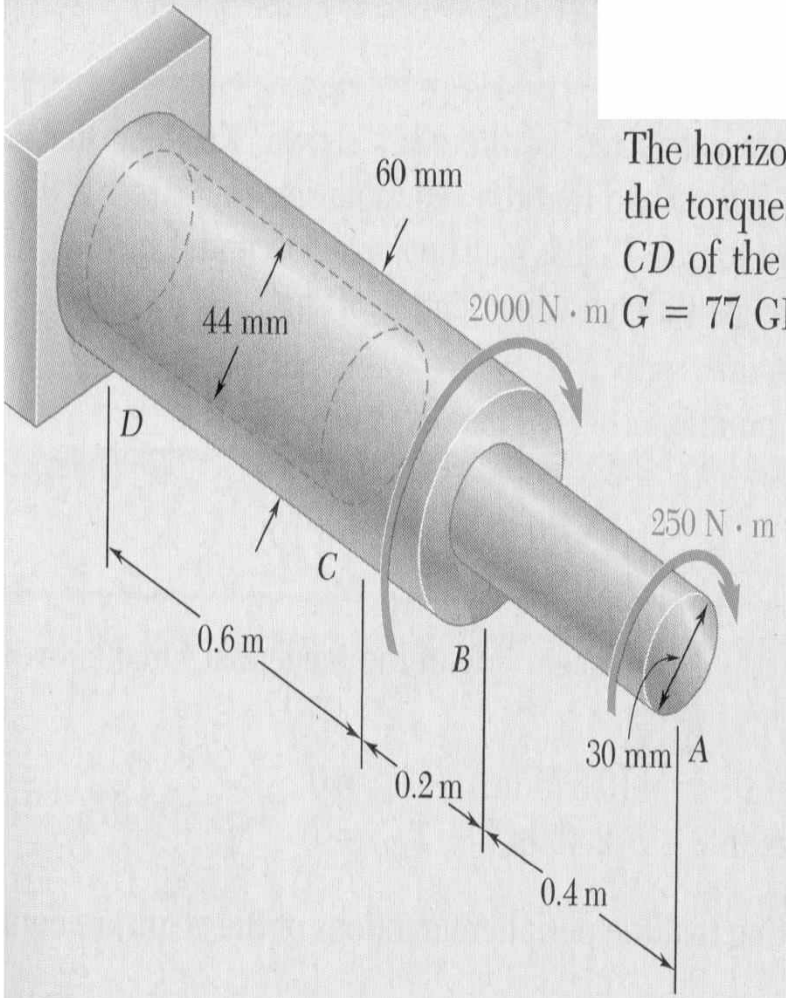
(a) $12 = \dfrac{T(6/2)}{\frac{\pi}{32}(6^4 - 4^4)} \Rightarrow T = 408.4$ kip·in ①

(b) $d = \sqrt{6^2 - 4^2} \Rightarrow 12 = \dfrac{T(d/2)}{\frac{\pi}{32} d^4} \Rightarrow T = 210.7$ kip·in

(c) $d_i^2 = 8^2 - (6^2 - 4^2) \Rightarrow 12 = \dfrac{T(8/2)}{\frac{\pi}{32}(8^4 - d_i^4)} \Rightarrow T = 636.2$ kip·in

Example 3.5

The horizontal shaft AD is attached to a fixed base at D and is subjected to the torques shown. A 44-mm-diameter hole has been drilled into portion CD of the shaft. Knowing that the entire shaft is made of steel for which $G = 77$ GPa, determine the angle of twist at end A.



Example 3.5

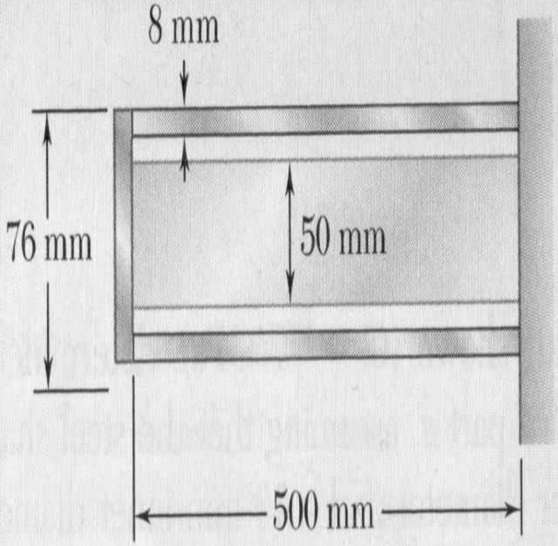
Ex 3.5

$$\theta_A = \theta_{A/B} + \theta_{B/C} + \theta_{C/D} + \cancel{\theta_D} \to 0$$

$$= \frac{(250)(0.4)}{(77E9)\frac{\pi}{32}\left(\frac{30}{1000}\right)^4} + \frac{(2250)(0.2)}{(77E9)\frac{\pi}{32}\left(\frac{60}{1000}\right)^4} + \frac{(2250)(0.6)}{(77E9)\frac{\pi}{32}\left[\left(\frac{60}{1000}\right)^4 - \left(\frac{44}{1000}\right)^4\right]}$$

$$= 0.0403 \text{ rad.}$$

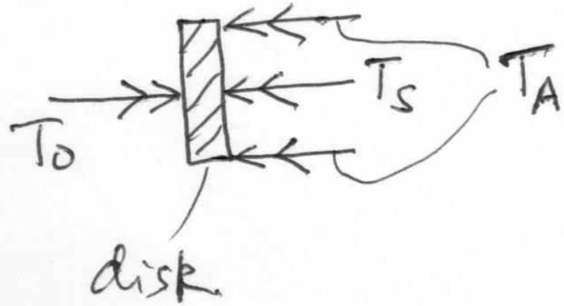
Example 3.6



A steel shaft and an aluminum tube are connected to a fixed support and to a rigid disk as shown in the cross section. Knowing that the initial stresses are zero, determine the maximum torque $\mathbf{T}_0$ that can be applied to the disk if the allowable stresses are 120 MPa in the steel shaft and 70 MPa in the aluminum tube. Use G = 77 GPa for steel and G = 27 GPa for aluminum.

Example 3.6

Ex 3.6



Equilibrium: $T_0 = T_A + T_S \rightarrow (1)$

Compatibility: $\theta_S = \theta_A$

$$\frac{T_A(.5)}{(27E9)\frac{\pi}{32}\left(\frac{76^4-60^4}{1000^4}\right)} = \frac{T_S(.5)}{(77E9)\frac{\pi}{32}\left(\frac{50^4}{1000^4}\right)} \rightarrow (2)$$

We see that $T_A/J_A = \frac{27}{77} T_S/J_S$.

$$\Rightarrow (\tau_{max})_A = \frac{27}{77}\underbrace{\left(\frac{T_S(25)}{J_S}\right)}_{(\tau_{max})_S}\frac{1}{25}\cdot(76/2) = 0.53(\tau_{max})_S$$

and $\frac{(\tau_{all})_A}{(\tau_{all})_S} = \frac{70}{120} = 0.58$

*So failure in steel first.

$\Rightarrow T_S = (\tau_{all})_S J_S/(25mm) = 2945.2$ N.m

Example 3.6

(2)

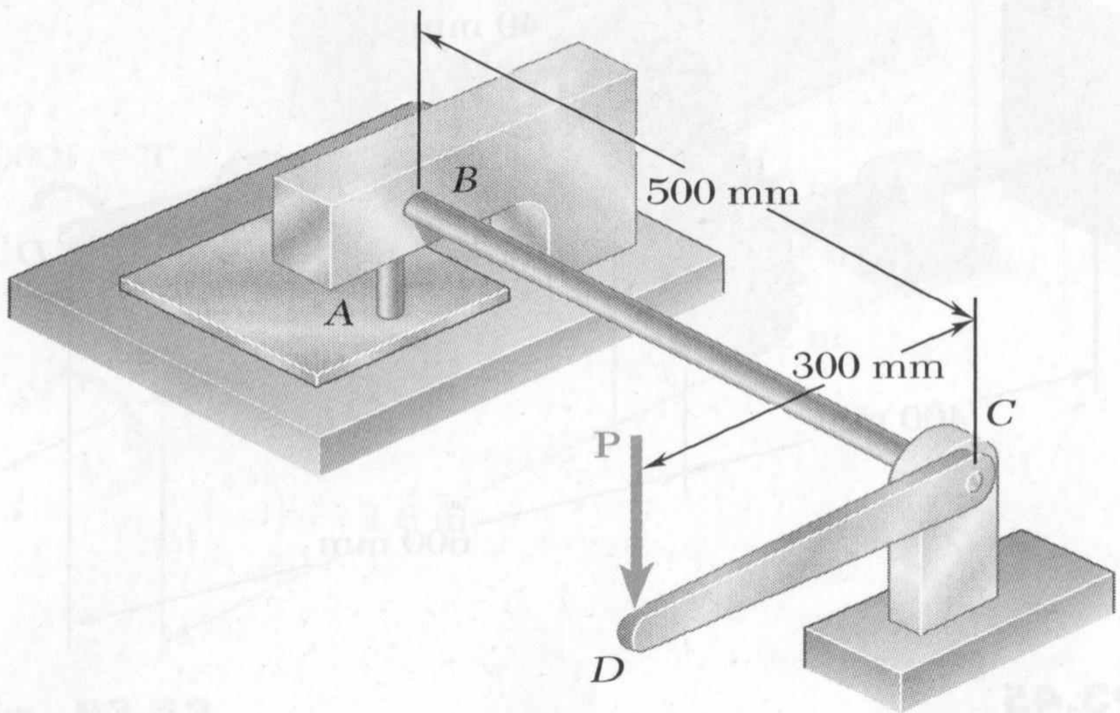
From (2) $\rightarrow T_A = 3371.2$

From (1) $\rightarrow T_0 = 6316.4$ N.m.

<u>Note</u>: we concluded failure in steel is critical $\because$ $\dfrac{(\tau_{max})_A}{(\tau_{max})_s} < \dfrac{(\tau_{all})_A}{(\tau_{all})_s}$

Example 3.7

A hole is punched at A in a plastic sheet by applying a 600-N force **P** to end D of lever CD, which is rigidly attached to the solid cylindrical shaft BC. Design specifications require that the displacement of D should not exceed 15 mm from the time the punch first touches the plastic sheet to the time it actually penetrates it. Determine the required diameter of shaft BC if the shaft is made of a steel with $G = 77$ GPa and $\tau_{all} = 80$ MPa.



Example 3.7

Ex 3.7 ———————— x ————————

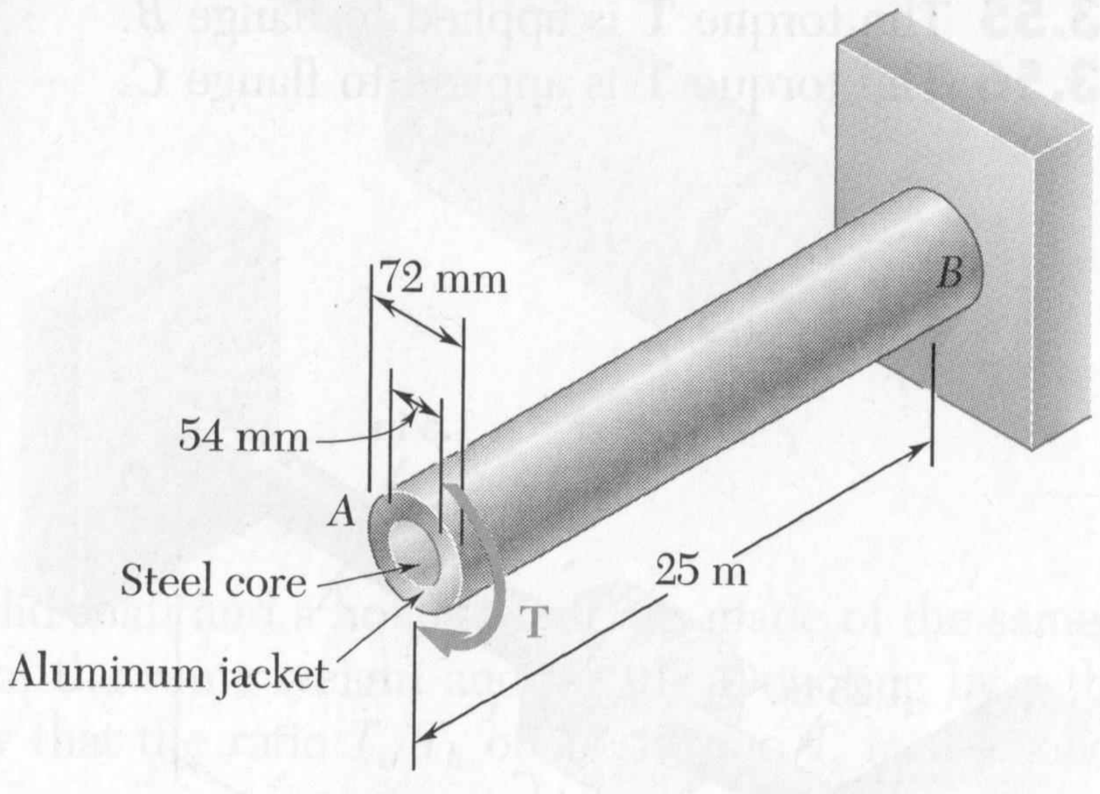
So, For shaft BC, end B is fixed.

$$(\theta_c)_{max} = \frac{15}{300} = \frac{1}{20} \text{ rad.}$$ (ie, angle during deformation stage, this excludes rigid body rot.)

$$= \frac{(600)(300/1000)(0.5)}{(77E9)\frac{\pi}{32}d^4} \Rightarrow d = 22.09 \text{ mm.}$$

Example 3.8

A torque of magnitude $T = 4$ kN · m is applied at end A of the composite shaft shown. Knowing that the modulus of rigidity is 77 GPa for the steel and 27 GPa for the aluminum, determine (*a*) the maximum shearing stress in the steel core, (*b*) the maximum shearing stress in the aluminum jacket, (*c*) the angle of twist at A.



Example 3.8

Ex 3.8 —— x ——

Applied Torque distributes itself to act partly on steel & partly on aluminium.

Equilibrium $\longrightarrow$ $4000 = T_s + T_A \longrightarrow$ (1)

Compatibility $\longrightarrow$ $\theta_S = \theta_A \Rightarrow \dfrac{T_S(25)}{(77E9)\frac{\pi}{32}\left(\frac{54}{1000}\right)^4} = \dfrac{T_A(25)}{\left\{27E9 * \frac{\pi}{32} * \left(\frac{72^4-54^4}{1000^4}\right)\right\}}$

(1), (2) $\longrightarrow T_S = 2275.9\ N.m$

$T_A = 1724.1\ N.m$ $\longleftarrow T_A = \left(\frac{27}{77}\right)\left(\frac{72^4-54^4}{54^4}\right)T_S \Longleftarrow$ (2)

$$(\tau_{max})_S = \frac{(2275.9)\left(\frac{54}{2}\cdot\frac{1}{1000}\right)}{\left(\frac{\pi}{32}\right)\left(\frac{54}{1000}\right)^4} = 73.61\ MPa$$

$$(\tau_{max})_A = \frac{(1724.1)\left(\frac{72}{2}\cdot\frac{1}{1000}\right)}{\left(\frac{\pi}{32}\right)\left(\frac{72^4-54^4}{1000^4}\right)} = 34.41\ MPa$$

$\theta = \theta_S = \theta_A$
$= 0.885\ rad.$

Example 3.8

$$(\tau_{max})_A = \frac{T_A}{J_A}\left(\frac{72}{2}\right) = \left(\underbrace{\frac{T_s}{J_s}\cdot\frac{54}{2}}_{(\tau_{max})_s}\cdot\frac{2}{54}\right)\cdot\frac{27}{77}\cdot\left(\frac{72}{2}\right) \quad (3)$$

$$= \frac{72}{54}\cdot\frac{27}{77}(\tau_{max})_s = 0.468(\tau_{max})_s.$$

$$\frac{(\tau_{all})_A}{(\tau_{all})_s} = \frac{45}{60} = 0.75.$$

$\because \dfrac{(\tau_{max})_A}{(\tau_{max})_s} < \dfrac{(\tau_{all})_A}{(\tau_{all})_s}$, steel will reach critical state first.

$$\Rightarrow T_s = \frac{(\tau_{all})_s\, J_s}{\left(\frac{54}{2}\cdot\frac{1}{1000}\right)} = 1855.1\ \text{N.m.}$$

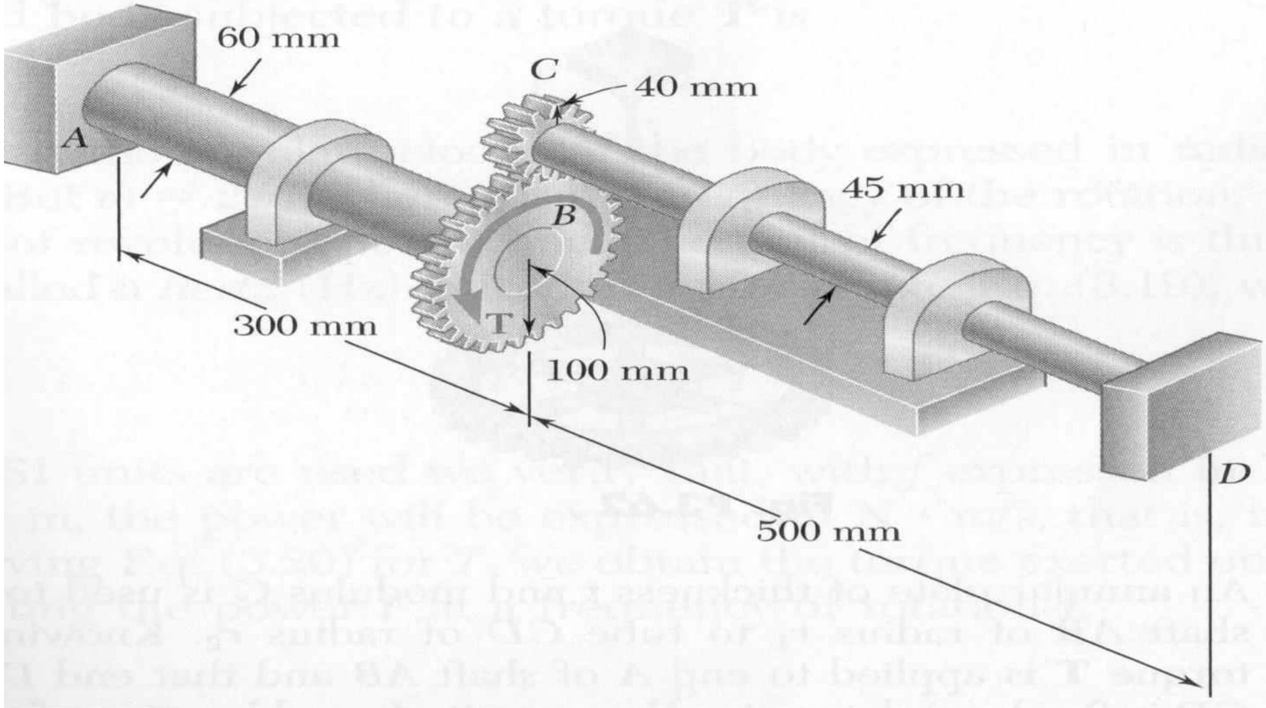
from (2), $\mathbf{T}_A = T_s\,\dfrac{J_A}{J_s}\cdot\dfrac{27}{77} = 1405.4$

$$T = T_A + T_s = 3260.5\ \text{N.m}$$

$$\theta = \theta_s = \theta_A = \frac{(1855.1)(25)}{(77E9)\frac{\pi}{32}\left(\frac{54}{1000}\right)^4} = 0.7215\ \text{rad}$$

Example 3.9

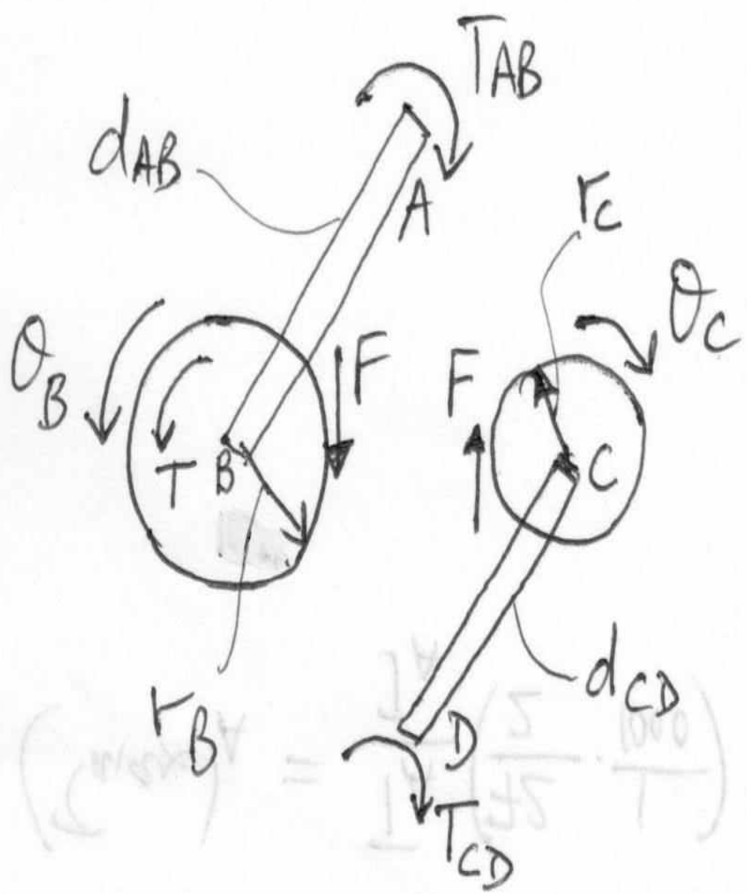
Ends A and D of the two solid steel shafts AB and CD are fixed, while ends B and C are connected to gears as shown. Knowing that a 4-kN · m torque **T** is applied to gear B, determine the maximum shearing stress (*a*) in shaft AB, (*b*) in shaft CD.



Ends A and D of the two solid steel shafts AB and CD are fixed, while ends B and C are connected to gears as shown. Knowing that the allowable shearing stress is 50 MPa in each shaft, determine the largest torque **T** that can be applied to gear B.

Example 3.9

Ex 3.9



Note: (1) Determine direction of F from impending motion concept, similar to friction. But even if its wrong, it wont matter, you will just get a negative number for F.

Example 3.9

(4)

Note (2): Assume directions of internal torques T_{AB}, T_{CD}, and remain consistent when writing equilibrium equations.

Note (3): θ_B, θ_C should be consistent with each other and with T_{AB}, T_{CD}.

Equilibrium:

$$\left.\begin{aligned} T &= T_{AB} + F \cdot r_B \quad (1) \\ 0 &= F \cdot r_C + T_{CD} \quad (2) \end{aligned}\right\} \text{3 unknowns, } F,\ T_{AB},\ T_{CD}.$$

Compatibility: $r_B \theta_B = r_C \theta_C$

$$\theta_B = \frac{T_{AB} L_{AB}}{G \frac{\pi}{32} (d_{AB})^4}, \quad \theta_C = (-) \frac{T_{CD} L_{CD}}{G \frac{\pi}{32} (d_{CD})^4}$$

imp!! for consistency

Example 3.9

$$\Rightarrow \frac{T_{AB} L_{AB} r_B}{G \frac{\pi}{32} (d_{AB})^4} = - \frac{T_{CD} L_{CD} r_C}{G \frac{\pi}{32} (d_{CD})^4} \longrightarrow (3).$$

Use $r_B = 100\,mm$, $r_C = 40\,mm$, $d_{AB} = 60\,mm$, $d_{CD} = 45\,mm$,
$L_{AB} = 300\,mm$, $L_{CD} = 500\,mm$, $T = 4000\,N.m$,

$$(1),(2),(3) \rightarrow \frac{T}{r_B} = \frac{T_{AB}}{r_B} - \frac{T_{CD}}{r_C} = T_{AB}\left(\frac{1}{r_B} + \frac{L_{AB}}{L_{CD}} \left(\frac{d_{CD}}{d_{AB}}\right)^4 \cdot \frac{1}{r_C} \frac{r_B}{r_C}\right)$$

$\Rightarrow T_{AB} = 1829.39\ N.m$, $T_{CD} = -\ 868.25\ N.m.$

expected ∵ F, T_{CD} chosen positive in same direction.

$$\tau_{AB} = \frac{(1829.39)(30/1000)}{\frac{\pi}{32}\left(\frac{60}{1000}\right)^4} = 43.13\ MPa$$

$$\tau_{CD} = \frac{(868.25)\left(\frac{22.5}{1000}\right)}{\frac{\pi}{32}\left(\frac{45}{1000}\right)^4} = 48.53\ MPa$$

Example 3.9

(5)

From previous part, for given torque T applied,

$(\tau_{AB})_{max} < (\tau_{CD})_{max}$.

$$\Rightarrow T_{CD} = \frac{(\tau_{All}) \cdot J_{CD}}{(d_{CD}/2)} = \frac{(50E6)\frac{\pi}{32}\left(\frac{45}{1000}\right)^4}{\left(\frac{45}{2}\cdot\frac{1}{1000}\right)} = -894.62 \text{ N.m.}$$

Also from (3), $T_{AB} =$ 1884.96 N.m

From (1), (2), eliminating F, $T = r_B\left(\frac{T_{AB}}{r_B} - \frac{T_{CD}}{r_C}\right) =$ 4121.50 N-m

Example 3.10

A solid shaft and a hollow shaft are made of the same material and are of the same weight and length. Denoting by n the ratio c_1/c_2, show that the ratio T_s/T_h of the torque T_s in the solid shaft to the torque T_h in the hollow shaft is (*a*) $\sqrt{(1 - n^2)}/(1 + n^2)$ if the maximum shearing stress is the same in each shaft, (*b*) $(1 - n^2)/(1 + n^2)$ if the angle of twist is the same for each shaft.

Example 3.10

Ex 3.10

Solid shaft radius $= r$
Hollow shaft radius $= r_o$ (outer) & r_i (inner)
Given $n = \frac{r_i}{r_o}$. Also $\because$ lengths & weights are same,
$\Rightarrow$ areas same $\Rightarrow r^2 = r_o^2 - r_i^2$

For max shear stresses same,

$$\tau_s = \frac{T_s r}{G \frac{\pi}{2} r^4} = \tau_h = \frac{T_h r_o}{G \frac{\pi}{2} (r_o^4 - r_i^4)}$$

$$\frac{T_s}{T_h} = \frac{r^3 r_o}{(r_o^2 - r_i^2)(r_o^2 + r_i^2)} = \frac{\left(\sqrt{r_o^2 - r_i^2}\right) r_o}{(r_o^2 + r_i^2)} = \frac{\sqrt{1-n^2}}{1+n^2}$$

For angle of twist same,

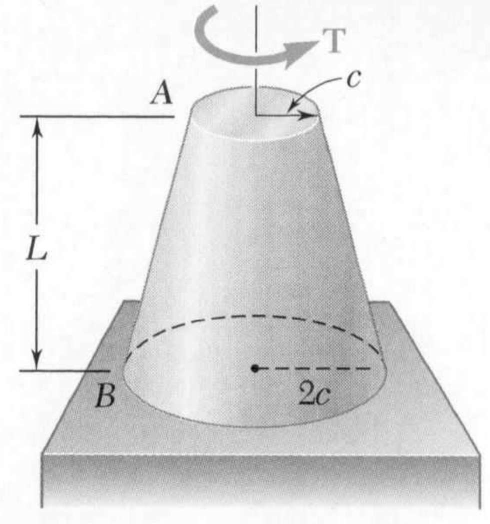
$$\frac{T_s \cancel{L_s}}{\cancel{G} J_s} = \frac{T_h \cancel{L_h}}{\cancel{G} J_h} \Rightarrow \frac{T_s}{T_h} = \frac{J_s}{J_h} = \frac{r^4}{r_o^4 - r_i^4} = \frac{r_o^2 - r_i^2}{r_o^2 + r_i^2}$$

$$= \frac{1-n^2}{1+n^2}$$

Example 3.11

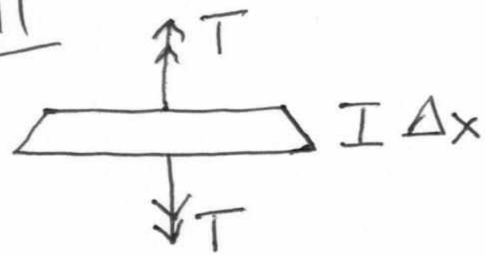
A torque **T** is applied as shown to a solid tapered shaft AB. Show by integration that the angle of twist at A is

$$\phi = \frac{7TL}{12\pi G c^4}$$



Example 3.11

Ex 3-11 (6)



Equilibrium of element shown.

$$\Delta\theta = \frac{T\,\Delta x}{G\,J(x)} \Rightarrow \int_0^{\theta} d\theta = \int_0^{L} \frac{T\,dx}{G\,J(x)}$$

where $J(x) = \frac{\pi}{2} r^4 = \frac{\pi}{2}\left(2c - \frac{c}{L}x\right)^4$

$$\Rightarrow \theta = \frac{T}{G}\int_0^L \frac{2}{\pi c^4}\,\frac{1}{\left(2-\frac{x}{L}\right)^4}\,dx = \frac{2T}{G\pi c^4}\int_2^1 \frac{1}{y^4}(-L\,dy)$$

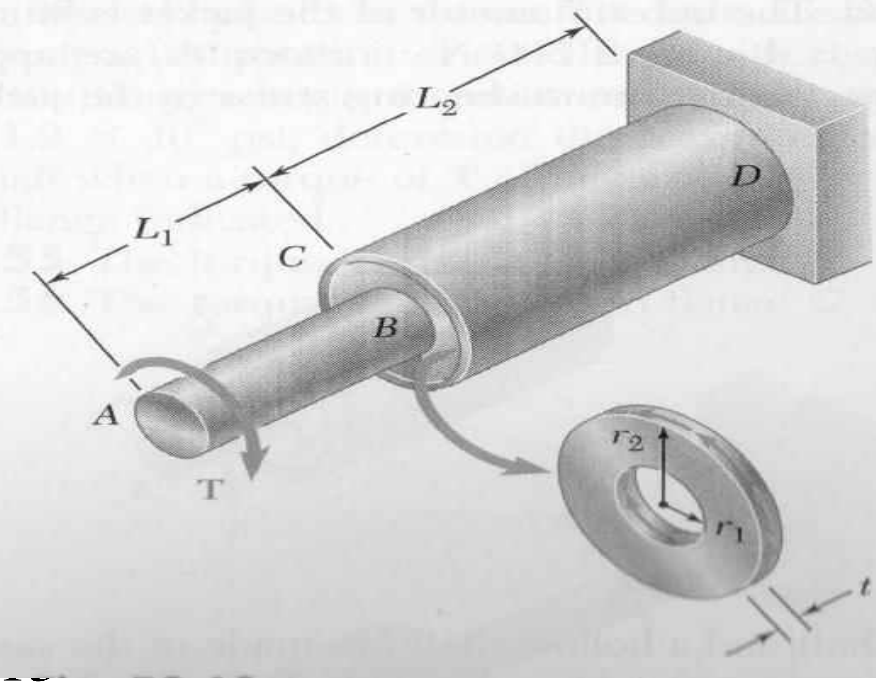
$$= \frac{2T}{G\pi c^4}\left[-\frac{1}{y^3}\,\frac{1}{3}\right]_2^1 \cdot (-L) = \frac{2TL}{3G\pi c^4}\left(1-\frac{1}{8}\right)$$

$$= \frac{7}{12}\,\frac{TL}{G\pi c^4}$$

Example 3.12

An annular plate of thickness t and modulus G is used to connect shaft AB of radius r_1 to tube CD of radius r_2. Knowing that a torque $\mathbf{T}$ is applied to end A of shaft AB and that end D of tube CD is fixed, (a) determine the magnitude and location of the maximum shearing stress in the annular plate, (b) show that the angle through which end B of the shaft rotates with respect to end C of the tube is

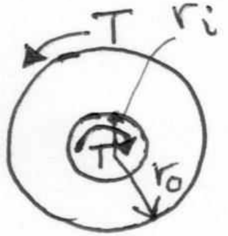
$$\phi_{BC} = \frac{T}{4\pi G t}\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right)$$



Example 3.12

Ex 3.12

Consider annular plate as rigid.



Torque balance on annular plate is shown. These torques are resultants of shear stress distribution on inner & outer radii.

$$T = \int(\tau_i\, dA) r_i = \int(\tau_o\, dA) r_o = \tau_i (2\pi r_i^2 t) = \tau_o (2\pi r_o^2 t)$$

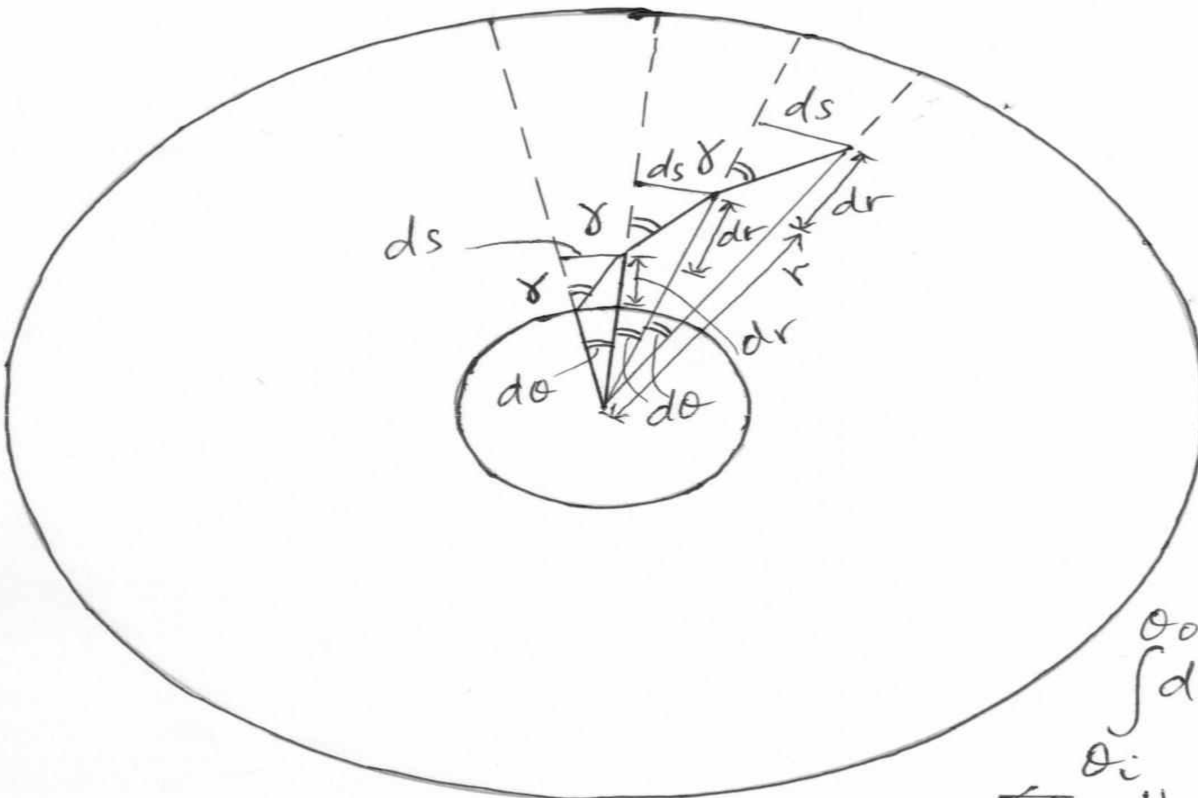
$$\Rightarrow \tau_{max} = \tau_i = T / (2\pi r_1^2 t) \qquad \begin{pmatrix} r_i = r_1 \\ r_o = r_2 \end{pmatrix}$$

Thus for any r,

$$T = \int (\tau\, dA) r = \tau(2\pi r^2 t)$$

(Contd)

Example 3.12



$$\gamma = \text{shear strain}$$

$$= \frac{\tau}{G} = \frac{T}{2\pi G t}\frac{1}{r^2}$$

$$ds = \gamma\, dr$$

$$= r\, d\theta$$

$$\Rightarrow d\theta = \frac{\gamma}{r} dr$$

$$= \frac{T}{2\pi G t}\frac{1}{r^3} dr$$

$$\int_{\theta_i}^{\theta_o} d\theta = \int_{r_i}^{r_o} \frac{T}{2\pi G t}\frac{1}{r^3} dr$$

$$\theta_o - \theta_i = \frac{T}{2\pi G t}\left[-\frac{1}{2}\frac{1}{r^2}\right]_{r_i}^{r_o}$$

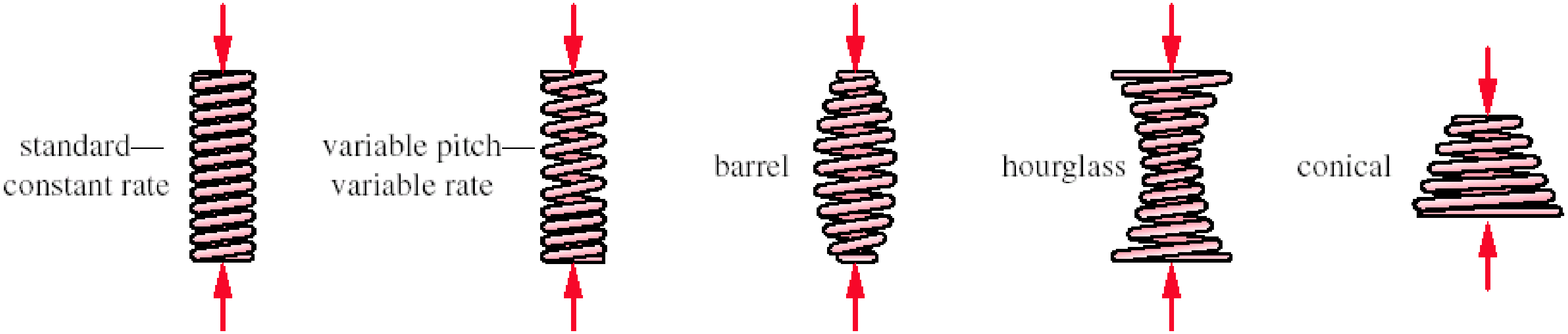
$$\theta_i - \theta_o = \theta_{BC} = \frac{T}{4\pi G t}\left(\frac{1}{r_i^2} - \frac{1}{r_o^2}\right)$$, Note $r_i = r_1$, $r_o = r_2$

Springs

Spring design

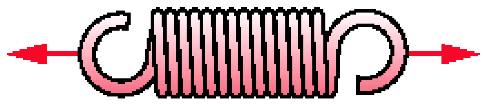
- Types
- Factors in spring design
- Materials
- Torsional

Types of Springs



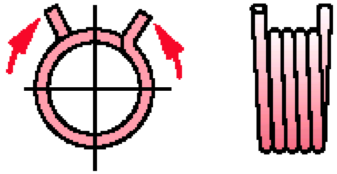
(a) Helical compression springs. *Push*—wide load and deflection range—round or rectangular wire. Standard has constant coil diameter, pitch, and rate. Barrel, hourglass, and variable-pitch springs are used to minimize resonant surging and vibration. Conical springs can be made with minimum solid height and with constant or increasing rate.

Types of spring cont.



(b) Helical extension springs. *Pull*—wide load and deflection range—round or rectangular wire, constant rate.

(c) Drawbar springs. *Pull*—uses compression spring and drawbars to provide extension pull with fail-safe, positive stop.

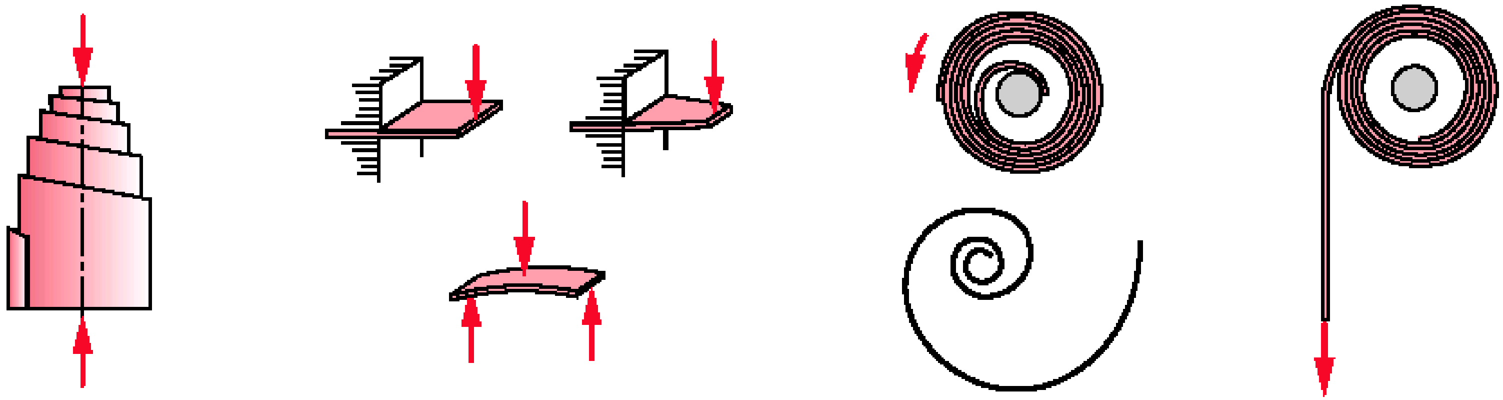


(d) Torsion springs. *Twist*—round or rectangular wire—constant rate.

Types of springs cont.

(e) Spring washers. *Push*—Belleville has high loads and low deflections—choice of rates (constant, increasing, or decreasing). Wave has light loads, low deflection, uses limited radial space. Slotted has higher deflections than Belleville. Finger is used for axial loading of bearings. Curved is used to absorb axial end play.

Types of springs cont.



(f) Volute spring. *Push*—may have an inherently high friction damping.

(g) Beam springs. *Push* or *Pull*—wide load but low deflection range—rectangular or shaped cantilever, or simply supported.

(h) Power or motor springs. *Twist*—exerts torque over many turns. Shown in, and removed from, retainer.

(i) Constant Force. *Pull*—long deflection at low or zero rate.

FIGURE 13-2

Spring Configurations (Adapted from: *Design Handbook*: *Engineering Guide to Spring Design*, 1987, Associated Spring, Barnes Group Inc., 10 Main St., Bristol, Conn., with permission)

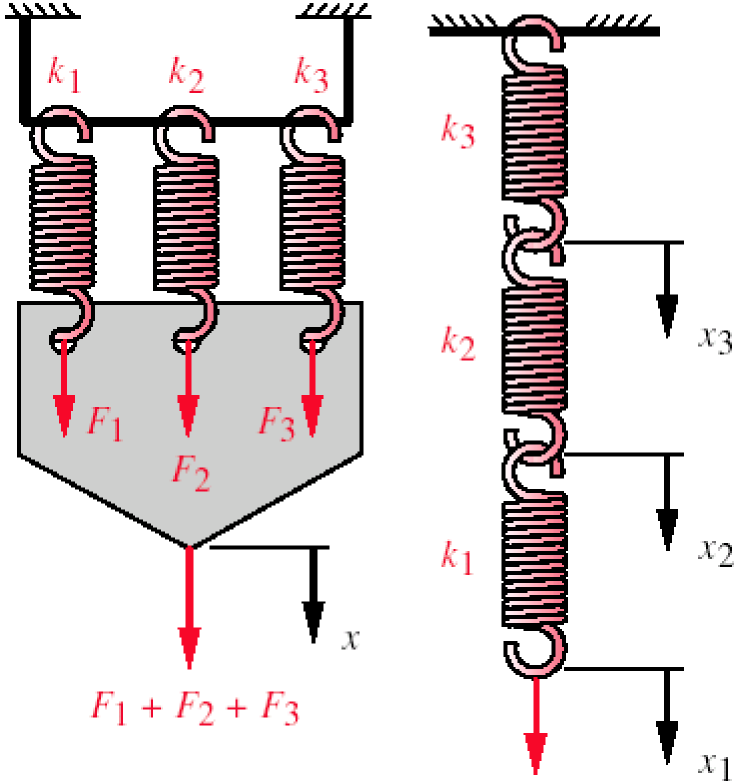
Spring Design

$$F = ky$$

$$k = F / y$$

$$\frac{1}{k_{series}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$

$$k_{parallel} = k_1 + k_2 + k_3$$



(b) Parallel

(a) Series

Factors in spring design

- High strength
- High yield
- Modulus may be low for energy storage
- Cost
- Environmental factors
  - Temperature resistance (e.g. valve springs)
  - Corrosion resistance

Common materials for springs

Table 13-1 Common Spring Wire Materials

Source: Reference 2

ASTM #	Material	SAE #	Description
A227	Cold-drawn wire ("hard-drawn")	1066	Least expensive general-purpose spring wire. Suitable for static loading but not good for fatigue or impact. Temperature range 0°C to 120°C (250°F).
A228	Music wire	1085	Toughest, most widely used material for small coil springs. Highest tensile and fatigue strength of all spring wire. Temperature range 0°C to 120°C (250°F).
A229	Oil-tempered wire	1065	General-purpose spring steel. Less expensive and available in larger sizes than music wire. Suitable for static loading but not good for fatigue or impact. Temperature range 0°C to 180°C (350°F)
A230	Oil-tempered wire	1070	Valve-spring quality—suitable for fatigue loading.
A232	Chrome vanadium	6150	Most popular alloy spring steel. Valve-spring quality—suitable for fatigue loading. Also good for shock and impact loads. For temperatures to 220°C (425°F). Available annealed or pretempered.
A313 (302)	Stainless steel	30302	Suitable for fatigue applications.

Influence of diameter on ultimate stress

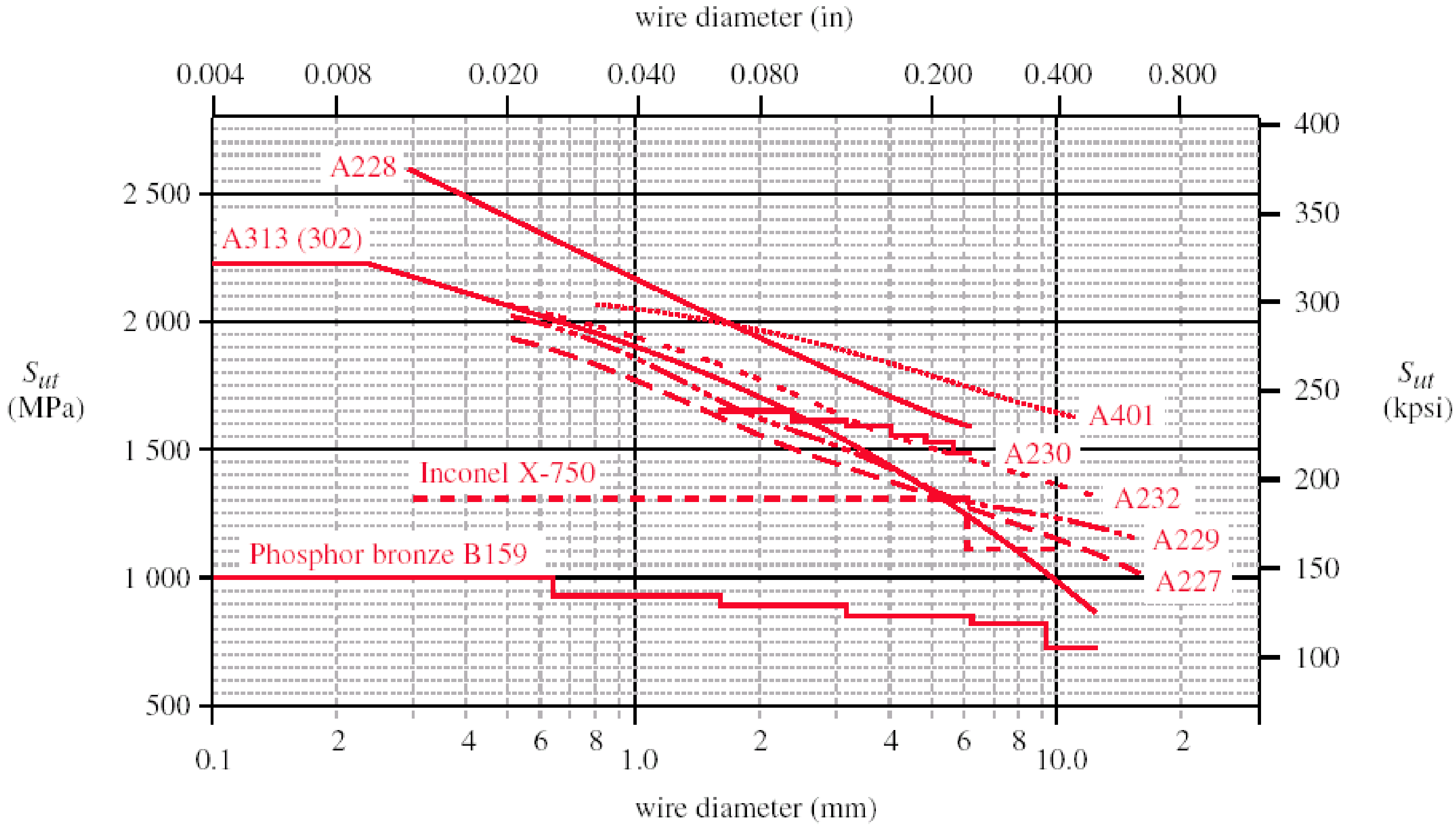


FIGURE 13-3

Minimum Tensile Strengths of Spring Wire—Identified by ASTM Number—See Table 13-1 Source: *Design Handbook: Engineering Guide to Spring Design*, 1987, Associated Spring, Barnes Group Inc., Bristol, Conn.

Influence of diameter on ultimate stress cont.

Table 13-4 Coefficients and Exponents for Equation 13.3

Source: Reference 1

ASTM #	Material	Range mm	Range in	Exponent b	Coefficient A MPa	Coefficient A psi	Correlation Factor
A227	Cold drawn	0.5–16	0.020–0.625	–0.182 2	1 753.3	141 040	0.998
A228	Music wire	0.3–6	0.010–0.250	–0.1625	2 153.5	184 649	0.9997
A229	Oil tempered	0.5–16	0.020–0.625	–0.183 3	1 831.2	146 780	0.999
A232	Chrome-v.	0.5–12	0.020–0.500	–0.145 3	1 909.9	173 128	0.998
A401	Chrome-s.	0.8–11	0.031–0.437	–0.093 4	2 059.2	220 779	0.991

$$S_{ut} = Ad^b$$

$$S_{us} \cong 0.67 S_{ut}$$

Design of helical compression springs

- Length nomenclature
  - Free
  - Assembled
  - Solid or shut height
  - Working deflection

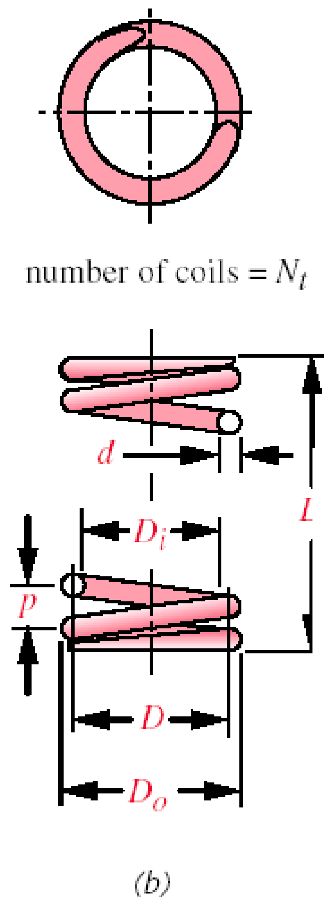


FIGURE 13-7

(a) Sample Springs and (b) Dimensional

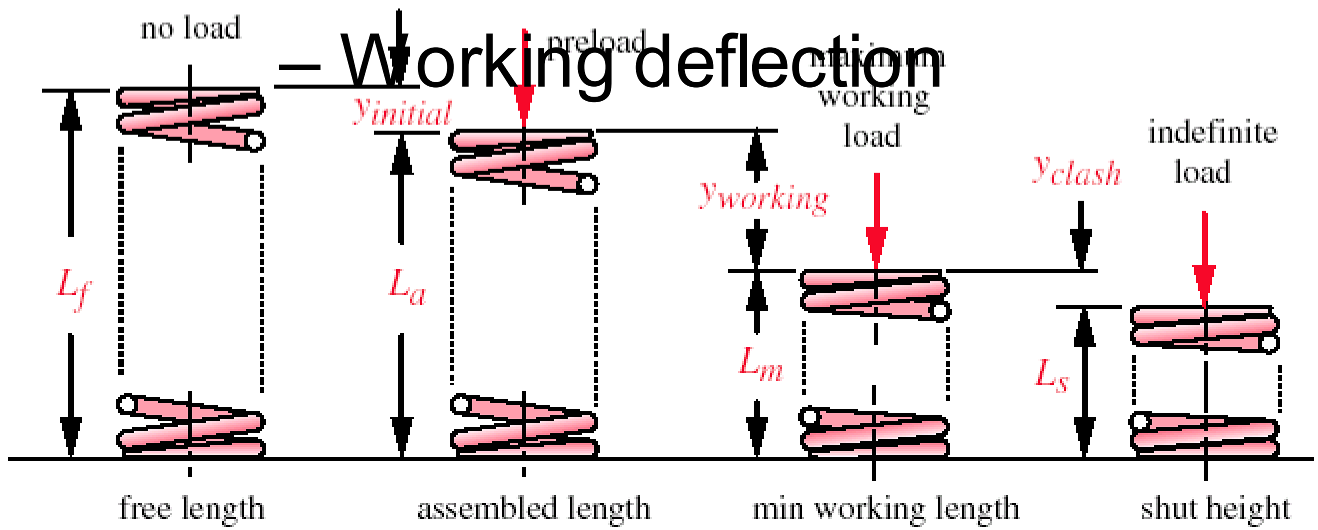


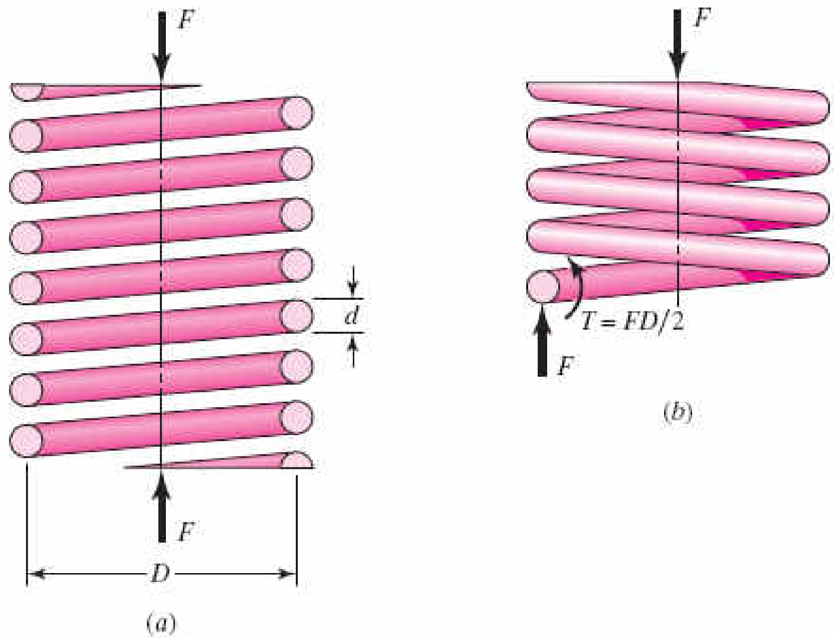
FIGURE 13-8

Various Lengths of a Helical Compression Spring in Use

Stresses in Helical Spring

Figure 10–1

(*a*) Axially loaded helical spring; (*b*) free-body diagram showing that the wire is subjected to a direct shear and a torsional shear.



Stresses in Helical springs cont.

At the inside of the spring

$$\tau_{\max} = \frac{Tr}{J} + \frac{F}{A}$$

Substituting for

$$\tau_{\max} = \tau\,,\ T = FD/2,\ r = d/2,$$
$$J = \pi d^4/32, \quad A = \pi d^2/4$$

Gives

$$\tau = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

Defining the spring index $C = \frac{D}{d}$ 4<C<12

Therefore the stress is

$$\tau = K_s \frac{8FD}{\pi d^3}$$

Equation(1)

K_s is a *shear-stress correction factor*

$$K_s = \frac{2C+1}{2C}$$

Effect of curvature on Stress

- Equation (1) is based on the wire being straight
- However the curvature increases the stress on the inside of the wire
- For static stress the effect of curvature can be neglected
- For fatigue the effect of curvature is important

Effect of curvature cont.

$$K_W = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

Wahl factor

$$K_B = \frac{4C+2}{4C-3}$$

Bergstrasser factor

The results of the two equations differ by less than 1%. Bergstrasser factor is preferred due to simplicity

Deflection

- The external work done on an elastic member in deforming it is transformed into *strain,* or *potential, energy*. If the member is deformed a distance *y*, and if the force-deflection relationship is linear, this energy is equal to the product of the average force and the deflec

$$U = \frac{F}{2}y = \frac{F^2}{2k}$$

- This equation is general in the sense that the force *F* can also mean torque, or moment, provided, that consistent units are used for *k*.

Deflection cont..

- By substituting appropriate expressions for *k*, strain-energy formulas for various simple loadings may be obtained. For tension and compression and for torsion,

$$U = \frac{F^2 l}{2AE} \qquad \text{tension and compression} \qquad (4\text{–}15)$$

$$U = \frac{T^2 l}{2GJ} \qquad \text{torsion} \qquad (4\text{–}16)$$

Deflection of a helical spring

- Using Castigliano's theorem, strain energy is equal to

$$U = \frac{T^2 l}{2GJ} + \frac{F^2 l}{2AG}$$

- Substituting

$$T = FD/2,\ l = \pi DN,\ J = \pi d^4/32, \text{ and } A = \pi d^2/4$$

$$U = \frac{4F^2 D^3 N}{d^4 G} + \frac{2F^2 DN}{d^2 G} \qquad \text{where } N = N_a = \text{number of active coils.}$$

$$y = \frac{\partial U}{\partial F} = \frac{8FD^3 N}{d^4 G} + \frac{4FDN}{d^2 G}$$

Deflection cont.

- Using the spring index

$$y = \frac{8FD^3N}{d^4G}\left(1 + \frac{1}{2C^2}\right) \doteq \frac{8FD^3N}{d^4G}$$

- Spring scale is $k = F/y,$

$$k \doteq \frac{d^4G}{8D^3N}$$

Spring design – end treatment

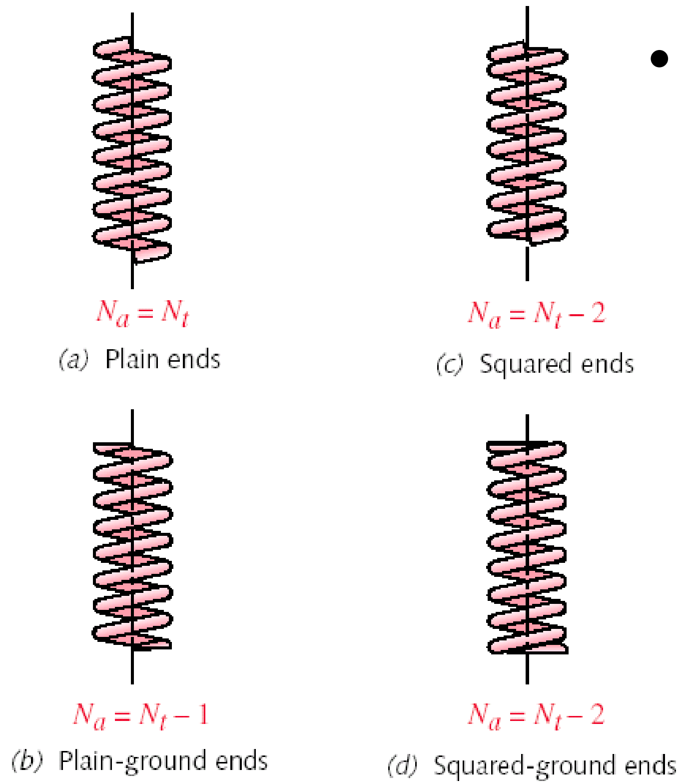


FIGURE 13-9

Four Styles of End-Coil Treatments for Helical Compression Springs

- End details affect active coils
  - Plain ends
  - Squared ends
  - Squared
  - Ground

Number of active coils

Figure 10–2

Types of ends for compression springs: (*a*) both ends plain; (*b*) both ends squared; (*c*) both ends squared and ground; (*d*) both ends plain and ground.

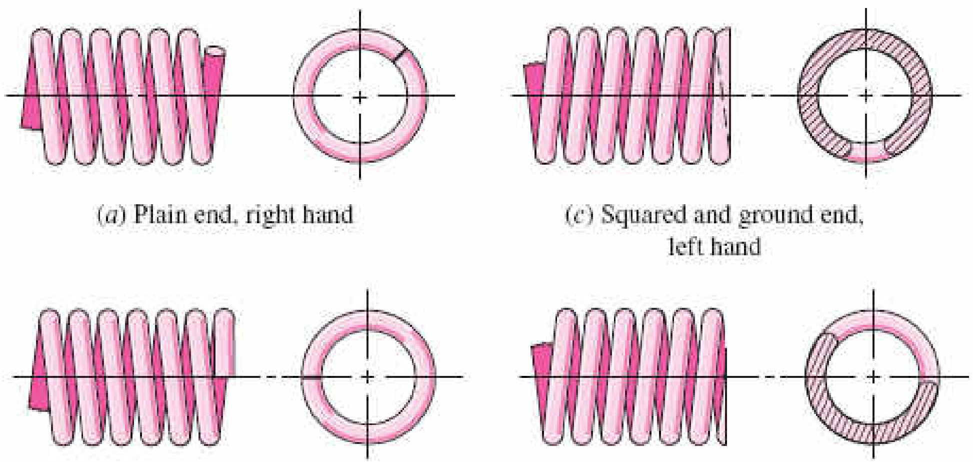


Table 10–1

Formulas for the Dimensional Characteristics of Compression-Springs. (N_a = Number of Active Coils)

Source: From *Design Handbook*, 1987, p. 32. Courtesy of Associated Spring.

	Type of Spring Ends			
Term	**Plain**	**Plain and Ground**	**Squared or Closed**	**Squared and Ground**
End coils, N_e	0	1	2	2
Total coils, N_t	N_a	$N_a + 1$	$N_a + 2$	$N_a + 2$
Free length, L_0	$pN_a + d$	$p(N_a + 1)$	$pN_a + 3d$	$pN_a + 2d$
Solid length, L_s	$d(N_t + 1)$	dN_t	$d(N_t + 1)$	dN_t
Pitch, p	$(L_0 - d)/N_a$	$L_0/(N_a + 1)$	$(L_0 - 3d)/N_a$	$(L_0 - 2d)/N_a$

Stability of a column

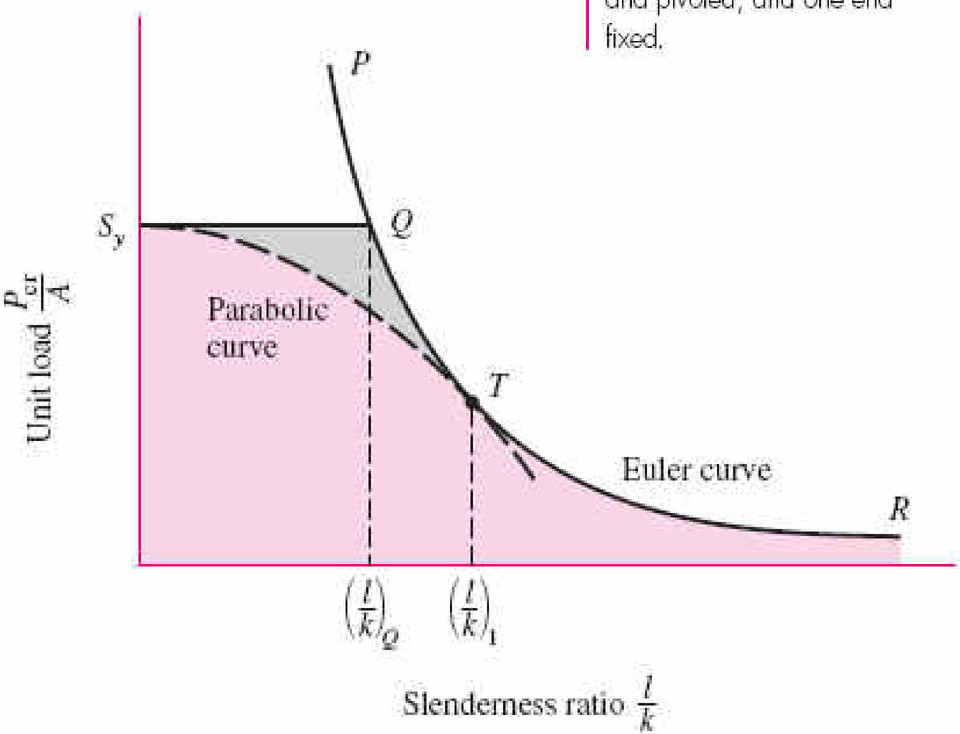
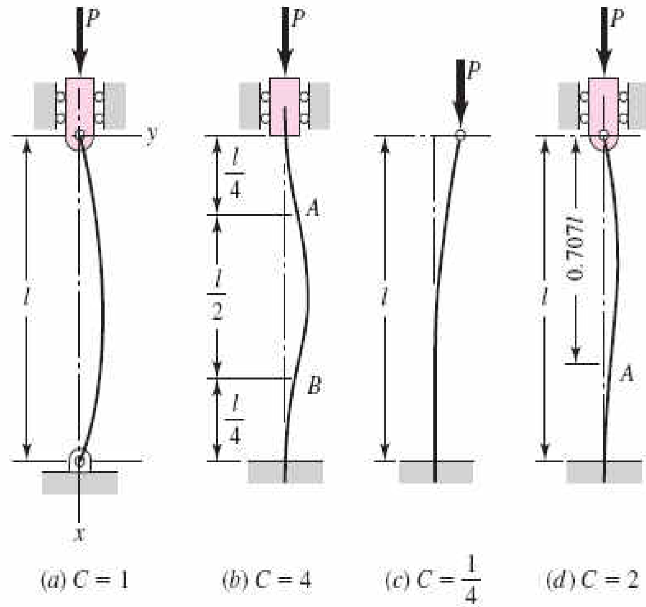


Figure 4–18

(*a*) Both ends rounded or pivoted; (*b*) both ends fixed; (*c*) one end free and one end fixed; (*d*) one end rounded and pivoted, and one end fixed.



Euler Formula

$$P_{cr} = \frac{C\pi^2 EI}{l^2}$$

Stability of a spring

- We know a column will buckle when the load is too large
- A compression coil spring will also buckle

$$y_{cr} = L_0 C_1' \left[1 - \left(1 - \frac{C_2'}{\lambda_{eff}^2} \right)^{1/2} \right]$$

- y_{cr} is the deflection corresponding to onset of instability

Deflection cont.

λ_{eff} Is called the effective slenderness ratio

$$\lambda_{\text{eff}} = \frac{\alpha L_0}{D}$$

Alpha = end condition constant

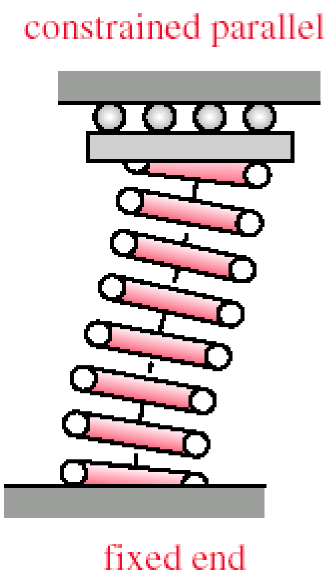
Lo is the spring length

D is the Coil diameter

C_1' and C_2' are elastic constants defined by the equations

$$C_1' = \frac{E}{2(E - G)}$$

$$C_2' = \frac{2\pi^2(E - G)}{2G + E}$$



(b) Parallel ends

FIGURE 13-13

End Conditions Determine Critical Buckling Situation

Adapted from Reference 1

Instability cont.

- End constraint alpha given by

Table 10–2

End-Condition Constants α for Helical Compression Springs*

End Condition	Constant α
Spring supported between flat parallel surfaces (fixed ends)	0.5
One end supported by flat surface perpendicular to spring axis (fixed); other end pivoted (hinged)	0.707
Both ends pivoted (hinged)	1
One end clamped; other end free	2

*Ends supported by flat surfaces must be squared and ground.

end-condition constant α.

Instability cont.

- For absolute stability

$$L_0 < \frac{\pi D}{\alpha}\left[\frac{2(E-G)}{2G+E}\right]^{1/2}$$

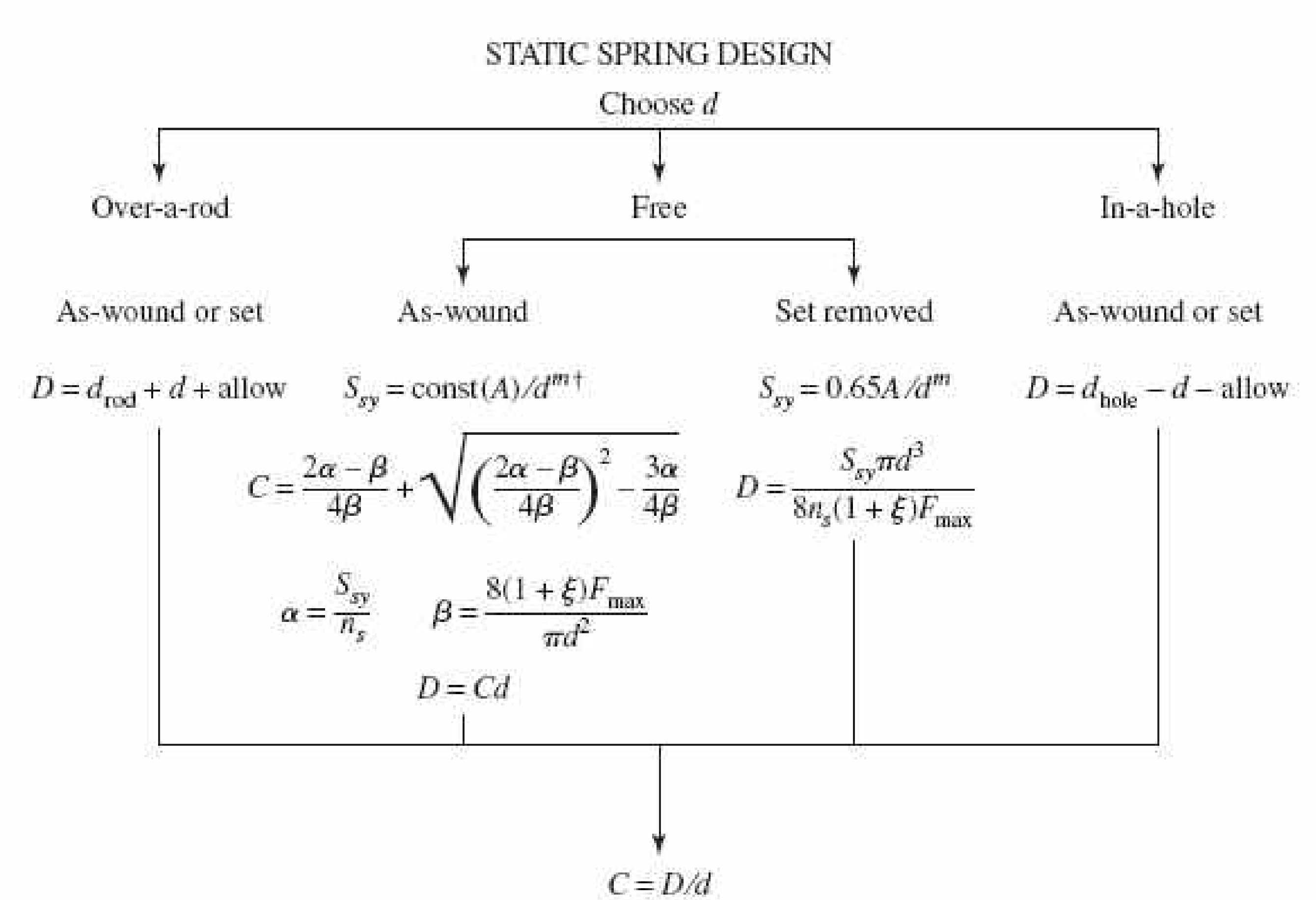
- For steels it turns out

$$L_0 < 2.63\frac{D}{\alpha}$$

- For square and ground ends

$$\alpha = 0.5 \text{ and } L_0 < 5.26D.$$

Static design flow chart



Flow chart cont.

$$K_B = (4C + 2)/(4C - 3)$$

$$\tau_s = K_B 8(1 + \xi)F_{\max} D/(\pi d^3)$$

$$n_s = S_{sy}/\tau_s$$

$$\mathrm{OD} = D + d$$

$$\mathrm{ID} = D - d$$

$$N_a = G d^4 y_{\max}/(8D^3 F_{\max})$$

N_t: Table 10–1

L_s: Table 10–1

L_O: Table 10–1

$$(L_O)_{cr} = 2.63D/\alpha$$

$$\mathrm{fom} = -(\mathrm{rel.\ cost})\,\gamma \pi^2 d^2 N_t D/4$$

Recommended design conditions

$$F_s = (1+\xi)F_{max} \qquad \xi = 1/7 = 0.143 \doteq 0.15.$$

$$4 \le C \le 12$$

$$3 \le N_a \le 15$$

$$\xi \ge 0.15$$

$$n_s \ge 1.2$$

Figure of merit (fom)

$$\text{fom} = -(\text{relative material cost})\frac{\gamma \pi^2 d^2 N_t D}{4}$$

Materials for springs

- Yield strength for static loading
  - Depends on set
  - Before set removed use Wahl factor
  - After set removed no stress concentration

Table 13-6 Maximum Torsional Yield Strength S_{ys} for Helical Compression Springs in Static Applications

Bending or Buckling Stresses Not Included. Source: Adapted from Ref. 1

Material	Maximum Percent of Ultimate Tensile Strength	
	Before Set Removed (Use Eq. 13.9*b*)	After Set Removed (Use Eq. 13.8*b*)
Cold-drawn carbon steel (e.g., A227, A228)	45%	60–70%
Hardened and tempered carbon and low-alloy steel (e.g., A229, A230, A232, A401)	50	65–75
Austenitic stainless steel (e.g., A313)	35	55–65
Nonferrous alloys (e.g., B134, B159, B197)	35	55–65

Properties for fatigue

- Fatigue Strength
  - Torsion is relevant loading- could use von Mises stress
  - Materials testing specific to helical compression springs is available, however
  - Correct for temp. reliability environment

Table 13-7 Maximum Torsional Fatigue Strength S_{fw}' for Round-Wire Helical Compression Springs in Cyclic Applications (Stress Ratio, R = 0)

No Surging, Room Temperature, and Noncorrosive Environment. Source: Ref. 1

Fatigue Life (cycles)	Percent of Ultimate Tensile Strength			
	ASTM 228, Austenitic Stainless Steel and Nonferrous		ASTM A230 and A232	
	Unpeened	Peened	Unpeened	Peened
10^5	36%	42%	42%	49%
10^6	33	39	40	47
10^7	30	36	38	46

Properties - endurance

- Endurance Strength (steels) unlimited cycles
  - For high ultimate strengths, endurance limits max out at 45 kpsi (unpeened) and 67.5 kpsi (peened)
  - Small wires have high ultimate strength
  - Tests have been done specific to spring wire
    - Temperature may require compensation
    - Corrosion
    - Reliability

S-N and Modified Goodman

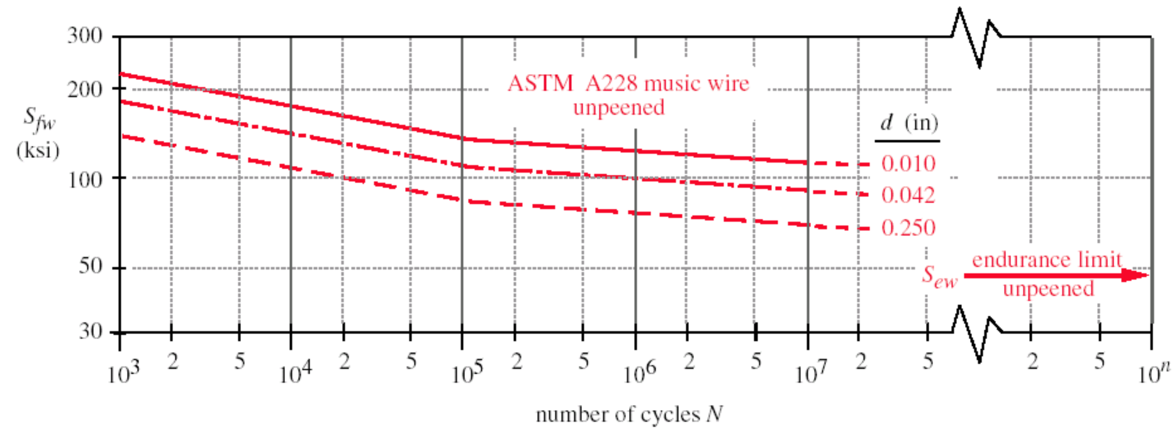


FIGURE 13-15

Torsional-Fatigue S-N Diagrams for Music Wire of Various Diameters

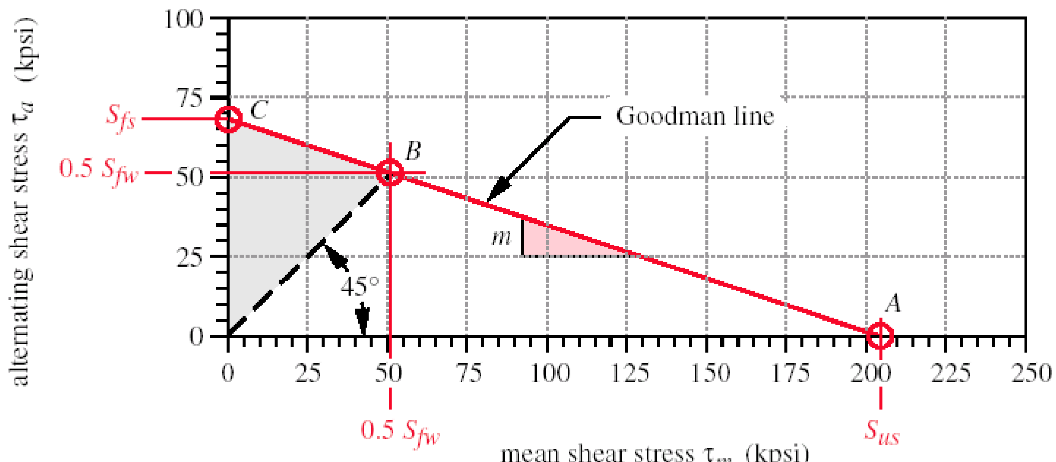


FIGURE 13-16

Torsional-Stress Modified-Goodman Diagram for 0.045-in Dia ASTM A228 Wire at N = 1E6 Cycles

Designing springs

Requirements

- Functionality
  - Stiffness
  - Lengths
  - Diameter
  - Forces
- Reliable operation
  - Static factor of safety
  - Fatigue factor of safety
  - Buckling and surge
- Manufacturability

Design Choices

- Index C
- Material
- Wire and coil diameter
- Number of turns
- End treatment and constraint
- Set and shot peen

Constraints (other)

- Bend radius

Helical extension spring

- Similar in most ways to compression springs
- Usually wound to be closed coil at zero force
- Thus a preload is required to stretch any, i.e. $y=k(F-F_i)$
- Spring hook is a source of failure in bending and torsion
- No set is used
- One coil not considered active

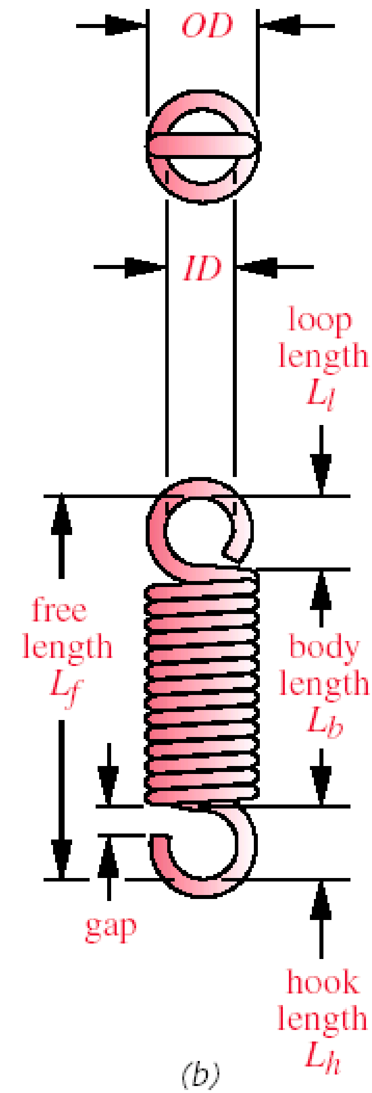


FIGURE 13-20

End stresses

Bending stress:

$$\sigma_A = K_b \frac{16DF}{\pi d^3} + \frac{4F}{\pi d^2}$$

$$K_b = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)}; \quad C_1 = \frac{2R_1}{d}$$

Torsional stress:

$$\tau_B = K_{w2} \frac{8DF}{\pi d^3}$$

$$K_{w2} = \frac{4C_2 - 1}{4C_2 - 4}; \quad C_2 = \frac{2R_2}{d}$$

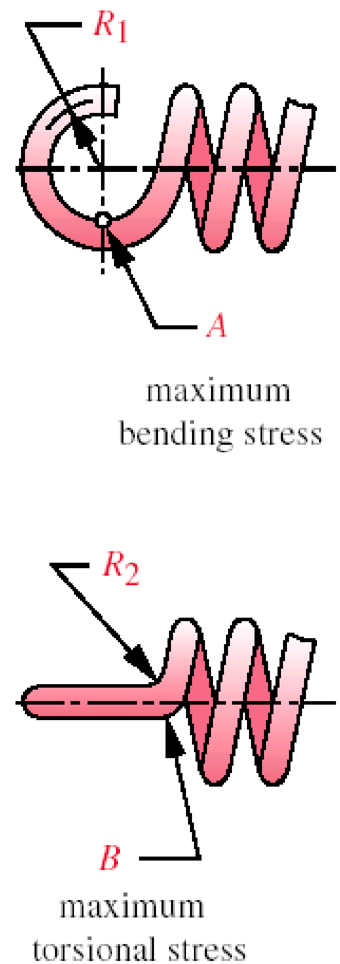


FIGURE 13-23

Points of Maximum Stress in Hook or Loop of an Extension Spring

Design for fatigue

- Data available for springs with loading from zero to some compresion value
- Application often has preload… how to use?
- First construct (or find) S-N curve
- Next construct Mod-Goodman chart
- Apply load line for given preload and design stress
- Find factor of safety to failure point

Goodman curve

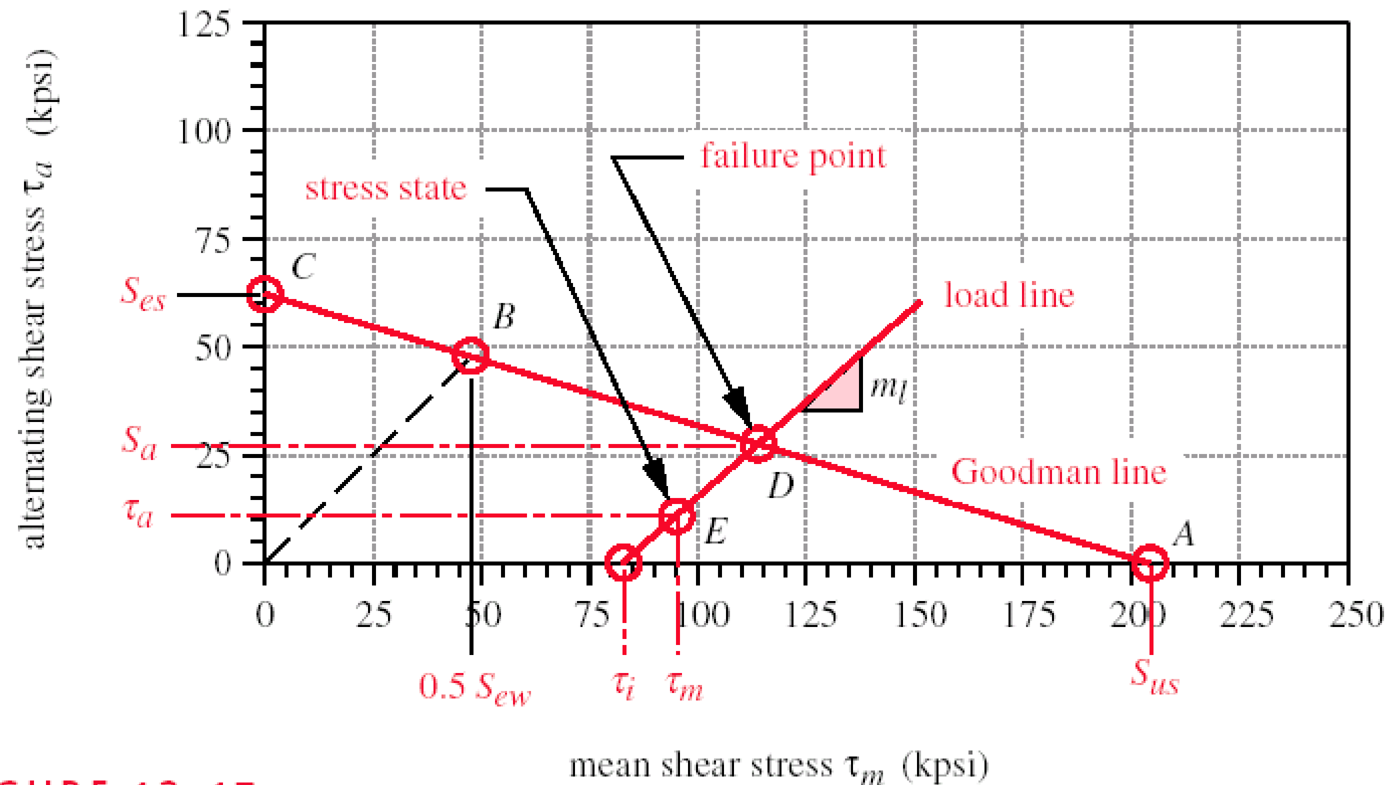


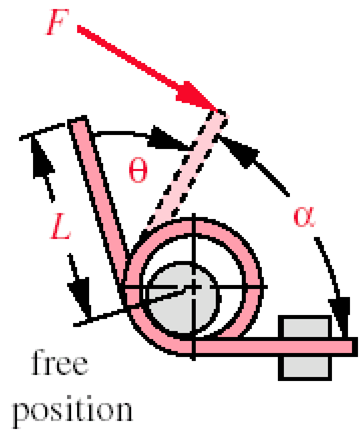
FIGURE 13-17

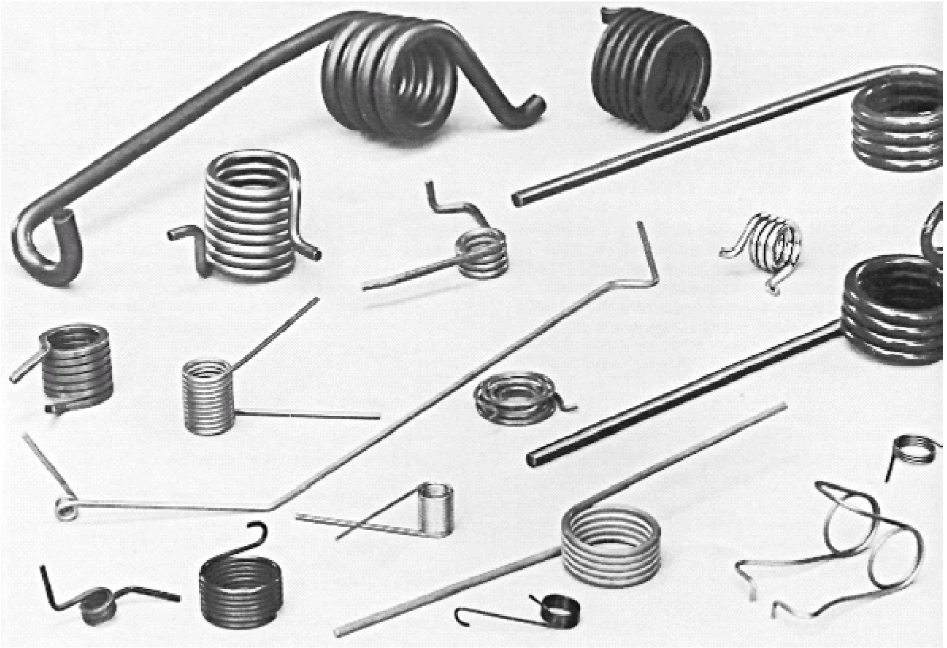
Modified-Goodman Diagram Showing Load Line and Data Needed for Safety-Factor Calculation of a Dynamically Loaded Compression Spring

A word about torsional springs

- The wire in a torsional spring is primarily in bending
- Spring constant is rotary $M=k\theta$
- Loading should act to wind up coil
- Design process resembles compression springs

Torsional





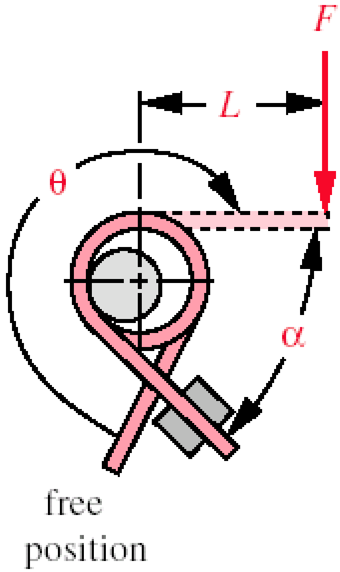


FIGURE 13-25

References

[Free strength of materials books –Craftsmanspace](https://www.craftsmanspace.com/free-books/strength-of-materials-books.html)

https://www.craftsmanspace.com/free-books/strength-of-materials-books.html

[Strength-of-material-by R K Bansal - PDF Drive](https://www.pdfdrive.com/strength-of-material-by-r-k-bansal-e22366752.html)

https://www.pdfdrive.com/strength-of-material-by-r-k-bansal-e22366752.html

Government Polytechnic Nanakpur

DEPARTMENT OF MECHANICAL ENGINEERING

E - NOTES

MECHANICAL ENGINEERING DRAWING

HOD/O.I. (Mechanical) : Er. **SHALANDER MOR**

Subject Teacher: Er. Amit Kumar

Semester: 3rd Sem

By: Er. Amit Kumar

Designed, Mechanical Editing & Developmental Editing

LEARNING OUTCOMES

After undergoing this subject, the students will be able to:

- Interpret different limits and fits of components
- Draw different kind of machine components like bearings, brackets, pulleys, pipe joints and lathe tool holder.
- Read and interpret drawings of mechanical components
- Interpret and draw the drawings of mechanical machine parts like jig, vices and screw jack
- Interpret and prepare the drawings of boiler and J.C. engine parts.
- Interpret gear terminology and draw spur gear teeth profile.

1

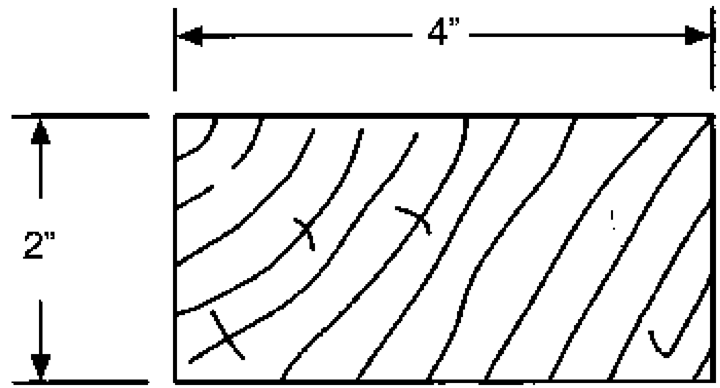
Limit, Fits and Tolerance

CONTENTS

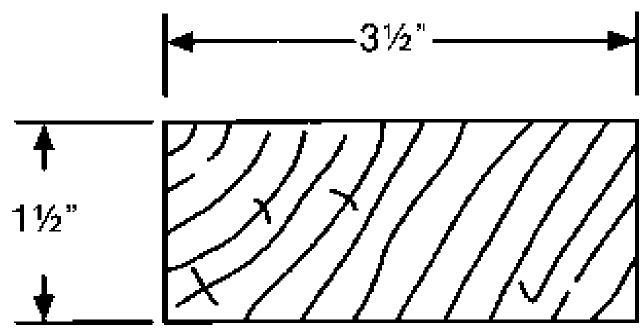
- LIMTS FITS AND TOLERANCES
- INSPECTION
- TYPES OF INSPECTION

TERMINOLOGY

- NOMINAL SIZE: It is the size of a part specified in the drawing.
- BASIC SIZE: It is the size of a part to which all limits of variation ar determined.
- ACTUAL SIZE: It is the actual measured dimension of a part. Nominal and basic size are often the same.



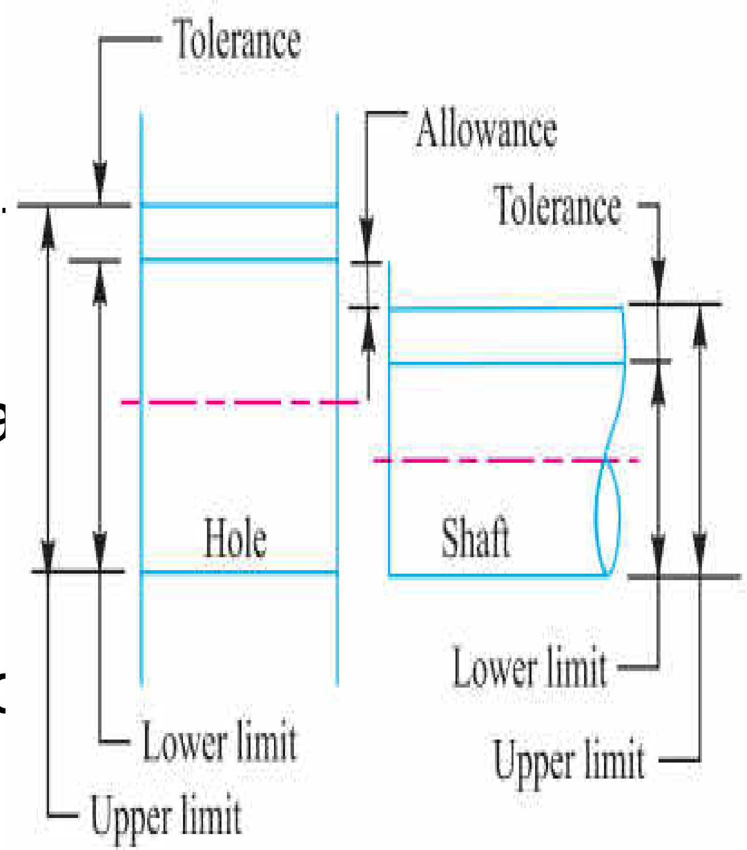
Nominal Size



Actual Size

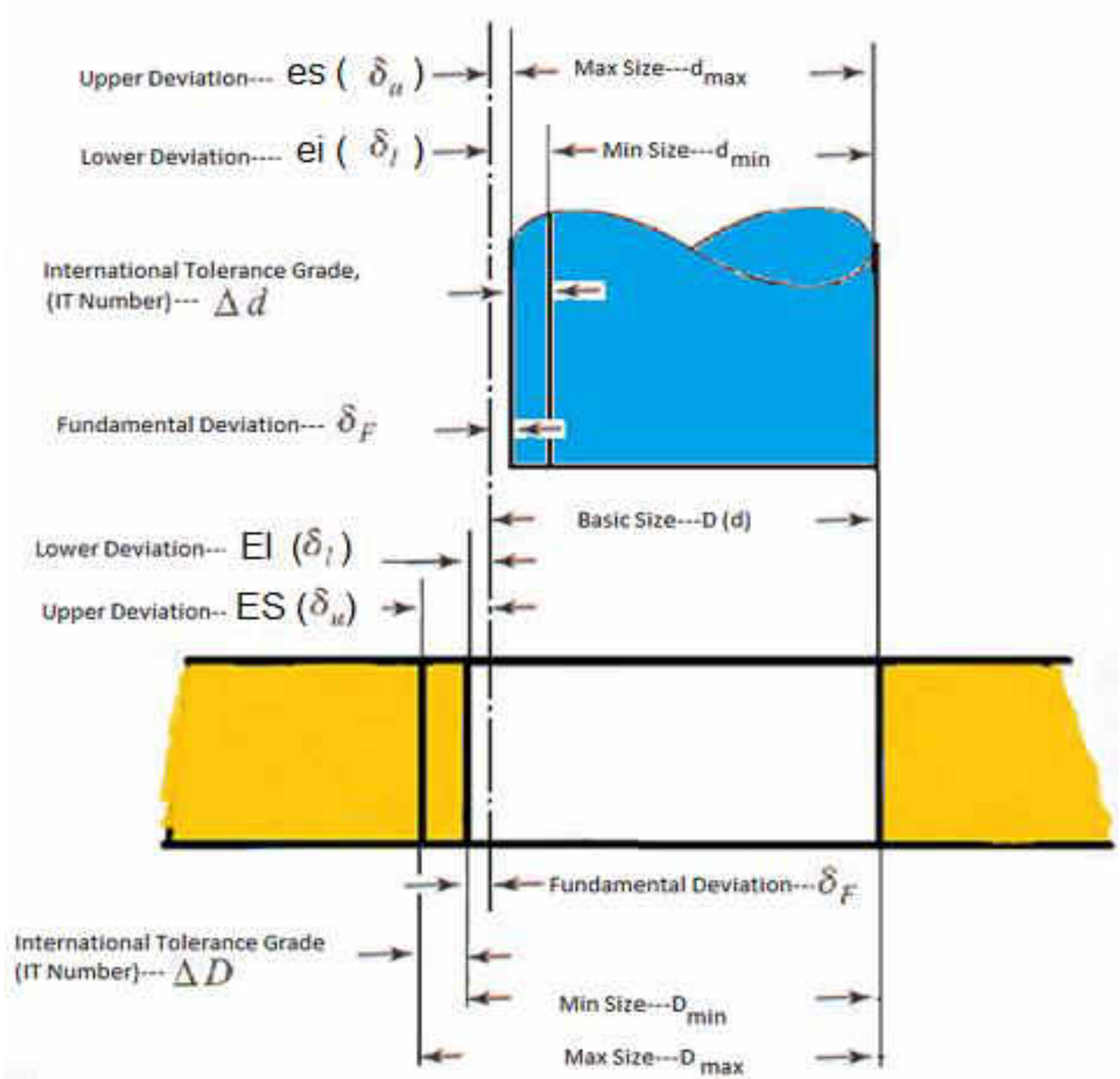
LIMIT OF SIZES

- There are two extreme possible sizes of a componen
- The largest permissible size for a component is called upper limit and smallest size is calle lower limit.



DEVIATION

- It is the algebraic difference between any given size and actual size.
- ACTUAL DEVIATION: It is the algebraic difference between the actual size and the basic size.

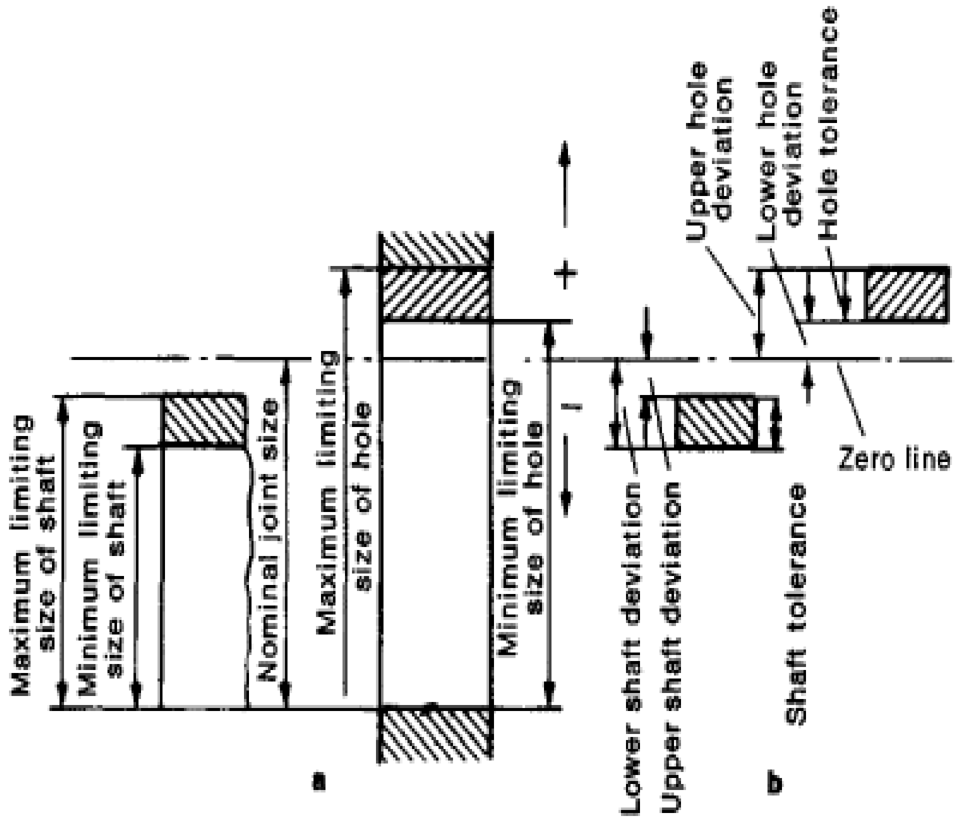


DEVIATION

- LOWER DEVIATION: It is the algebraic difference between the minimum limit of size and the basic size.
- UPPER DEVIATION: It is the algebraic difference between the maximum limit and the basic size.

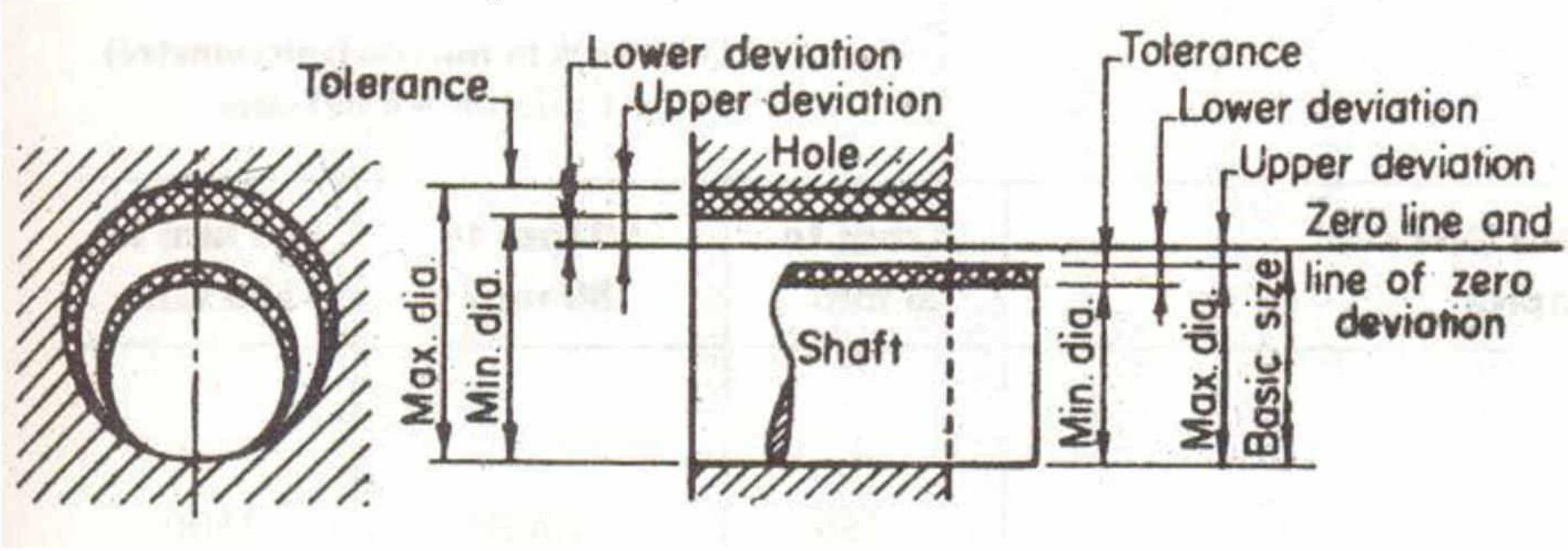
ZERO LINE

- It is the straight line corresponding to the basic size. The deviations are measured from this line.



Tolerance

- A tolerance is the total permissible variation from the specified basic size of the part.



NATURAL VARIABILITY OF PROCESS

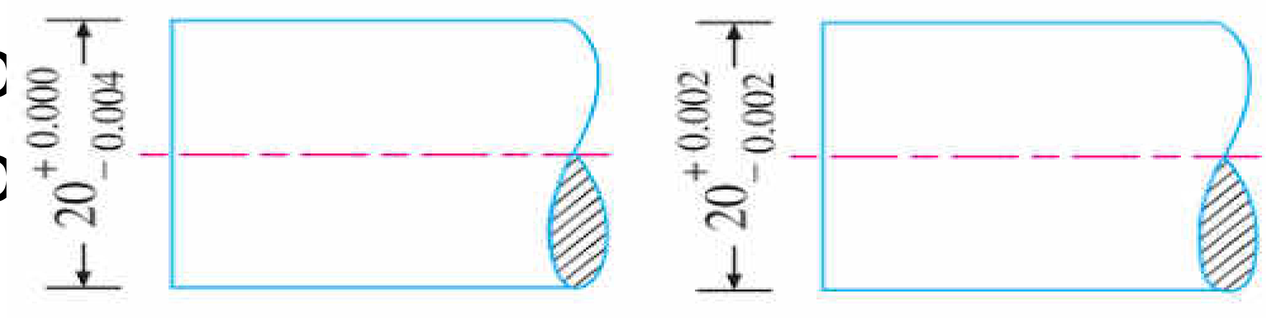
- It is the variation occurred in the size due to natural conditions like variations in material, environmental fluctuations, vibrations, human variability etc. It is an unavoidable process.

POSITIONAL TOLERANCES

- Two types of positional tolerances are used:

1. Unilateral tolerances
2. Bilateral tolerances

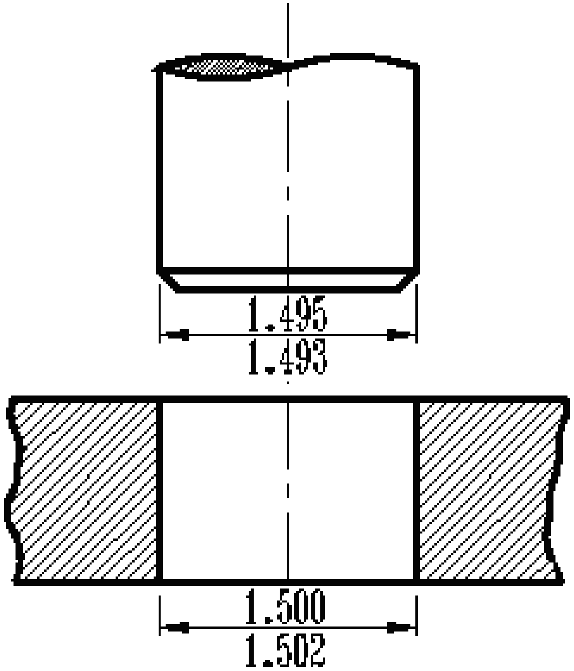
- When tolerance is on one side of basic size, it is called unilateral and if it is both in
p
b



(*a*) Unilateral tolerance (*b*) Bilateral tolerance.

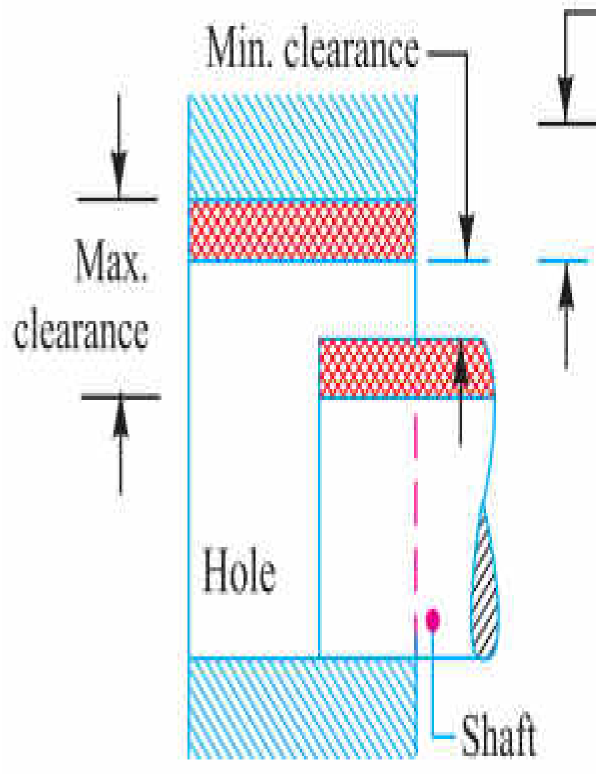
FITS

- The degree of tightness or looseness between two mating parts is called a fit.



TYPES OF FITS

- CLEARANCE FIT: There is a clearance or looseness in this type of fits. These fits maybe slide fit, easy sliding fit, running fit etc.



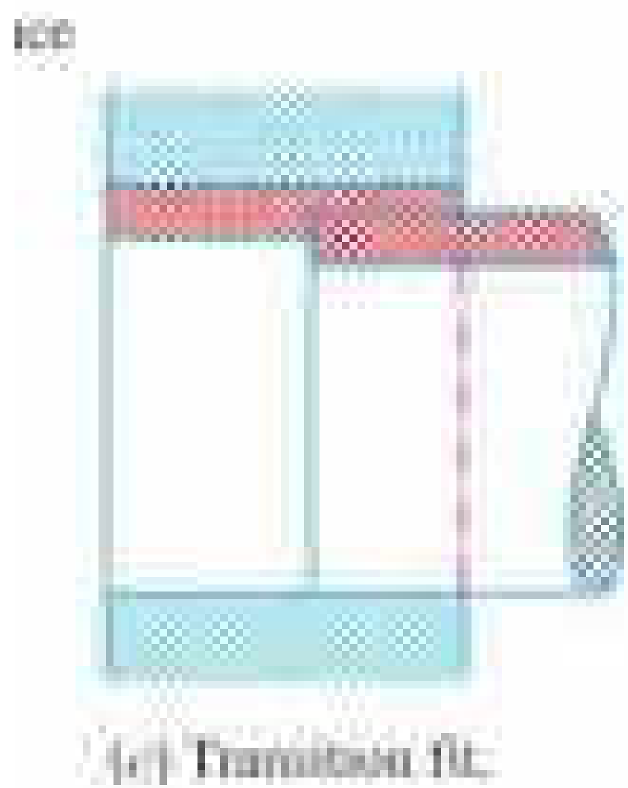
(*a*) Clearance fit.

TYPES OF FITS

- INTERFERENCE FIT: There is an interference or tightness in these type of fits. E.g. shrink fit, heavy drive fit etc.

TYPES OF FITS

- TRANSITION FIT: In this type of fit, the limits for the mating parts are so selected that either a clearance or interference may occur depending upon the actual size of the mating parts.



BASIS OF LIMIT SYSTEM

1. HOLE BASIS SYSTEM: In this system, the hole is kept as a constant member and different fits are obtained by varying the shaft size.

BASIS OF LIMIT SYSTEM

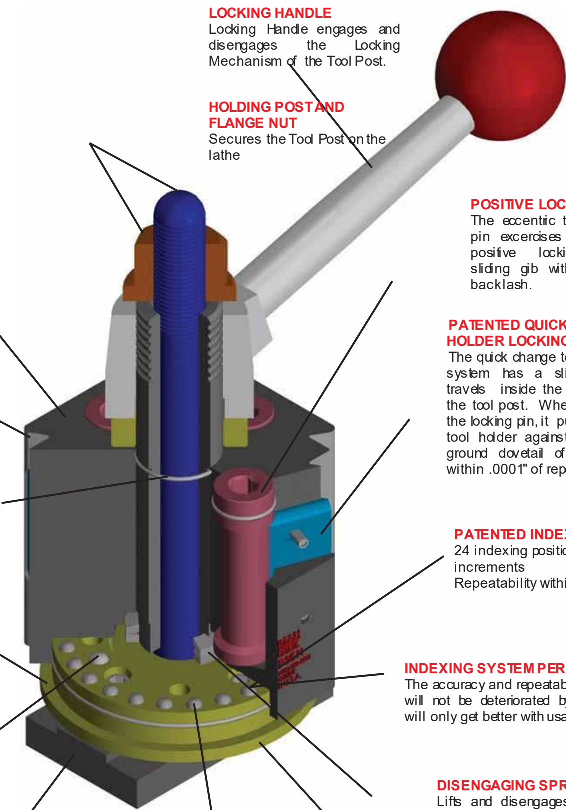
- SHAFT BASIS SYSTEM: In this system, the shaft is kept as constant member and different fits are obtained by varying the hole size.

DIFFERENT LIMIT SYSTEMS

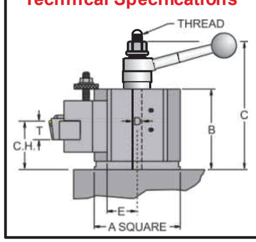
1. The Newall system
2. British Standard system
3. International Federation of National Standardization Association (ISA) system
4. ISO system
5. ISI system

Lathe Tool Holder

Anatomy of THE NEW QUADRA® Index Tool Post



Technical Specifications



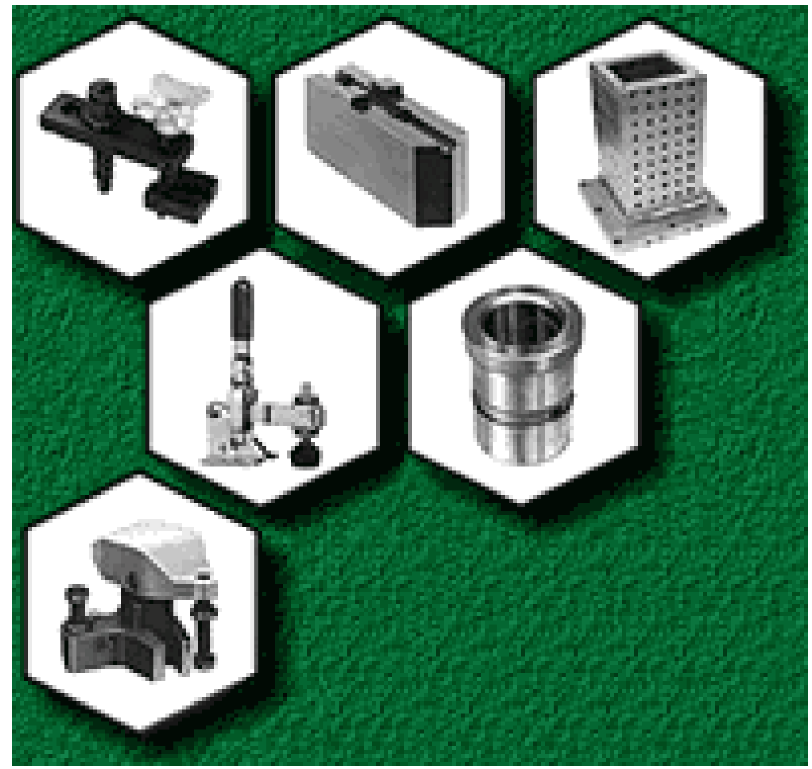
Description	QITP25N	QITP30N	QITP35N	QITP40N	QITP50N	QITP60N
Part No. 733101-	00000	00002	00004	00006	00008	00010
Lathe Swing Over Bed	in mm UP-12" 300,0	in mm 13-15" 320,0	in mm 14-17" 400,0	in mm 16-20" 450,0	in mm 17-32" 500,0	in mm 25-XHD XHD
A	2.500 63,5	3.000 76,2	3.500 88,9	4.000 101,6	5.000 127,0	6.000 152,4
B	2.710 68,8	3.205 81,4	3.460 87,9	4.070 103,4	5.230 132,8	5.615 142,6
C	5.210 132,3	5.720 145,3	6.415 162,9	7.525 191,1	9.135 232,0	9.855 250,3
D	0.500	0.500	0.625 16,0	0.750 19,0	1.000	1.125

Tool Post supplied standard with blank T-nut. For a machined T-nut, at additional cost, please fill out the "Toolpost Mounting Data" sheet.

Drilling Jig

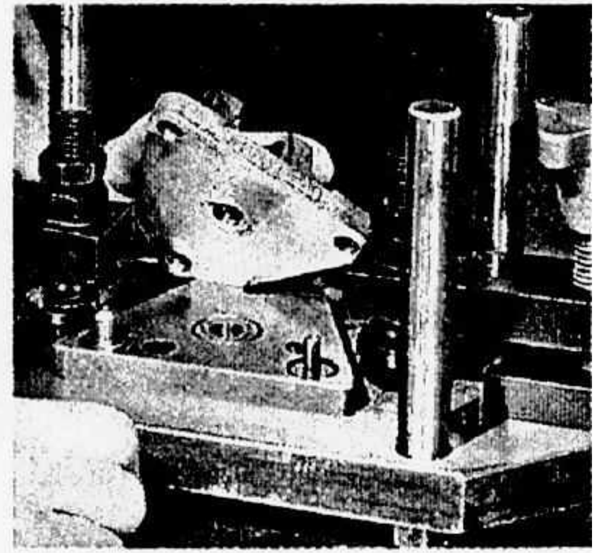
What are Jigs and Fixtures

- Anything used to hold a work piece in a desired location
  - Locate parts for precision
  - Repeating process on a series of parts
  - Holding parts for machining, painting, assembly

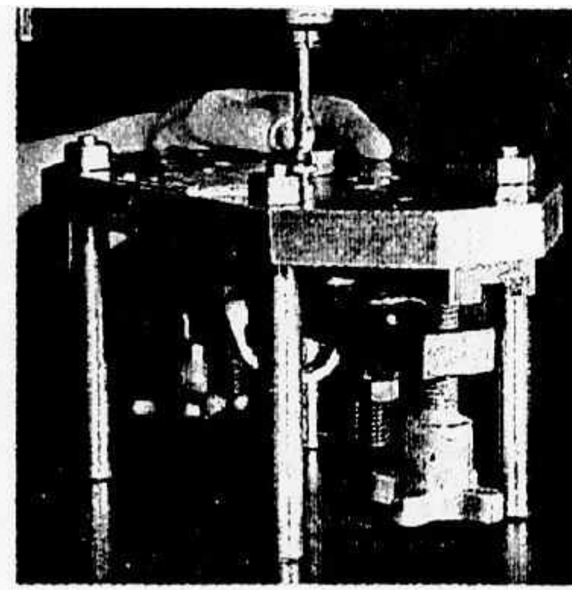


Two main types of jigs:

- *For machining purposes*
  - Locates the component, holds it firmly in place, and guides the cutting tool.
- *For assembly purposes*
  - Locates separate component parts and holds them rigidly in their correct positions while they are being connected.



(A) LOADING WORKPIECE

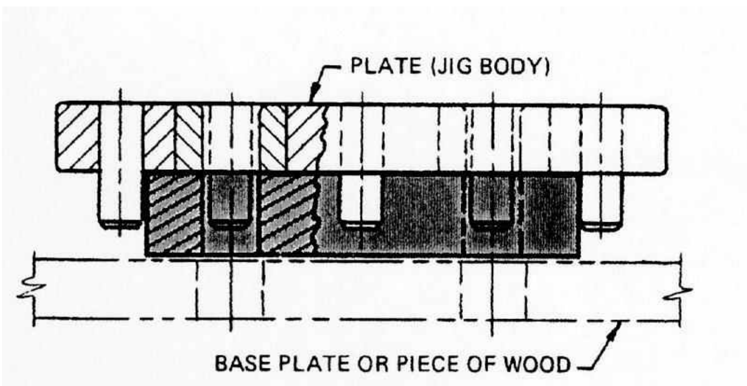


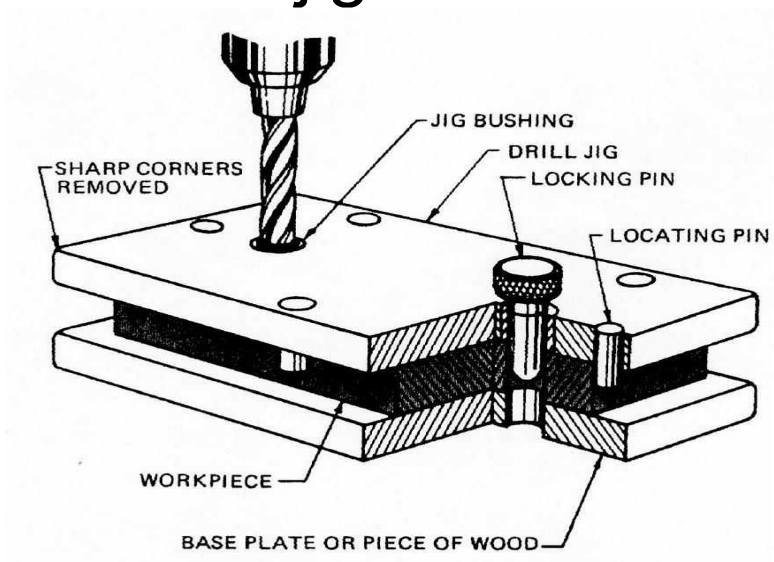
(B) TAPPING WORKPIECE

Northwestern Tools

Drill jig terms

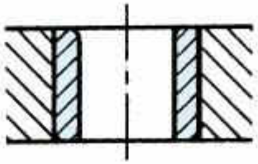
- *Open jig (also called plate jig or drill template)*
  - The simplest type of drill jig
  - Consists of a plate with holes to guide the drills, and may have locating pins that locate the workpiece on the jig



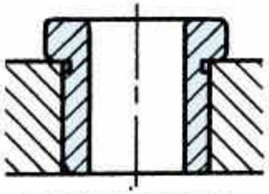


Drill jig terms

- *Drill bushings*
  - Precision tools that guide cutting tools such as drill and reamers into precise locations in a workpiece.



(A) HEADLESS

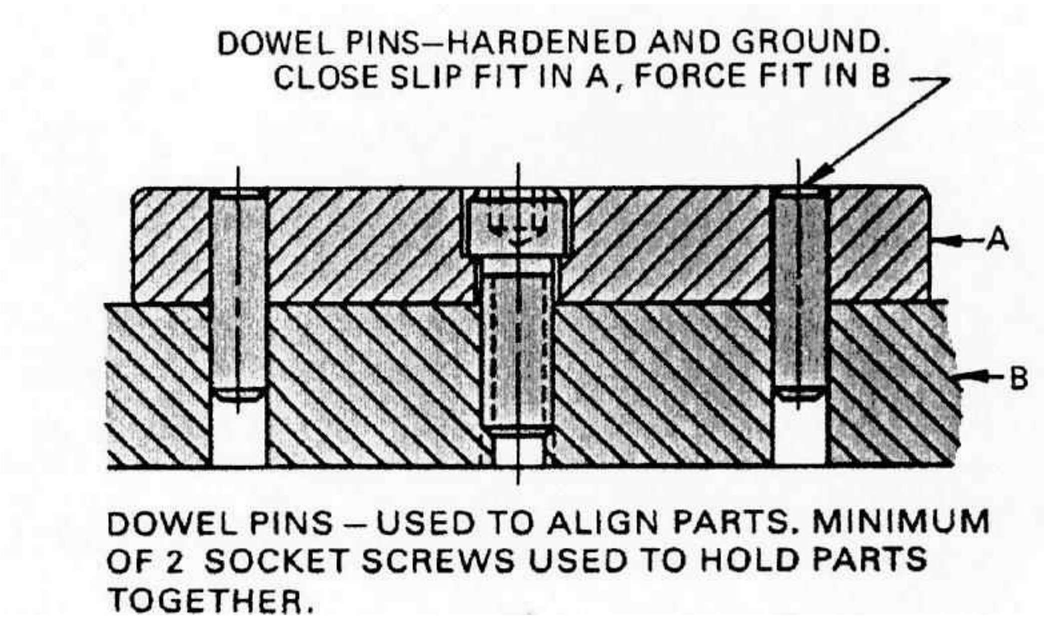


(B) HEAD TYPE

Accurate Bushing Co.

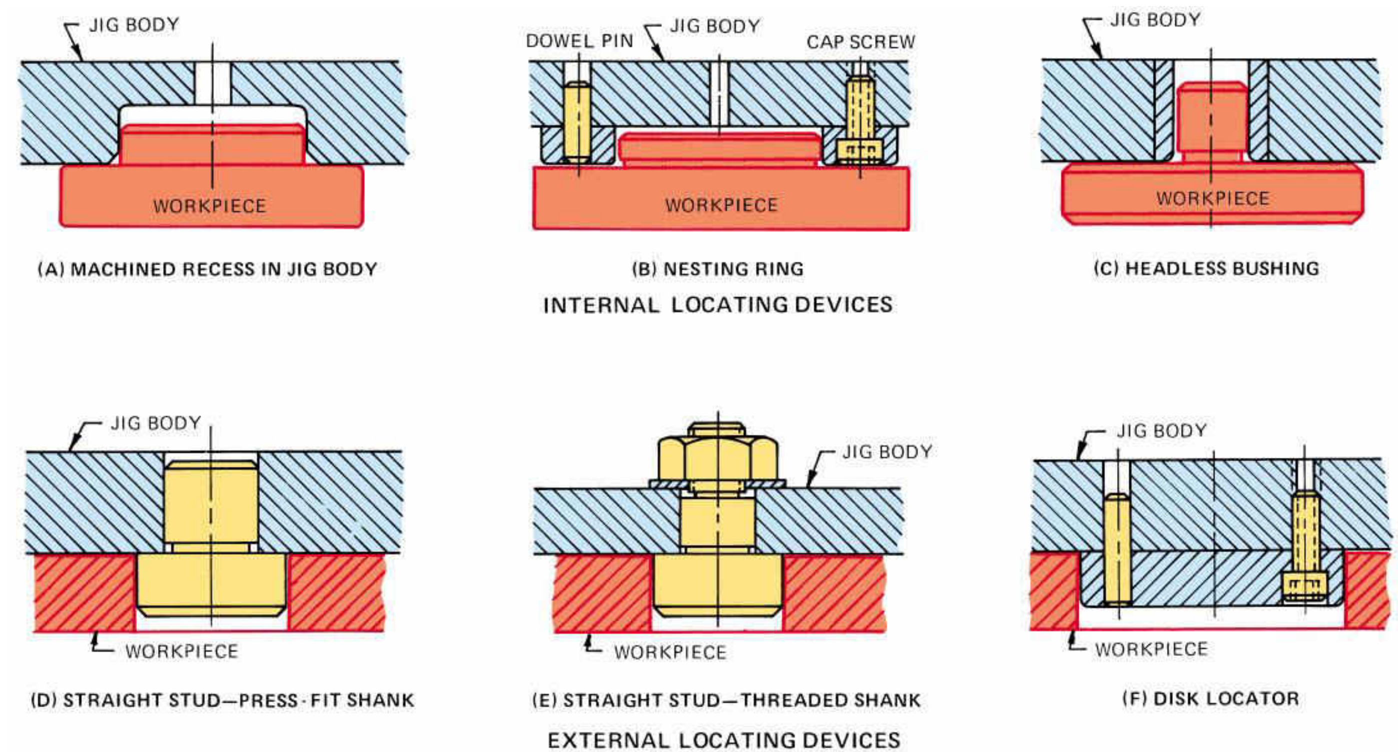
Drill jig components

- *Jig body*
  - Holds the various parts of a jig assembly.
- *Cap screws and dowel pins*
  - Hold fabricated parts together



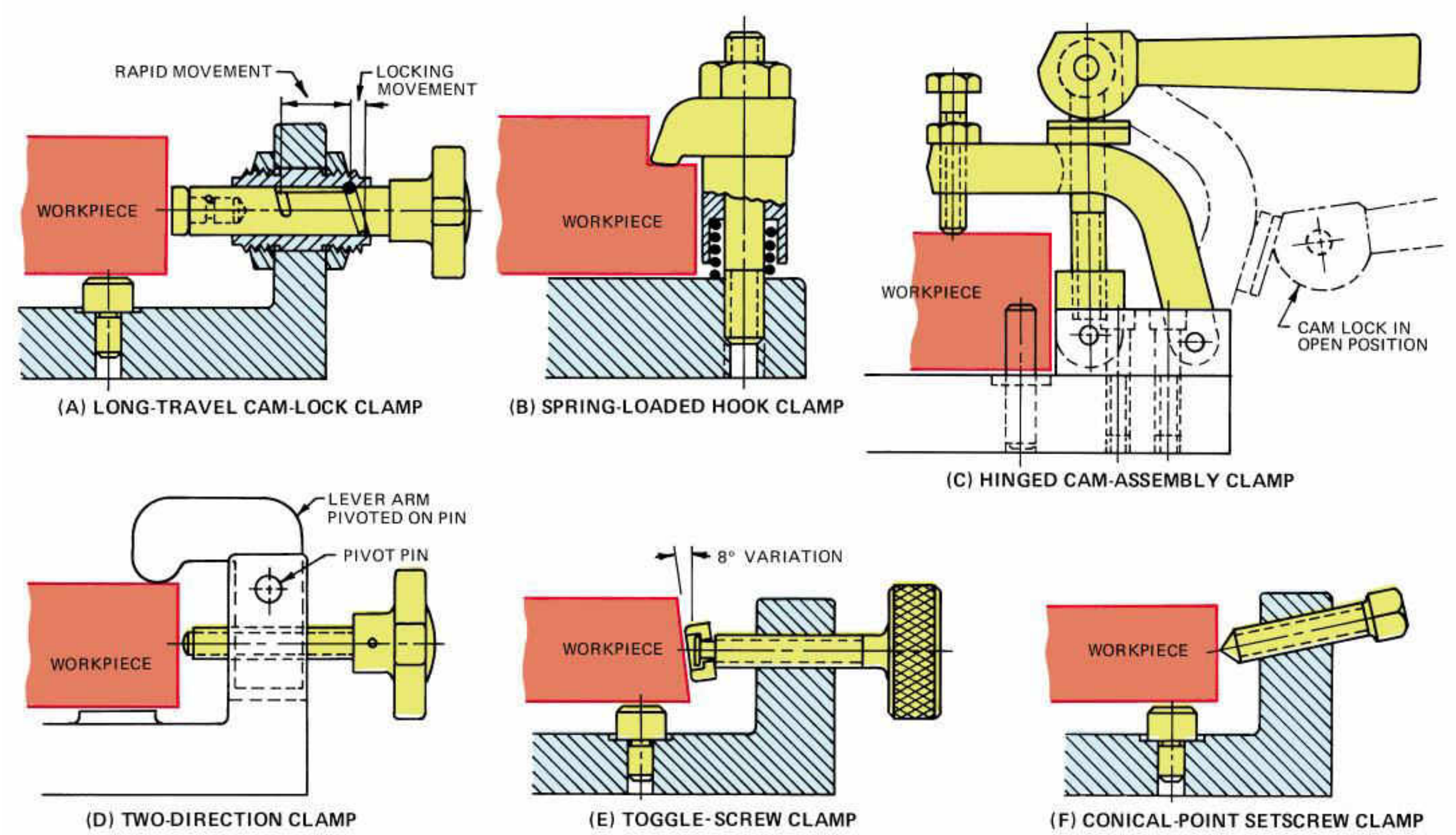
Drill jig components

- *Locating devices*
  - Pins, pads, and recesses used to locate the workpiece on the jig.



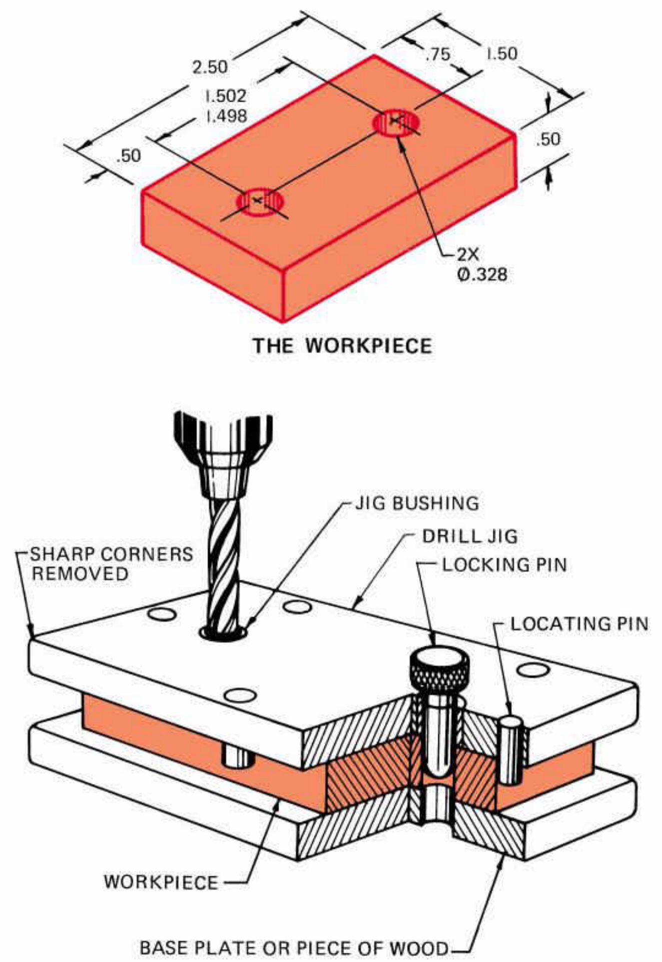
Drill jig components

- *Clamping devices*



Drill jig components

- *Locking pins*
  - Inserted to lock or hold the work piece securely to the jig plate while subsequent holes are being drilled.



(B) ADD LOCKING PIN BEFORE STARTING SECOND HOLE

Uses of Jig and fixture

- Reduce cost of production.
- Increase the production.
- To assure high accuracy of parts
- Provide for interchangeability
- Enable heavy and complex parts to machine
- Reduced quality control expenses.
- Increased versatility of machine tool.
- Less skilled labour.
- Saving labour.
- Partially automates the machine tools
- Use improve the safety, accidents low

Elements of Jig and fixture

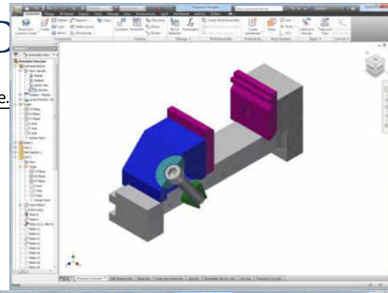
- Sufficiently rigid bodies (plate, box or frame structure
- Locating elements.
- Clamping elements.
- Tool guiding elements.
- Elements for positioning or fastening the jig or fixture.

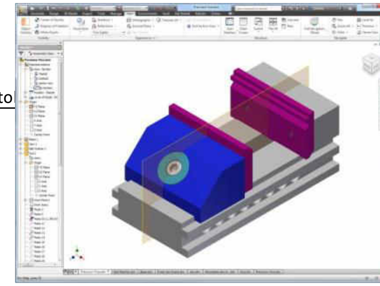
Machine Vice Assembly

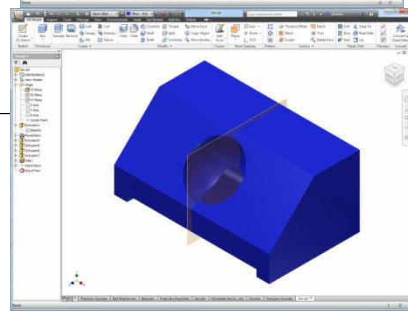
Machine Vice Models and

D

Re. . Berto





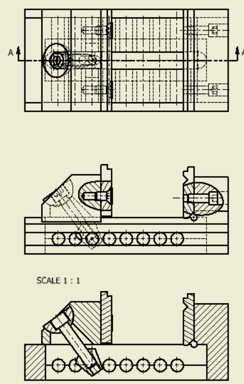


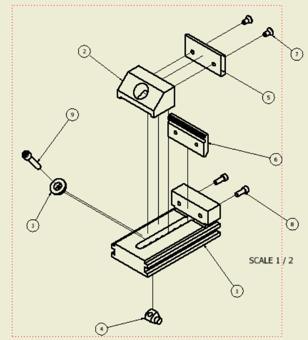
Modeling Considerations

1.Symmetry: Sketch Dims / Constraint

2.Centered Components: Constrain Origin Features

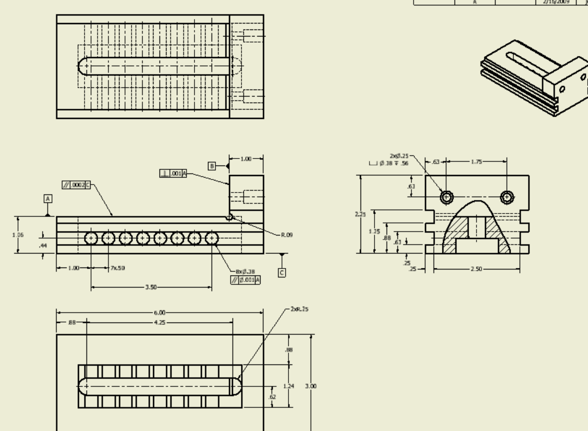
PARTS LIST				
ITEM	QTY	PART NUMBER	DESCRIPTION	MATERIAL
1	1	Base		Steel, Mild
2	1	Jaw		Steel, Mild
3	1	Ball Washer		Steel, Mild
4	1	Nut		Steel, Mild
5	1	Moveable Jaw Insert		Steel, High Strength Low Alloy
6	1	Fixed Jaw Insert		Steel, High Strength Low Alloy
7	2	ANSI B18.3 - 1/4-28 UNF x 0.5	Hexagon Socket Flat Countersunk Head Cap Screw	Steel, Mild
8	2	ANSI B18.3 1/4 28 UNF 0.625	Hexagon Socket Head Cap Screw	Steel, Mild
9	1	ANSI B18.3 - 5/16-24 UNF - 1.5 HS HCS	Hexagon Socket Head Cap Screw	Steel, Mild





DRAWN jshahan	2/16/2009	ISU - TSM 216
CHECKED		TITLE Working Drawings Project
QA		
MFG		
APPROVED		SIZE C / DWG NO Precision Vice / REV
		SCALE 1 : 1 / SHEET 1 OF 1

REVISION HISTORY				
ZONE	REV	DESCRIPTION	DATE	APPROVED
	A		2/16/2009	jcshahan



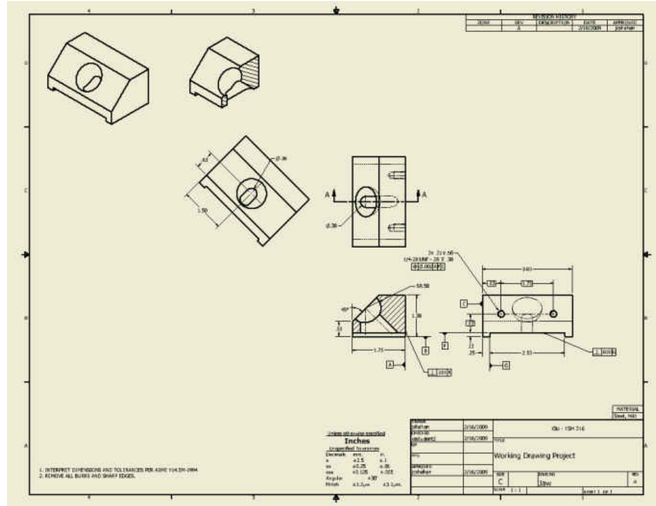
1. INTERPRET DIMENSIONS AND TOLERANCES PER ASME Y14.5M-1994
2. REMOVE ALL BURRS AND SHARP EDGES.

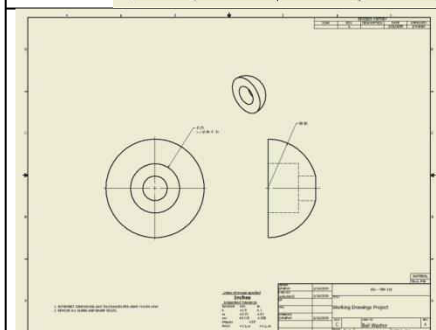
Unless otherwise specified
Inches
Unspecified Tolerances

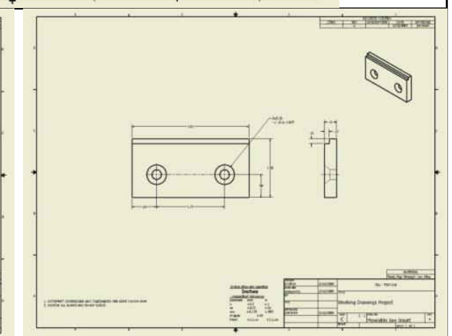
Decimals	mm.	in.
x	±2.5	±.1
xx	±0.25	±.01
xxx	±0.125	±.005
Angular	±30'	
Finish	±3.2μm	±3.2μIN.

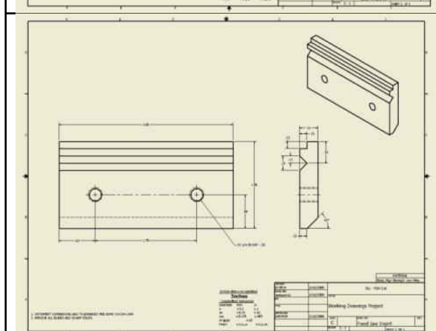
MATERIAL
Steel, Mild

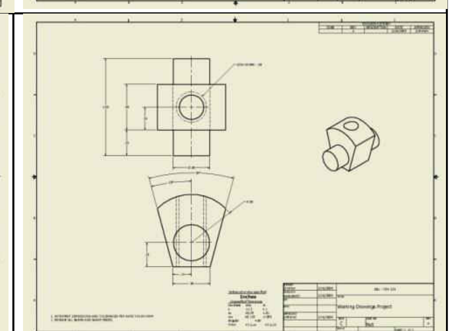
DRAWN jshahan	2/16/2009	ISU - TSM 216
CHECKED rastudent2	2/16/2009	TITLE Working Drawings Project
QA		
MFG		
APPROVED jcshahan	2/16/2009	SIZE C / DWG NO Base / REV A
		SCALE 1 : 1 / SHEET 1 OF 1


1. INTERPRET DIMENSIONS AND TOLERANCES PER ASME Y14.5M-1994
2. REMOVE ALL BURRS AND SHARP EDGES.
Inches
Working Drawing Project
2/16/2009
C
Jaw
MATERIAL

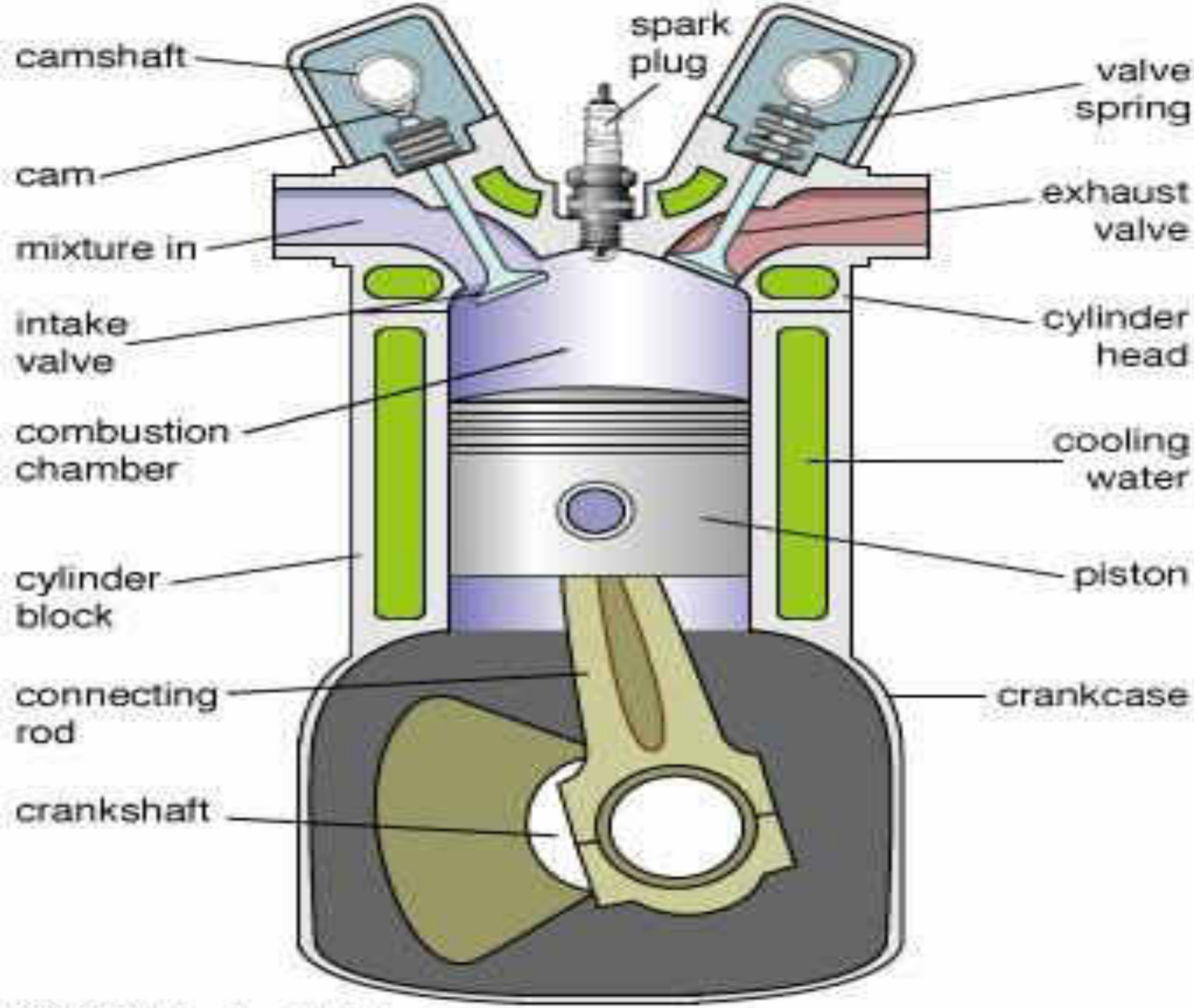

Inches
Working Drawings Project
Ball Washer


Inches
Working Drawings Project
Moveable Jaw Insert


Inches
Working Drawings Project
Fixed Jaw Insert


Inches
Working Drawings Project

I.C. Engine Parts


spark plug
camshaft
valve spring
cam
exhaust valve
mixture in
intake valve
cylinder head
combustion chamber
cooling water
piston
cylinder block
connecting rod
crankcase
crankshaft

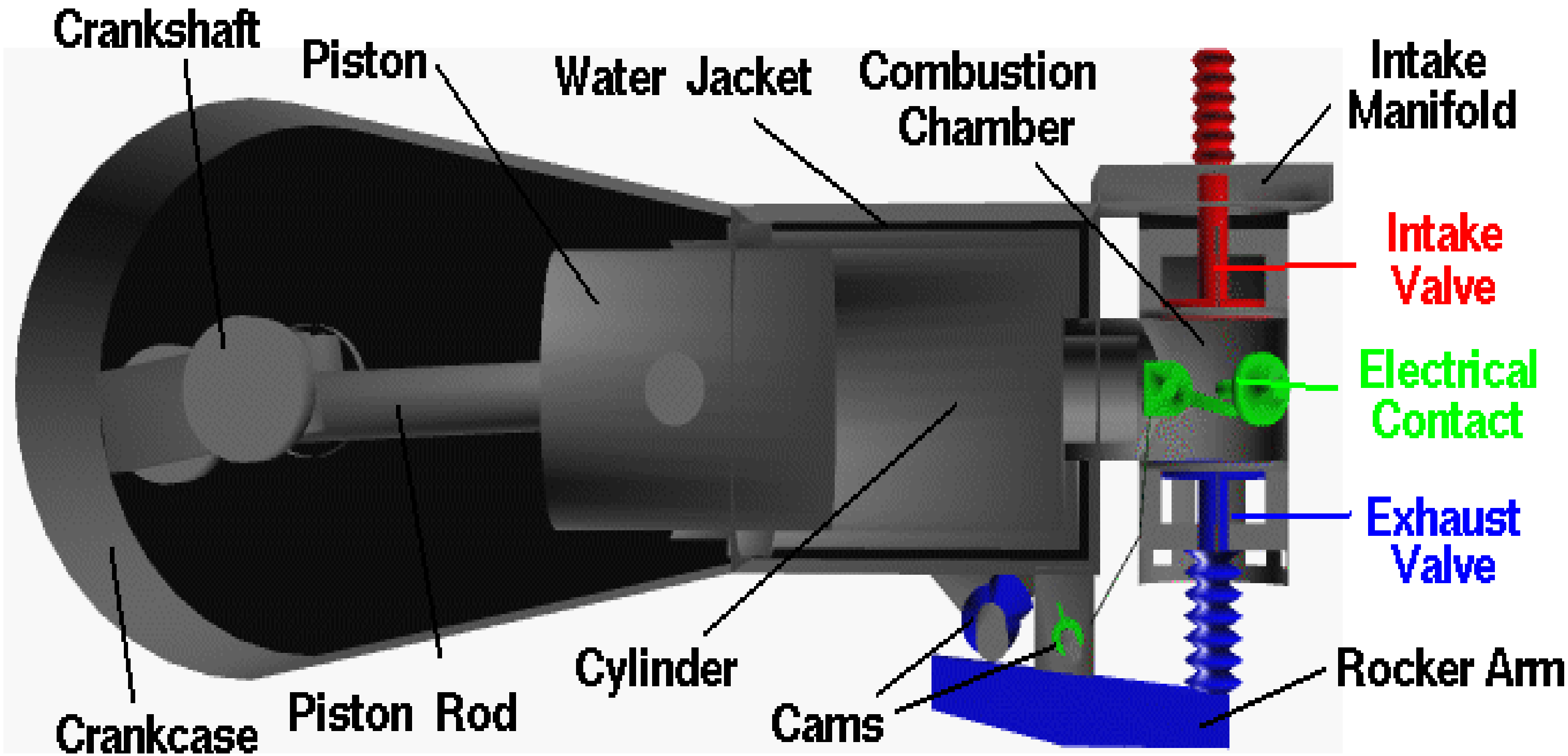


Figure2: Engine components

Internal combustion Engine Components:

- I.C. Engine components shown in figure1 and figure2 are defined as follows:
- **Block :** Body of the engine containing cylinders, made of cast iron or aluminium.
- **Cylinder :** The circular cylinders in the engine block in which the pistons reciprocate back and forth.
- **Head :** The piece which closes the end of the cylinders, usually containing part of the clearance volume of the combustion chamber.
- **Combustion chamber:** The end of the cylinder between the head and the piston face where combustion occurs.
  - The size of combustion chamber continuously changes from minimum volume when the piston is at TDC to a maximum volume when the piston at BDC.

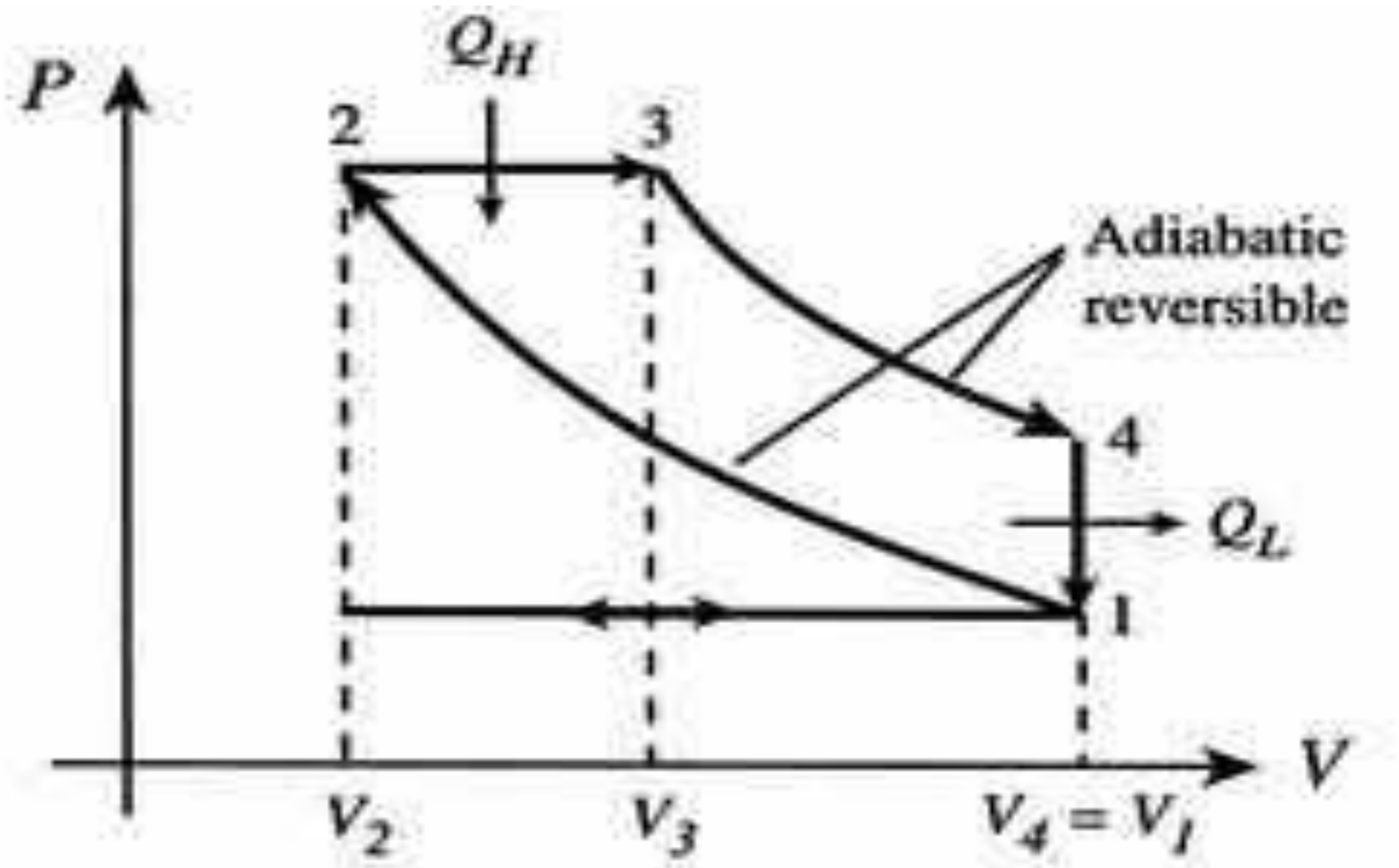
- **Crankshaft :** Rotating shaft through which engine work output is supplied to external systems.
  - The crankshaft is connected to the engine block with the main bearings.
  - It is rotated by the reciprocating pistons through the connecting rods connected to the crankshaft, offset from the axis of rotation. This offset is sometimes called crank throw or crank radius.
- **Connecting rod :** Rod connecting the piston with the rotating crankshaft, usually made of steel or alloy forging in most engines but may be aluminum in some small engines.
- **Piston rings:** Metal rings that fit into circumferential grooves around the piston and form a sliding surface against the cylinder walls.

- **Camshaft :** Rotating shaft used to push open valves at the proper time in the engine cycle, either directly or through mechanical or hydraulic linkage (push rods, rocker arms, tappets) .
- **Push rods :** The mechanical linkage between the camshaft and valves on overhead valve engines with the camshaft in the crankcase.
- **Crankcase :** Part of the engine block surrounding the crankshaft.
  - In many engines the oil pan makes up part of the crankcase housing.
- **Exhaust manifold :** Piping system which carries exhaust gases away from the engine cylinders, usually made of cast iron .

- **Intake manifold :** Piping system which delivers incoming air to the cylinders, usually made of cast metal, plastic, or composite material.
  - In most SI engines, fuel is added to the air in the intake manifold system either by fuel injectors or with a carburetor.
  - The individual pipe to a single cylinder is called runner.
- **Carburetor :** A device which meters the proper amount of fuel into the air flow by means of pressure differential.
  - For many decades it was the basic fuel metering system on all automobile (and other) engines.
- **Spark plug :** Electrical device used to initiate combustion in an SI engine by creating high voltage discharge across an electrode gap.

Compression Ignition Engine :

- We will deal with Compression Ignition engine.
- The ideal diesel cycle PV diagram is shown in following figure 8.



: Ideal diesel cycle P-V Diagram.

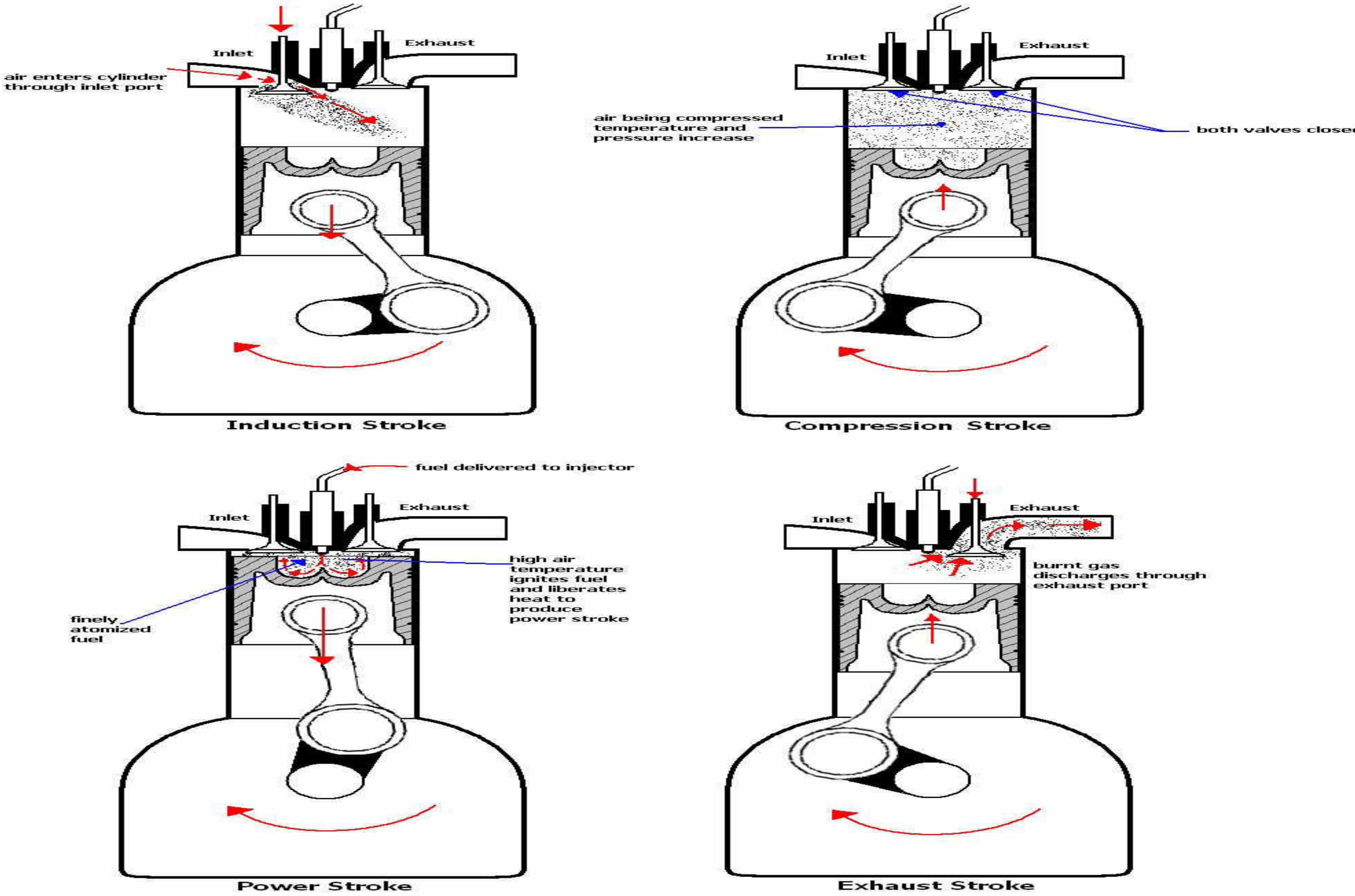
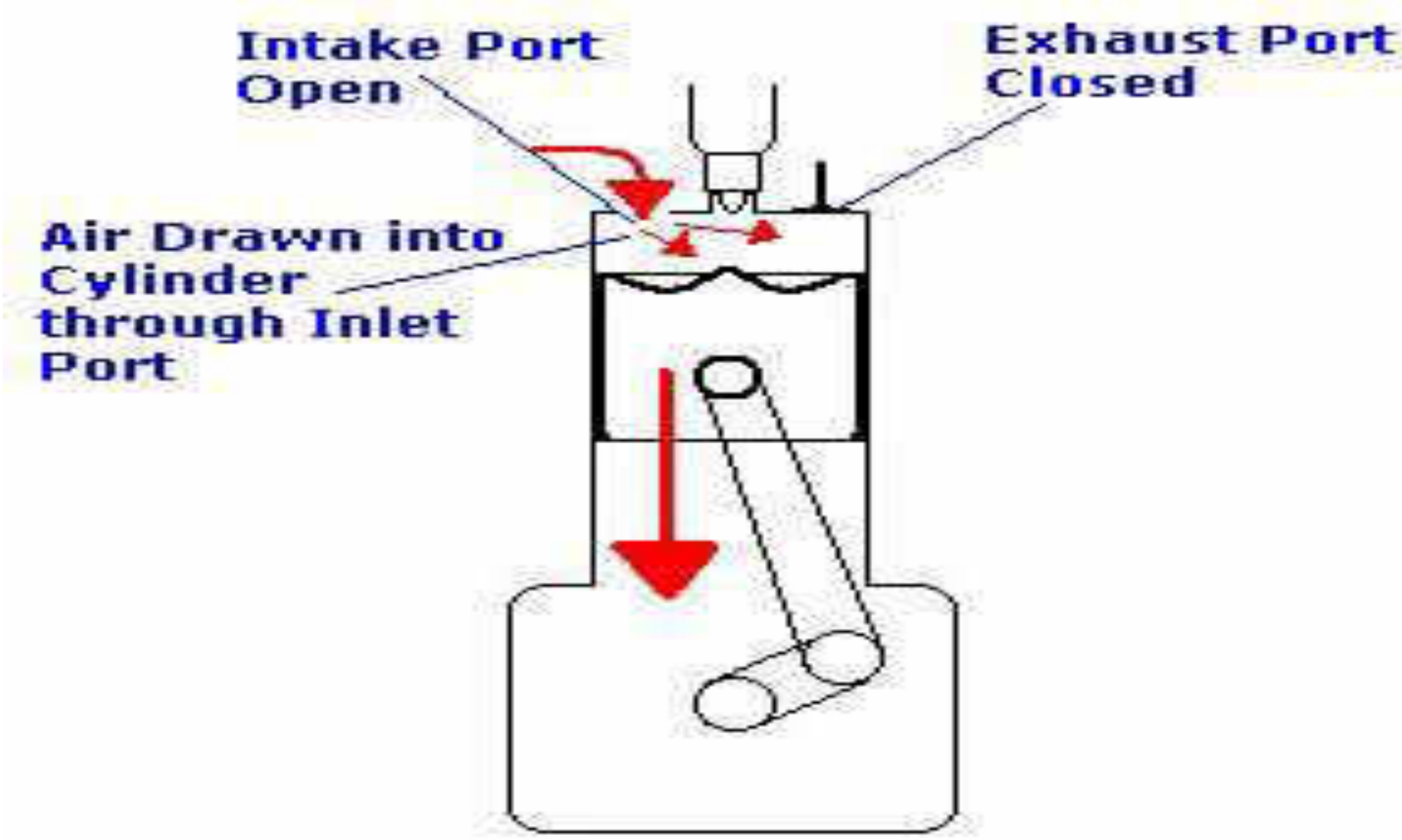


Figure9: Four strokes of ideal Diesel cycle.



:Suction stroke

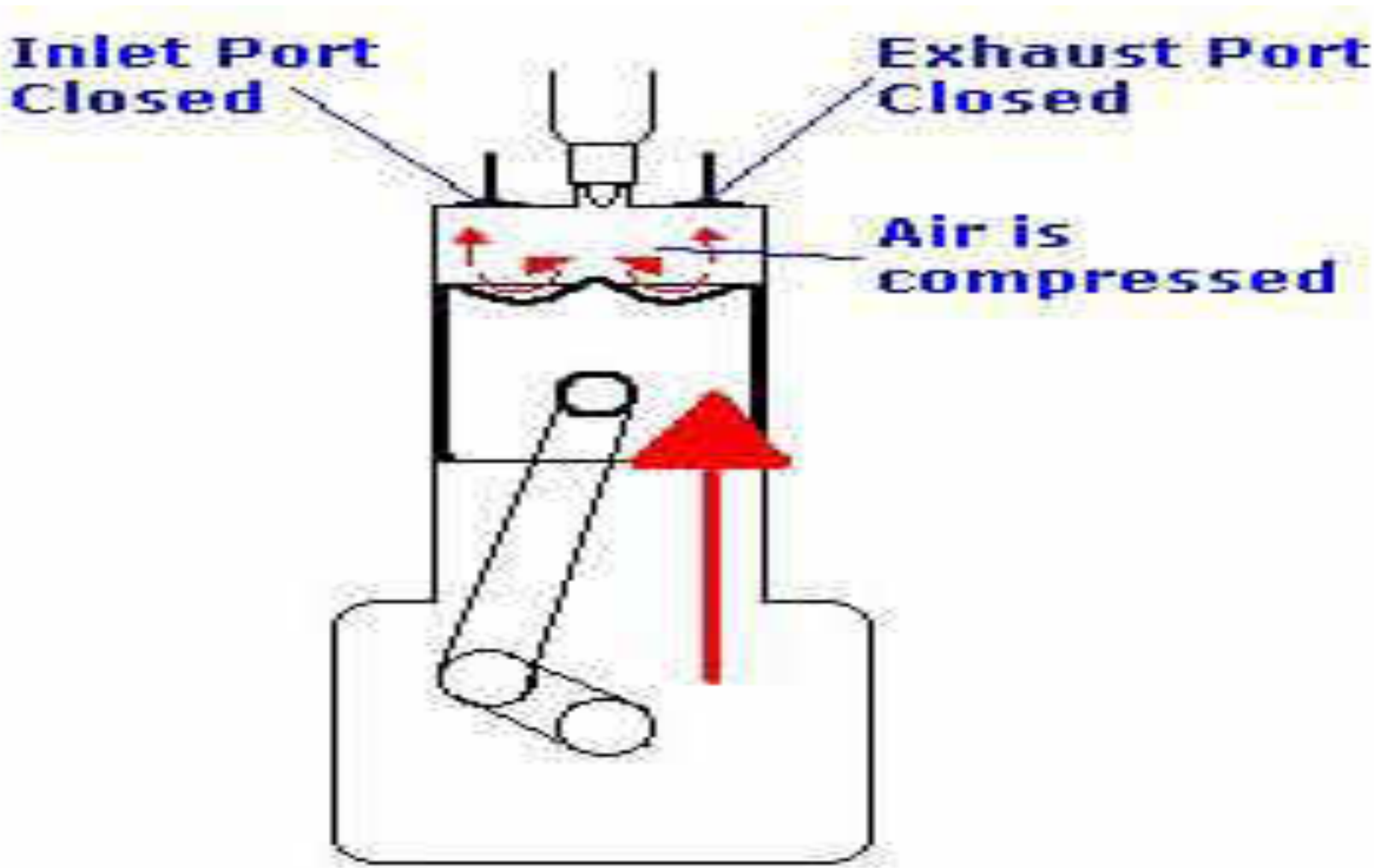


Figure11: Compression stroke

•Four strokes of CI Engine Cycle :

- **Intake/Suction Stroke :** The same as the intake stroke in an SI engine with one major difference : no fuel is added to the incoming air, refer figure 10.
- **Compression Stroke :** The same as in an SI engine except that only air is compressed and compression is to higher pressures and temperature, refer figure11.
  - Late in the compression stroke fuel is injected directly into the combustion chamber, where it mixes with very hot air.
  - This causes the fuel to evaporate and self ignite, causing combustion to start.

» **Combustion** is fully developed by TDC and continues at about constant pressure until fuel injection is complete and the piston has started towards BDC, refer figure12.

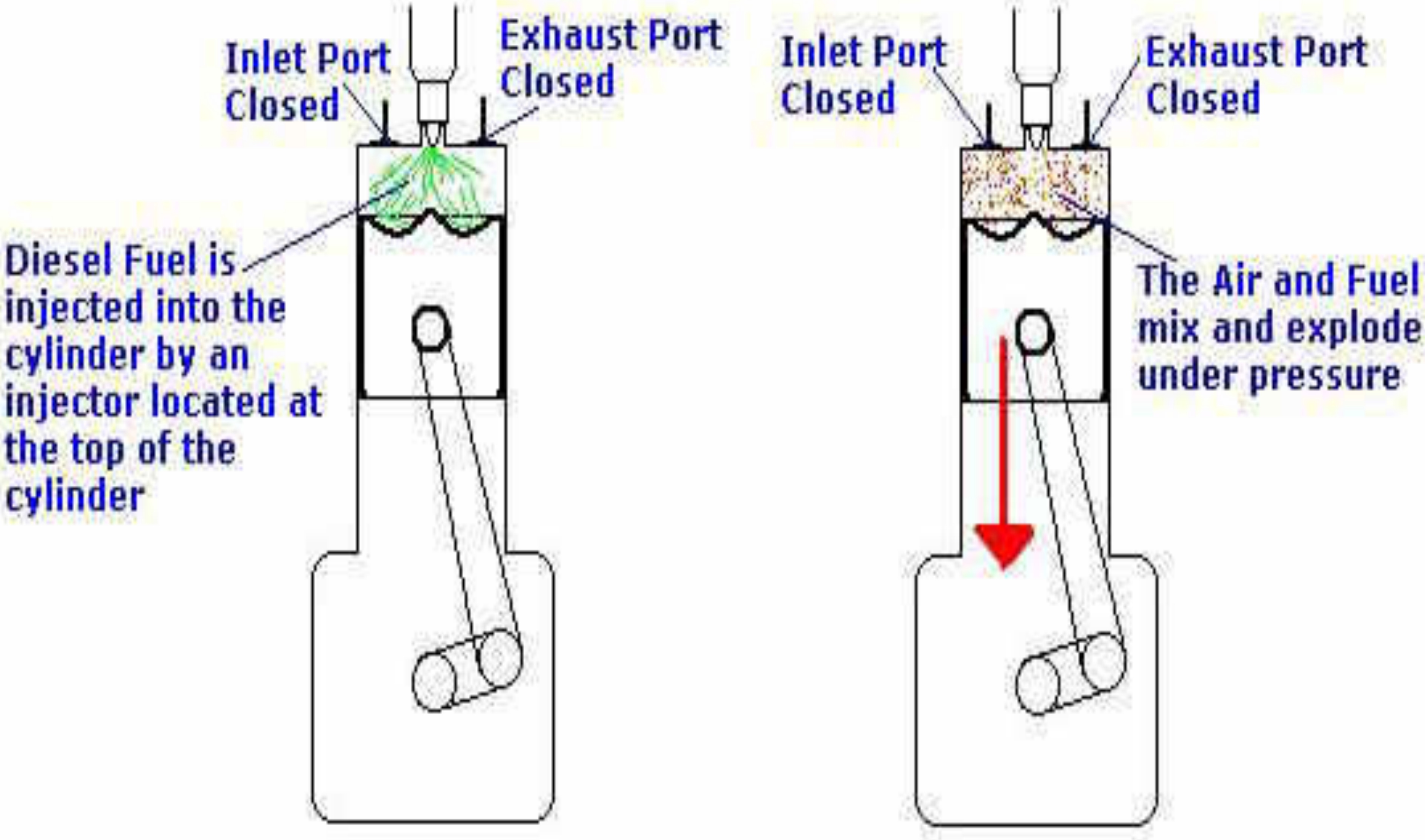


Figure12:Fuel injection and combustion followed by Expansion stroke .

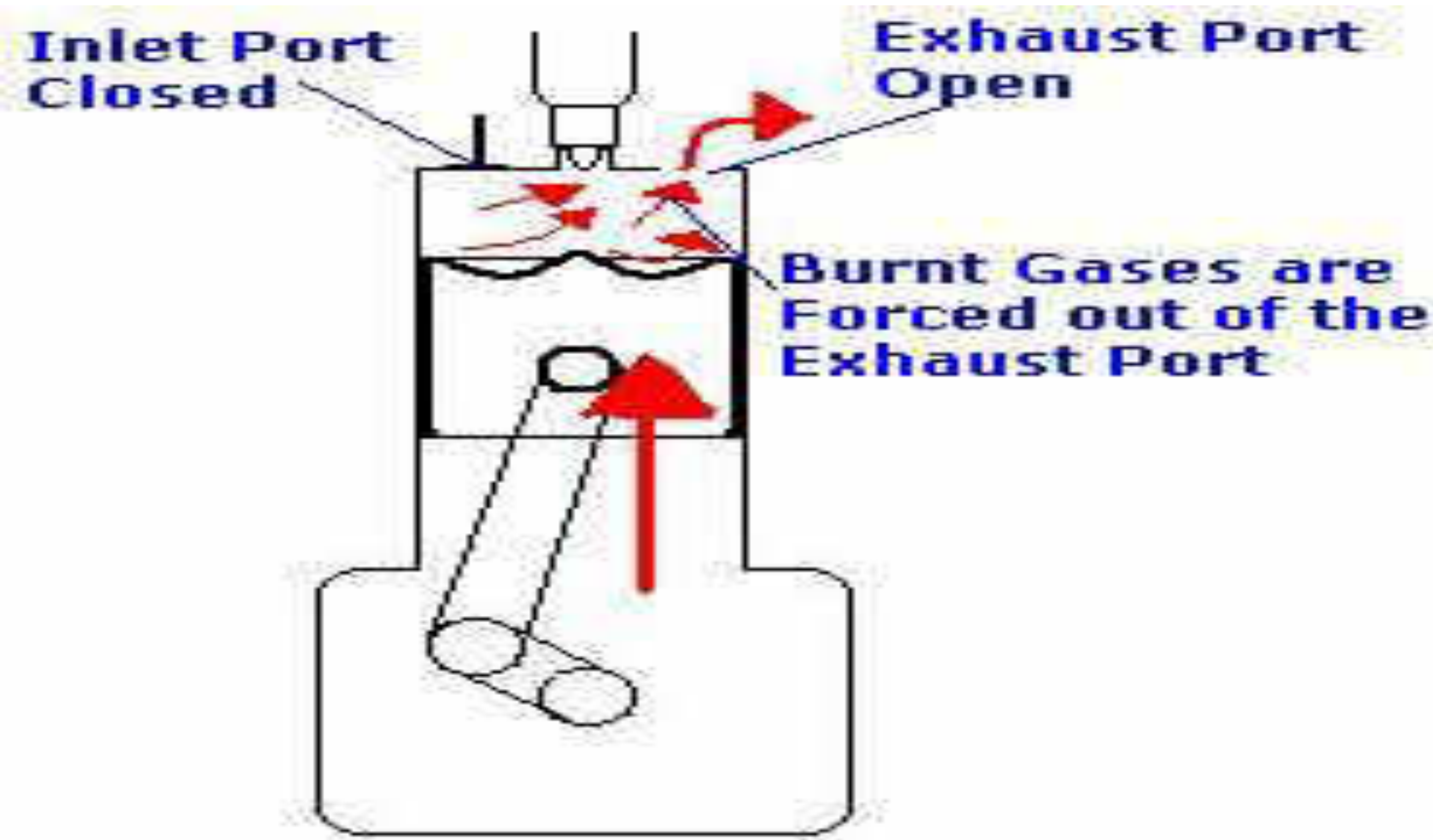
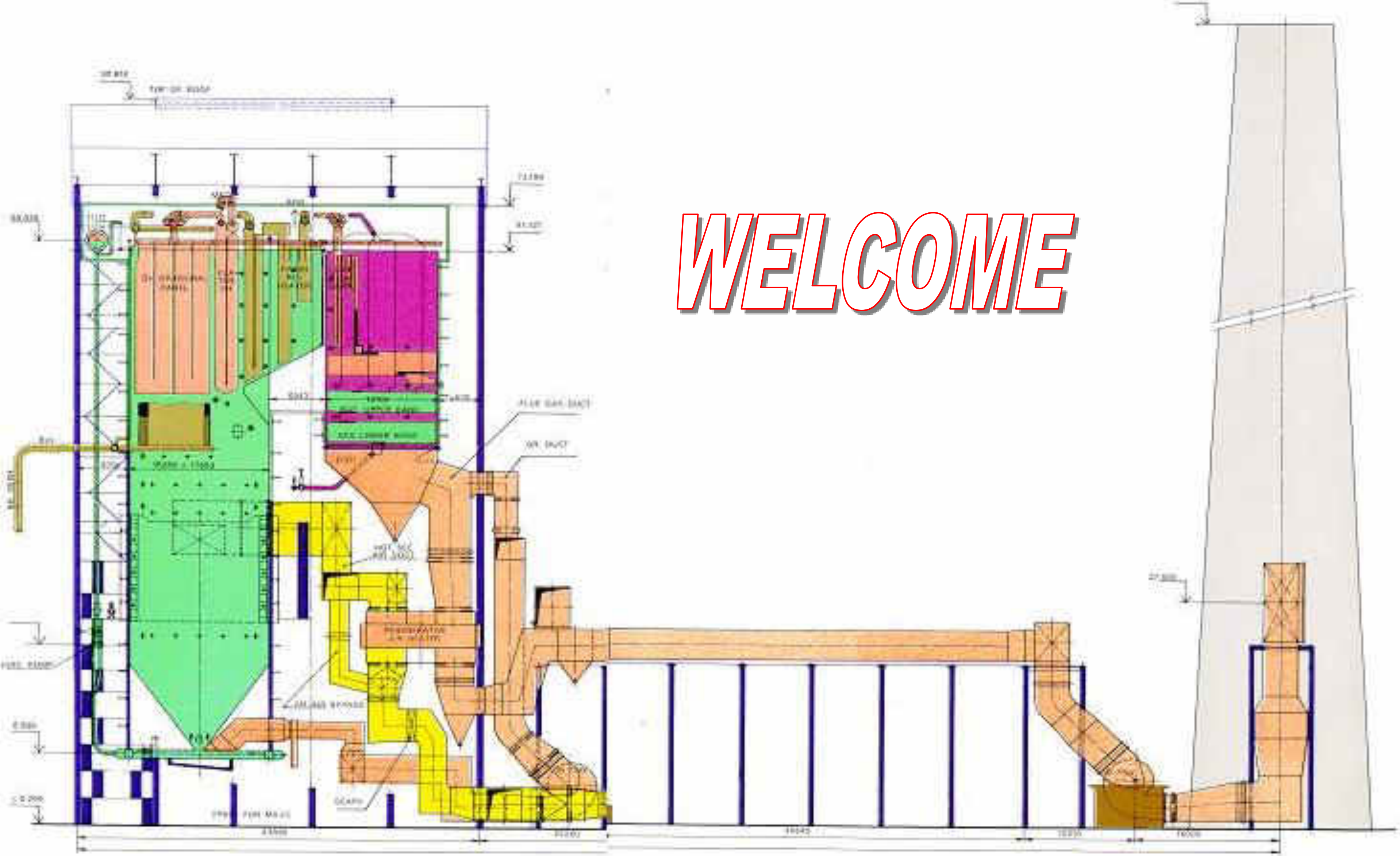


Figure13: Exhaust stroke followed by exhaust blowdown.

- **Expansion/Power stroke :** The power stroke continues as combustion ends and the piston travels towards BDC, refer figure 12.
  - **Exhaust blowdown** same as with an SI engine.
- **Exhaust stroke :** Same as with an SI engine, refer figure 13.

Boiler Parts

WELCOME



BOILER

"BOILER" MEANS ANY CLOSED VESSEL EXCEEDING 22.75 Litres IN CAPACITY WHICH IS USED FOR GENERATING STEAM UNDER PRESSURE.

BOILER CODES

Boiler Codes have been written by various nations in the past century to ensure safety of personnel and to avoid loss of property. Boiler codes cover the whole gamut of activities including Design, Fabrication, Testing, Construction and Operation. The various aspects of IBR Regulations are called out and consolidated against major items like drum, headers, lines & links, etc. The following codes have been used widely.

1. IBR 1950
2. ASME Section-I
3. BS 1113
4. DIN TRD 300.

TYPES OF BOILERS

(A) BASED ON APPLICATION :

1. UTILITY Boilers are large capacity steam generators used purely for electrical power generation.

2. INDUSTRIAL Boilers are small capacity boilers intended for use in the process industries.

Types of Boilers……contd

(B) BASED ON CONSTRUCTION:

1. Vertical Package- VP
2. Vertical Recovery-V2R
3. Vertical Unit 40-VU40
4. Vertical Unit 60-VU60
5. Modular Unit-MU
6. 2 Pass Single Arch
7. 2 Pass Double Arch
8. Close couple
9. Box Type
10. Tower Type

Types of Boilers……contd

(C) BASED ON FUEL:

1. OIL FIRED
2. OIL AND COAL FIRED
3. BLACK LIQUOR (For Paper Mills)
4. BAGGASE (Stoker Fired)

Types of Boilers.......contd

(D) BASED ON TYPE OF FIRING:

1. WALL FIRING
2. CORNER TANGENTIAL FIRING
3. STOKER

Types of Boilers……contd

(E) BASED ON NO. OF DRUMS:

1. SINGLE DRUM
2. BI- DRUM
3. NO DRUM (Vertical Separator)

Types of Boilers……contd

(F) BASED ON CIRCULATION:

1. NATURAL

2.1 FORCED Circulation (Pump)

2.2 CONTROLLED Circulation (+Orifice)

2.3 CC+ (Pump + Orifice + Rifled Tubing)

3. Once Through

BOILER PARAMETER:

(A) UTILITY BOILERS:

1. Main Steam Flow – T/Hr.
2. Main Steam Pressure – Kg/Sq.cm.(g)
3. Main Steam Temperature - ºC
4. Reheater Flow – T/Hr.
5. Reheater Pressure – Kg/Sq.Cm.(g)
6. Reheater Temperature - ºC

(B) INDUSTRIAL BOILERS:

Steam Flow – T/Hr.

(C) HEAT RECOVERY UNITS:

Fuel Used – T/Day.

FURNACE / BACK PASS WALLS:

Normally for boiler furnace enclosures membrane wall construction (fusion welded panels) is adopted in place of tangent tube construction which is not leak proof and increased erection work.

Back pass enclosures are formed by fin welded panels with wider pitch because the flue gas temperature is less compared to furnace.

SUPERHEATER AND REHEATER:

These heating surfaces are in the form of coils which are made by bending the tubes in cold or hot condition. The superheater is composed of four basic sections.

The platen section is located directly above the furnace in front of the furnace arch. It absorbs heat mainly by radiation. The pendant spaced section is located in back of the screen wall tubes. The mode of heat transfer is convection.

The horizontal section of the superheater is located in the rear gas pass above economiser. The steam cooled wall sections form the side, front and rear walls and roof of the vertical gas pass.

Desuperheaters:

Desuperheating - Steam temperature control.

Provided in

1. superheater connecting links
2. cold reheat line

to permit reduction of steam temperature when necessary

to maintain the temperatures

- The desuperheaters used in the reheater system is meant for emergency condition.
- The reheat steam temperature is controlled mainly by tilting burners.

Reheater

The reheater - Single stage – 2 Sections

Front & rear pendant vertical spaced.

The front section located between the rear water wall hanger tubes and the superheater platen section.

The rear section is located between water wall screen and rear wall hanger tubes.

SPACERS FOR SH & RH:

Spacer are used to maintain pitches along and across coil assemblies.

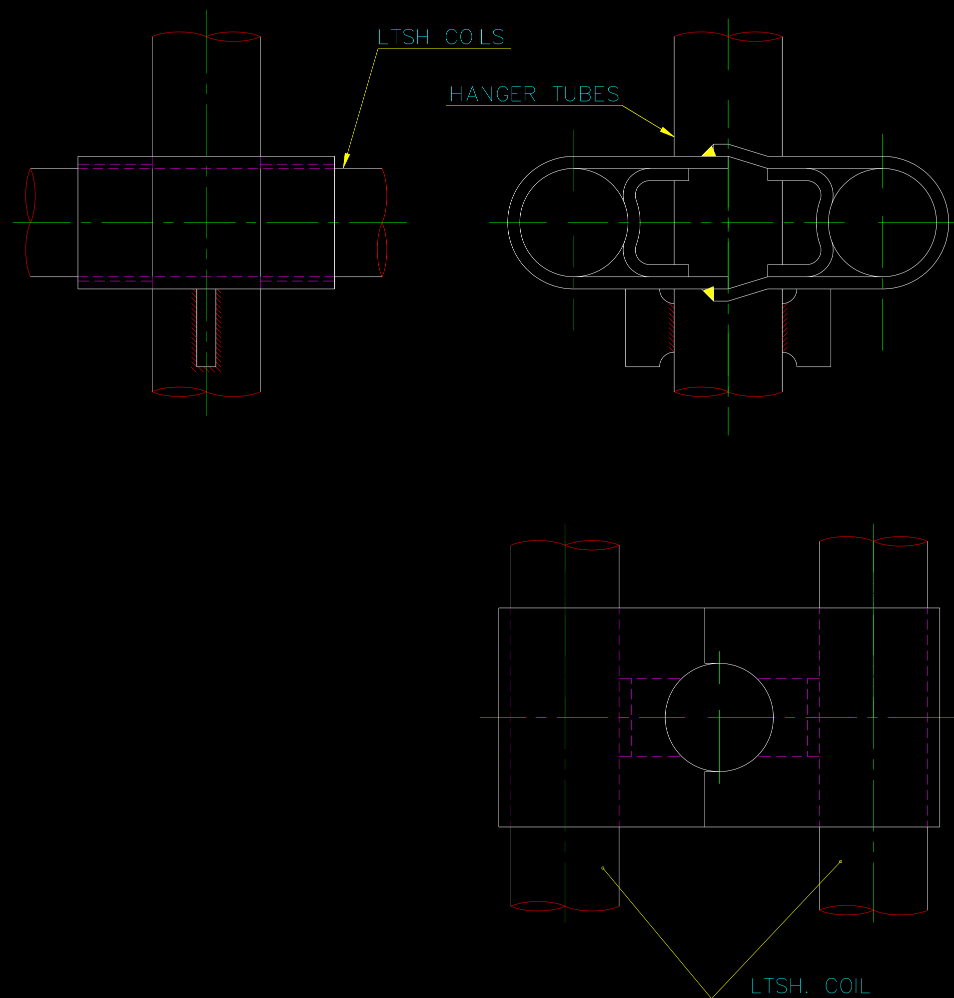
1. Transverse spacers and alignment ties.

2. Fluid cooled spacers or mechanical spacer bar are used as transverse spacers.

3. Flexible connector and alignment bands are used as alignment ties. to maintain pitch between tubes in the same assembly.

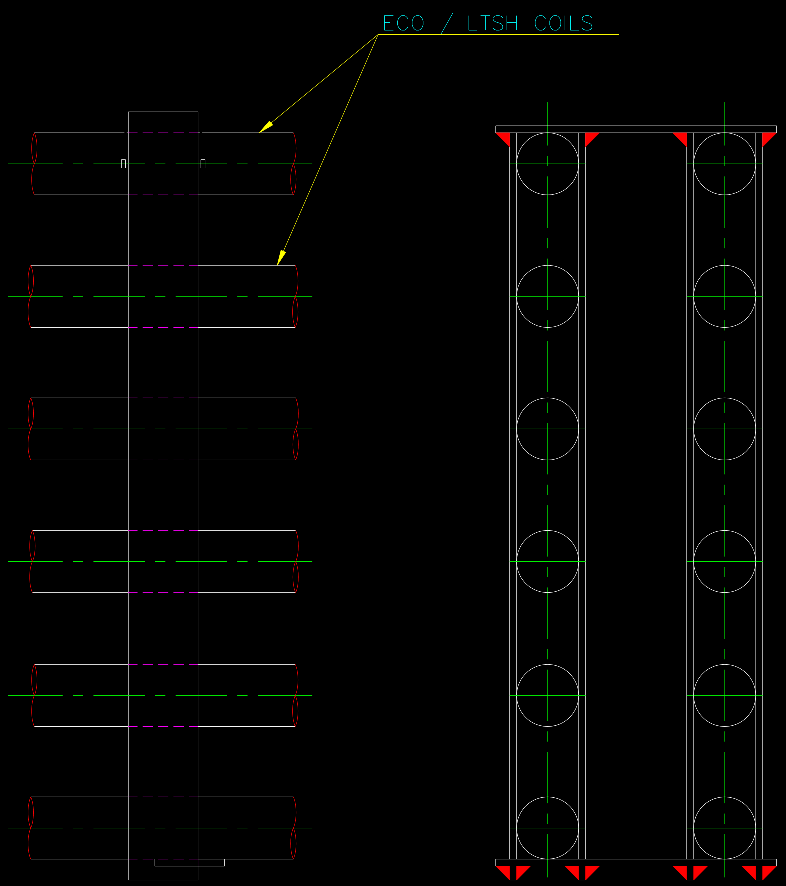
SUPPORTS & SUSPENSIONS FOR SH & RH:

1. Vertical Assemblies are suspended from the ceiling.
2. In pendant assemblies the tie lugs are welded in between tubes at the top row to transfer the load from centre to end terminals.
3. The horizontal superheaters are supported by economiser hanger tubes through strap supports.
4. The pendant coils are suspended by high crown supports. The high crown plates are welded on either side of seal band and the load is transferred through end bar.
5. The headers will be independently supported from the ceiling through tie rod assemblies with or without variable spring hangers as the case may be.



TYPICAL SUPPORTING ARRGT. OF LTSH COILS

(STRAP SUPPORT)

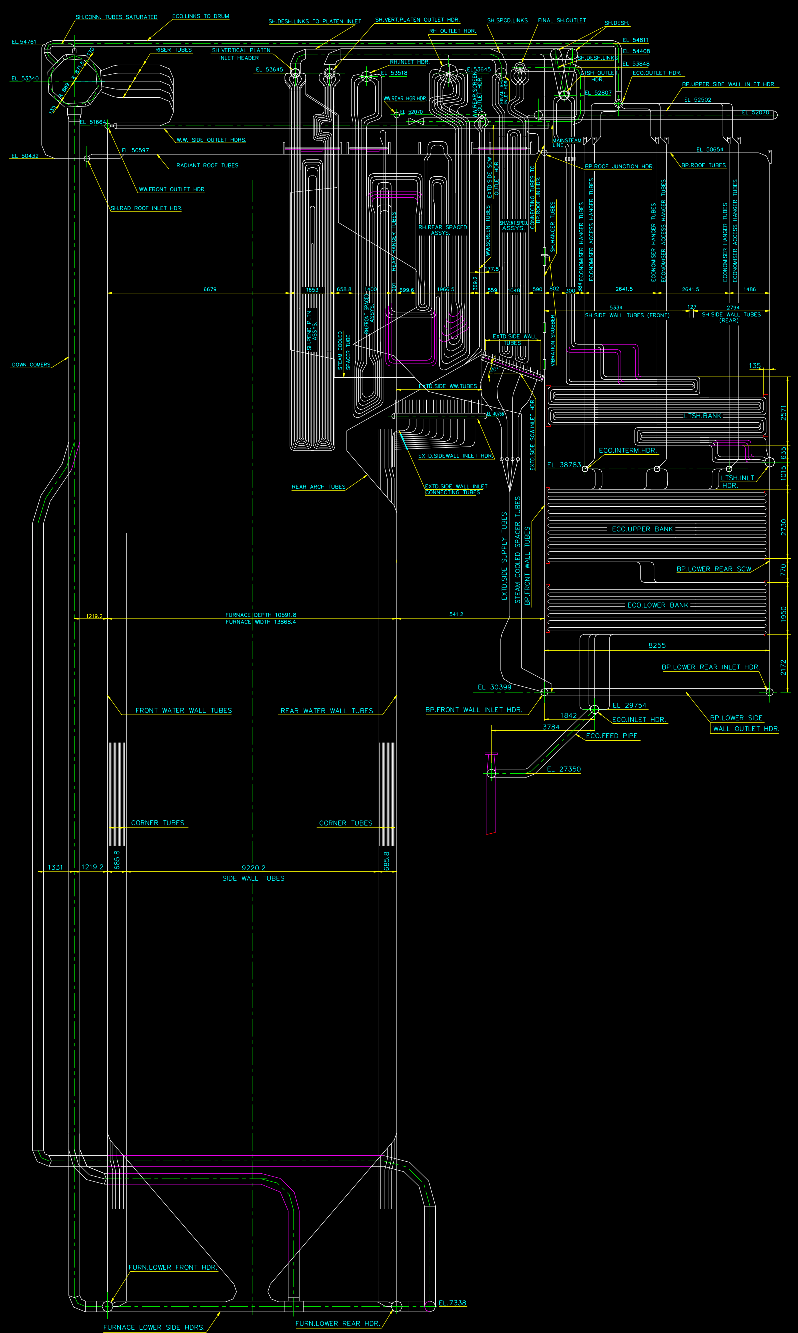


ALIGNMENT BAND
(FOR ECO / LTSH)

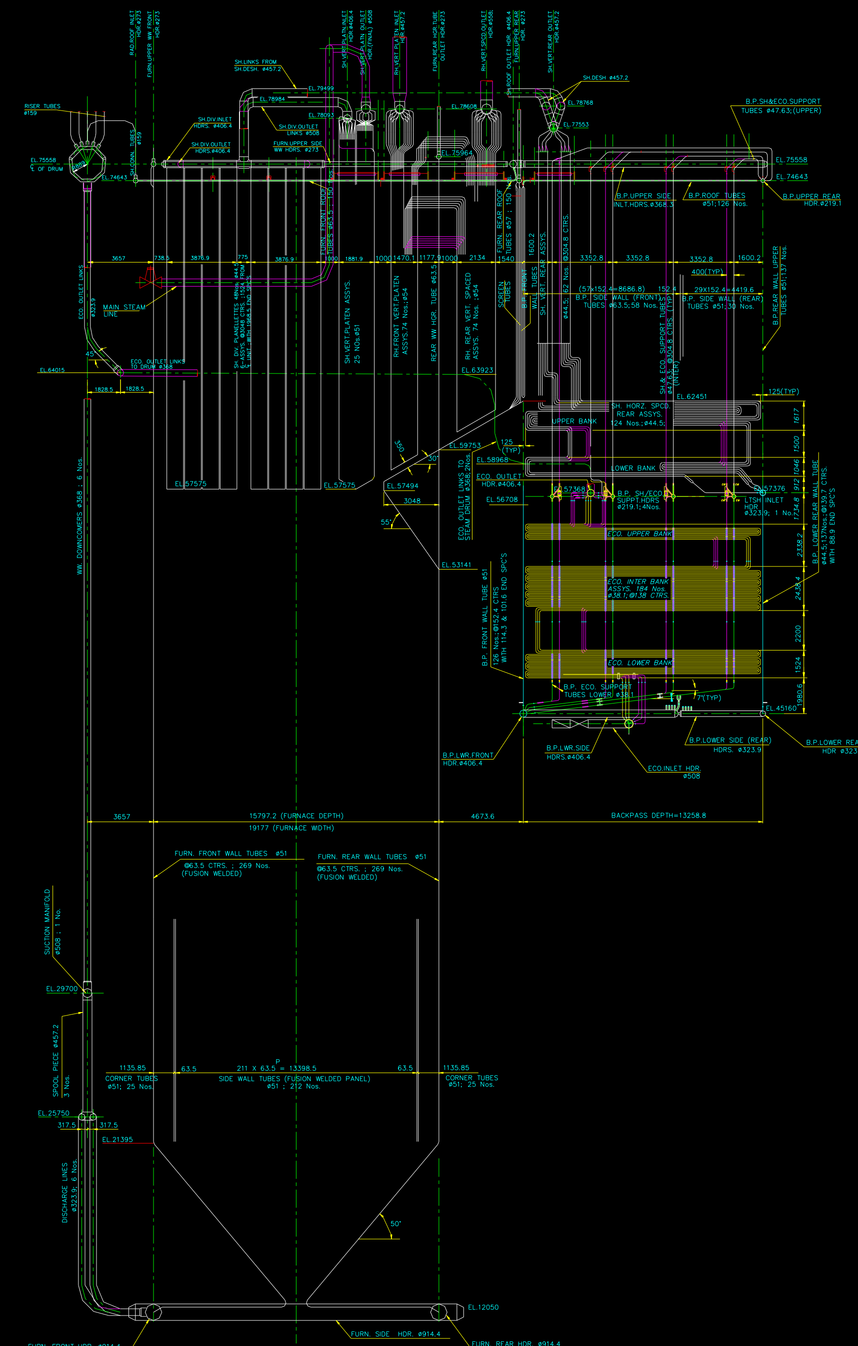
Pressure Parts Arrangement Comparison of 250 MW and 500 MW Boilers

- Circulation
  - Circulating Pump
  - Down comer Connections
  - Lower Ring header with Orifice Plates
- Divisional Panellette Super Heater
- Extended Pass
  - Split Extended pass
  - Steam Cooled side wall for Extended Pass
- Steam Cooled Eco. / LTSH Hangers
- Rifled Tubings in WaterWalls
- Wall Reheater (Presently not followed)

TYPICAL 210 MW/250 MW BOILER



TYPICAL 500 MW BOILER

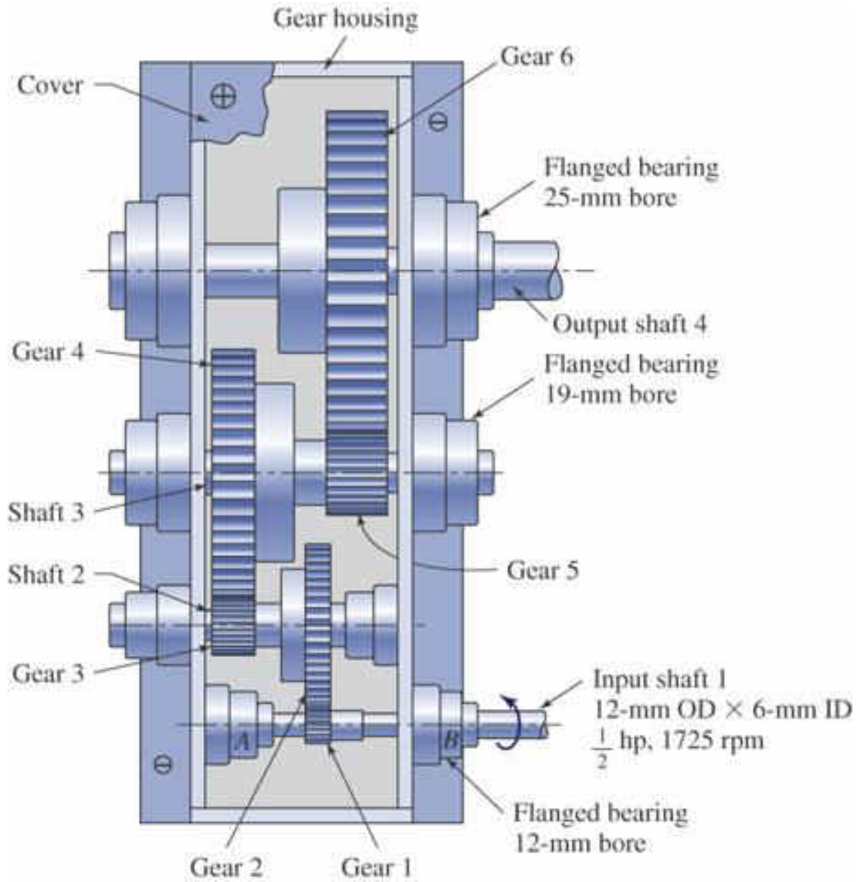


GEAR

Gears

What we need to Know about them.

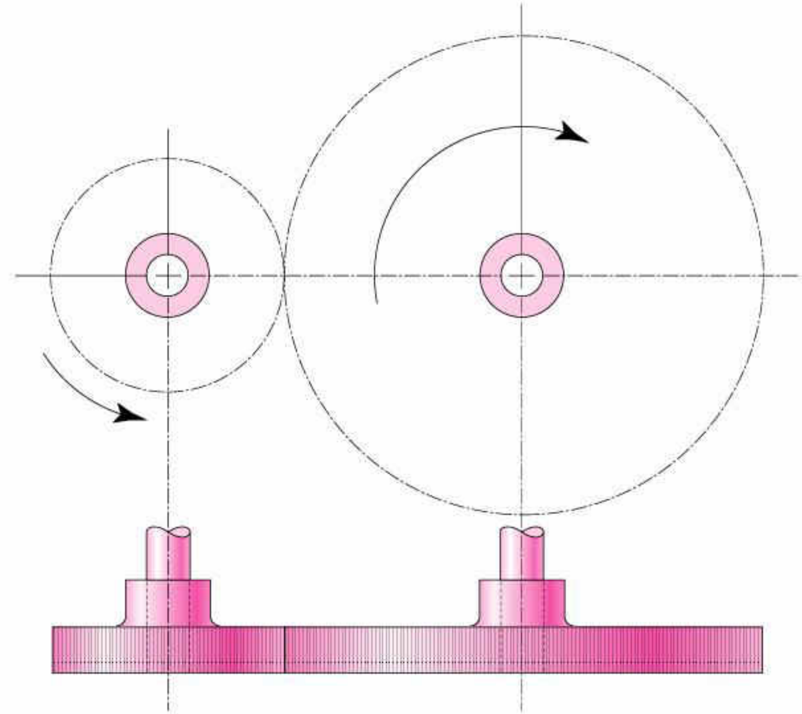
1. Type of gears
2. Terminologies or nomenclatures
3. Forces transmitted
4. Design of a gear box



Type of Gears

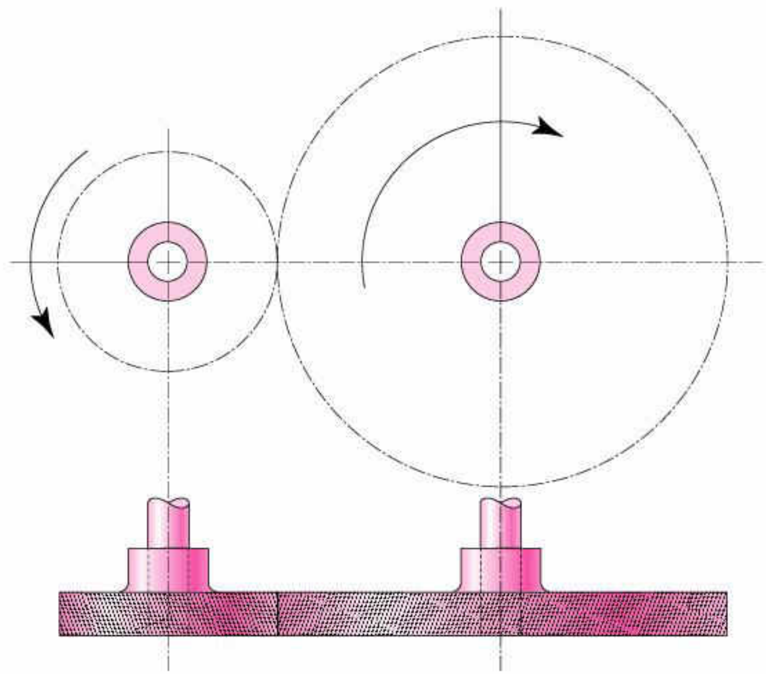
- Spurs
- Helical
- Bevel
- And Worm Gears

Spur Gears



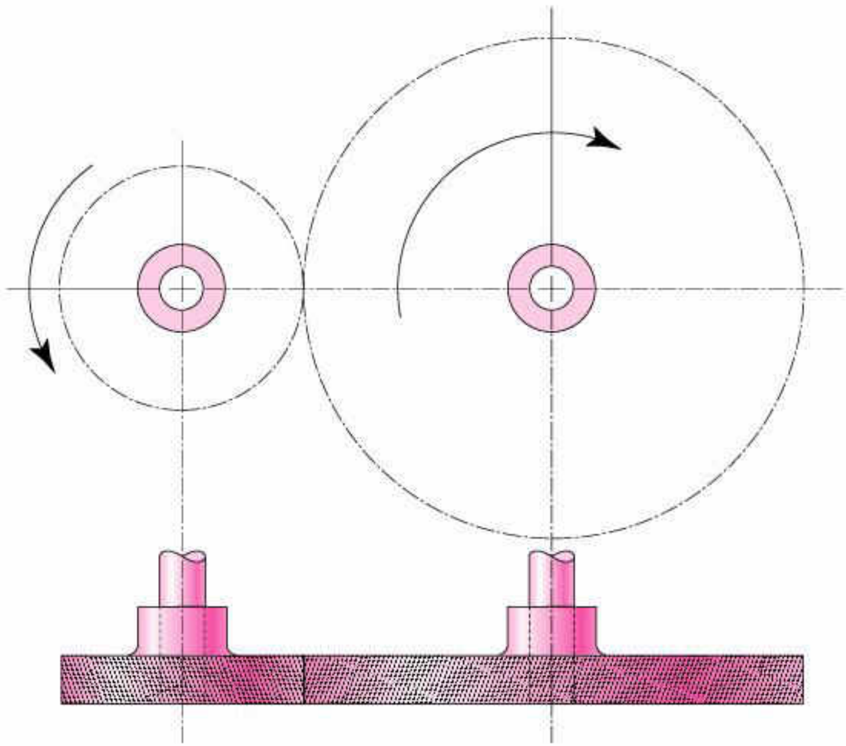
Are used in transmitting torque between parallel shafts

Helical Gears



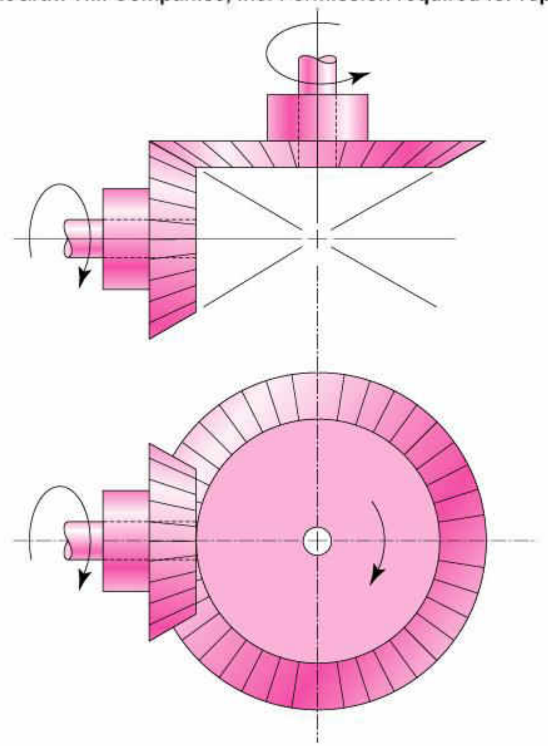
Are used in transmitting torques between parallel or non parallel shafts, they are not as noisy as spur gears

Fig. 13.2



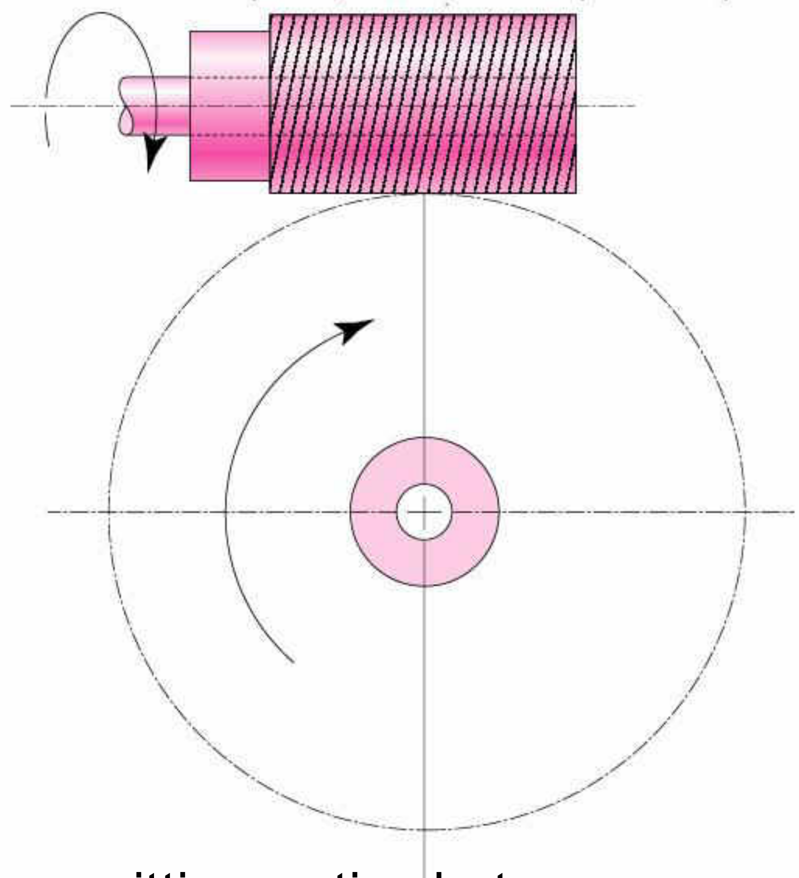
Bevel Gears

- Are used to transmit rotary motion between intersecting shafts



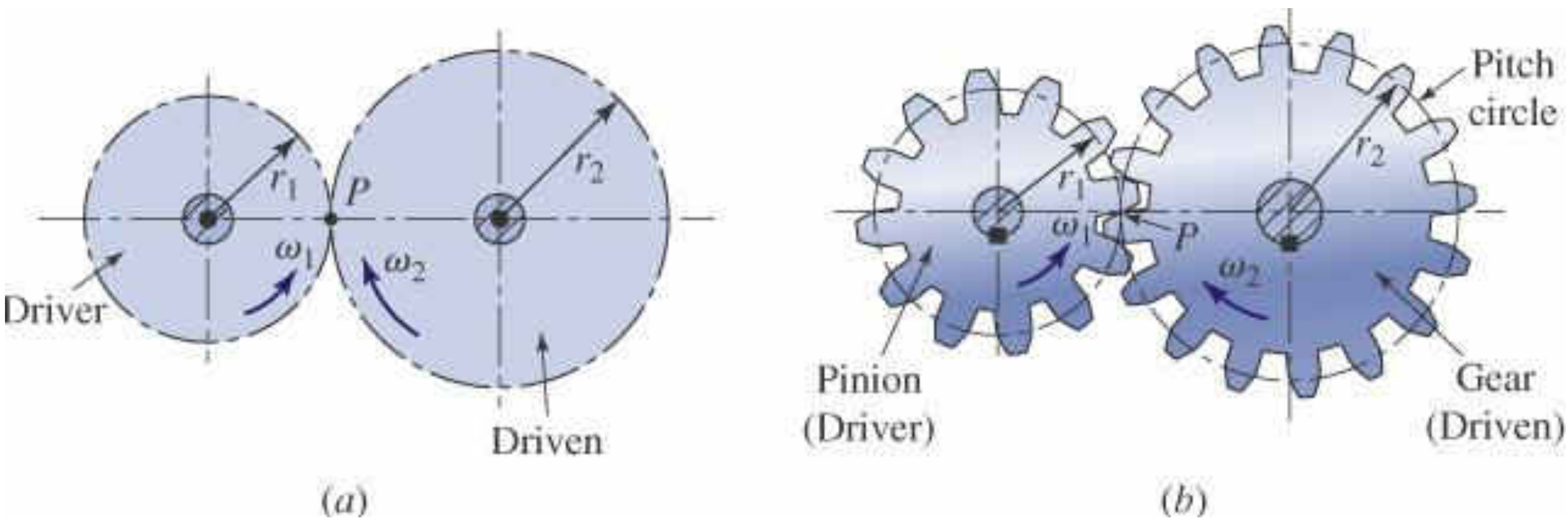
Teeth are formed on conical surfaces, the teeth could be straight or spiral.

Worm Gears



Are used for transmitting motion between non parallel and non transmitting shafts, Depending on the number of teeth engaged called single or double. Worm gear mostly used when speed ratio is quiet high, 3 or more.

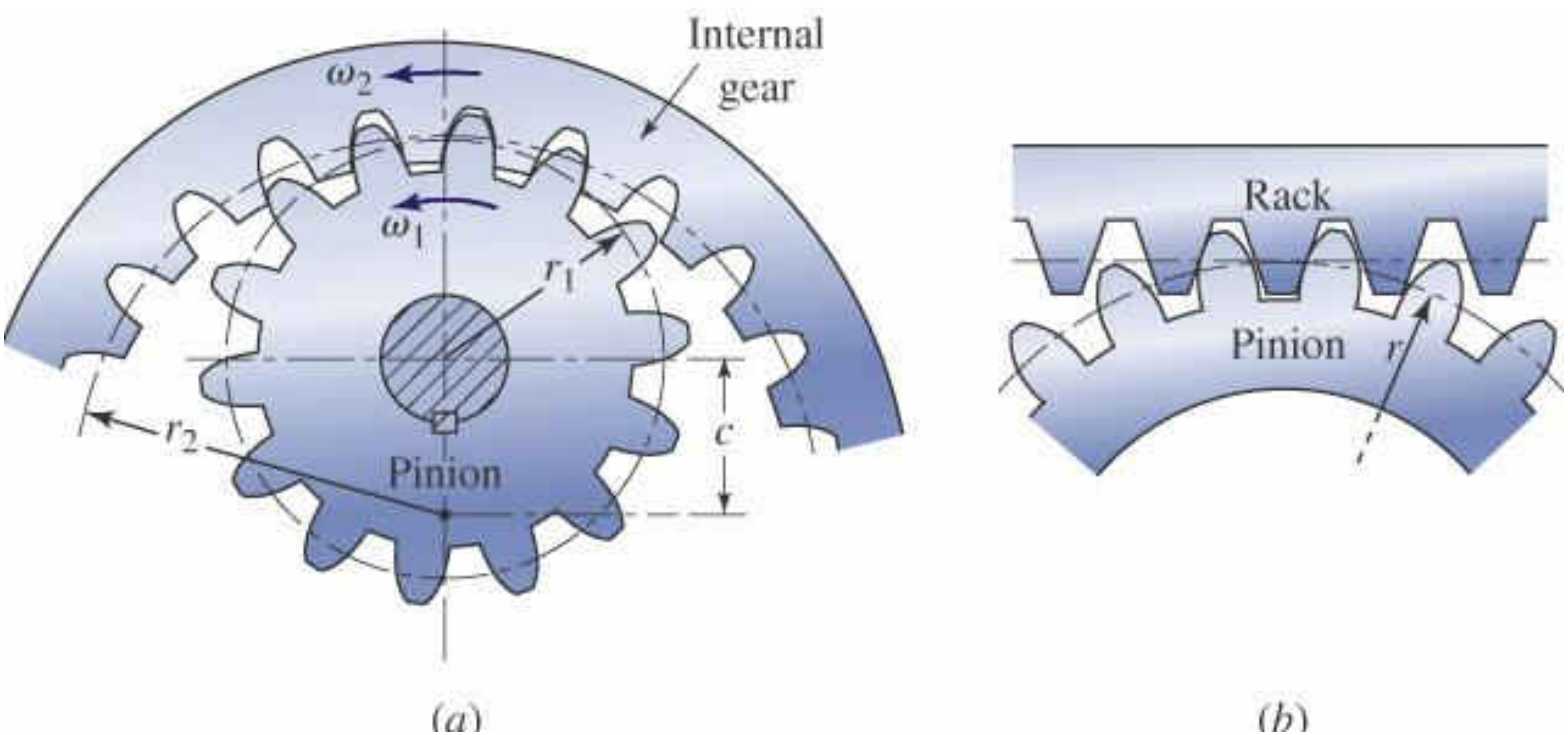
Nomenclature

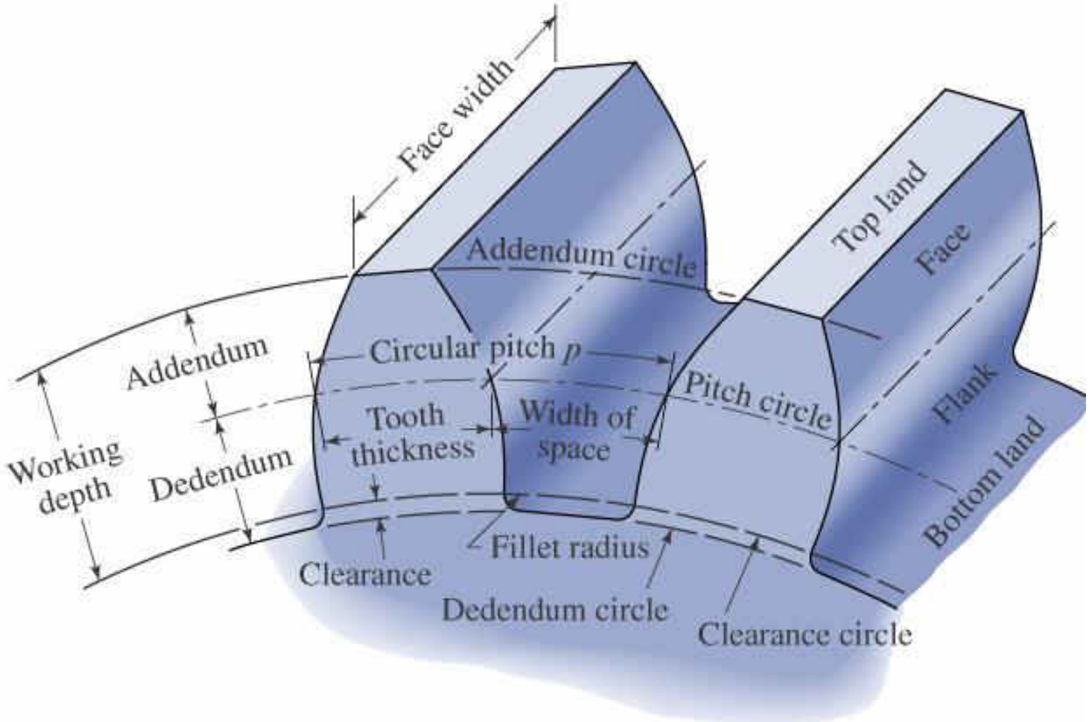


Smaller Gear is Pinion and Larger one is the gear

In most application the pinion is the driver, This reduces speed but it increases torque.

Internal Spur Gear System





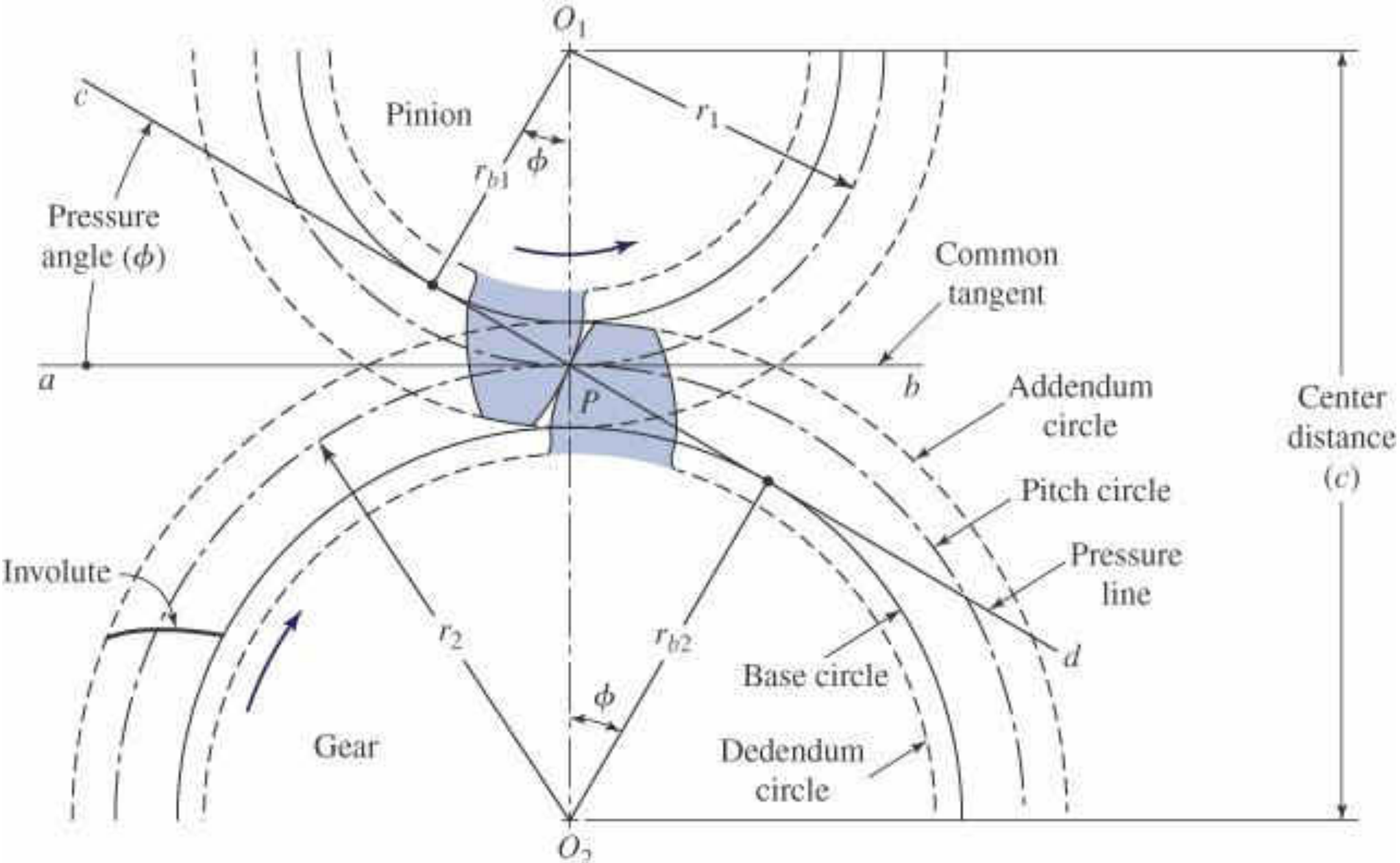
pitch circle, theoretical circle upon which all calculation is based

p, Circular pitch, p the distance from one teeth to the next, along the pitch circle. $p = \pi d / N$

m, module=d/N pitch circle/number of teeth

$p = \pi m$

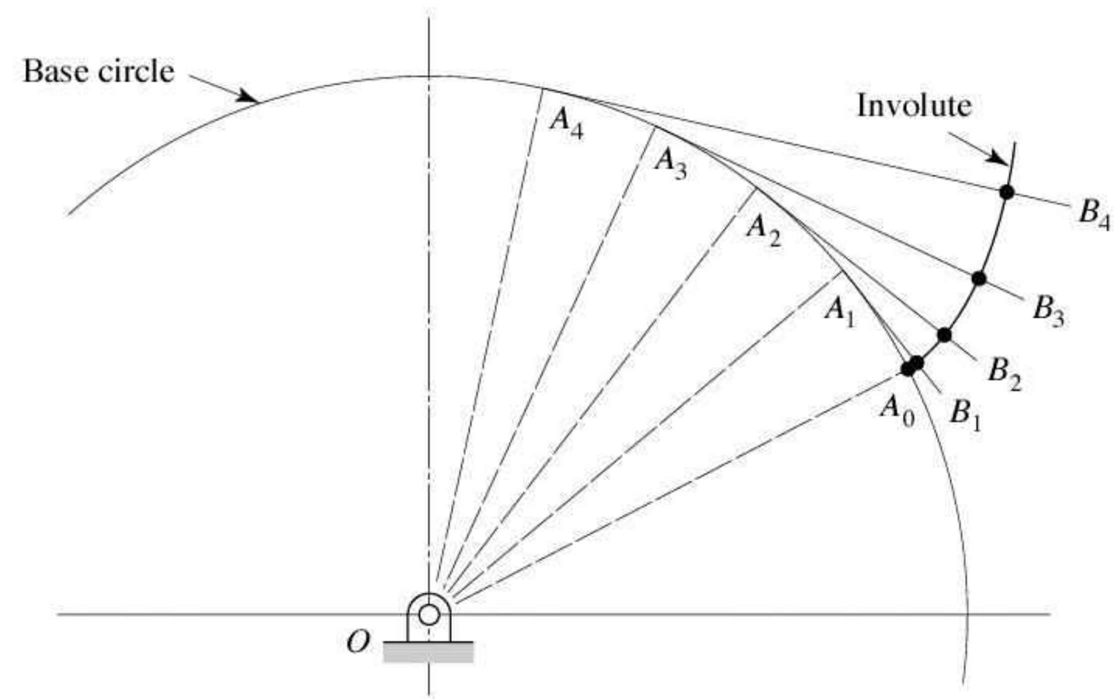
P, Diametral Pitch $P = N/d$



Angle Φ has the values of 20 or 25 degrees. Angle 14.5 have been also used.

Gear profile is constructed from the base circle. Then additional clearance are given.

How Gear Profile is constructed

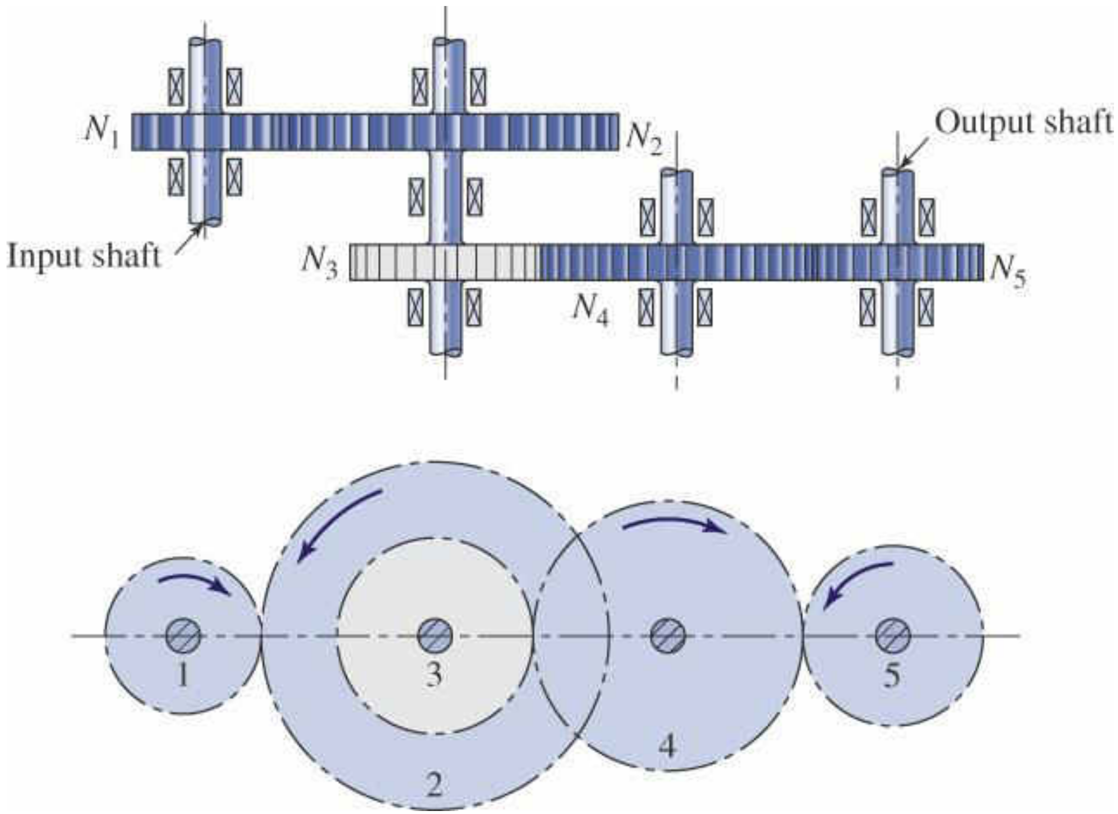


$A_1B_1 = A_1A_0$, $A_2B_2 = 2\,A_1A_0$, etc

Standard Gear Teeth

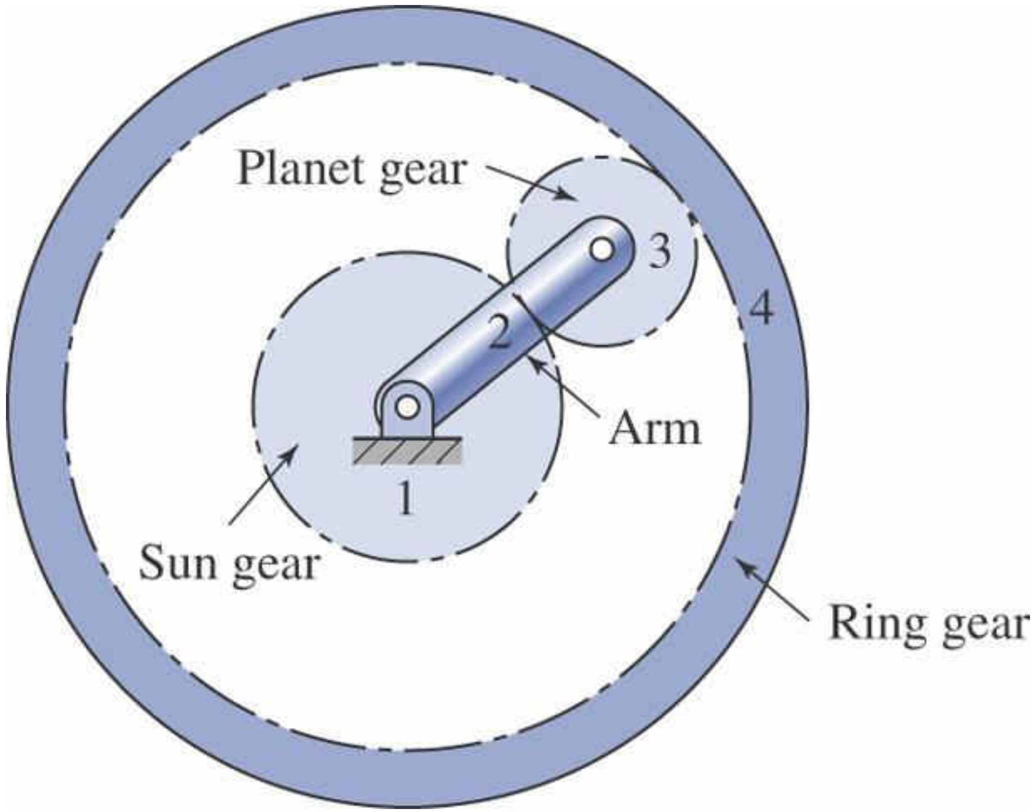
Item	20^o full depth	20^o Stub	25^o full depth
Addendum a	1/P	0.8/P	1/P
Dedendum	1.25/P	1/P	1.25/P
Clearance f	0.25/P	0.2/P	0.25/P
Working depth	2/P	1.6/P	2/P
Whole depth	2.25/P	1.8/P	2.25/P
Tooth thickness	1.571/P	1.571/P	1.571/P
Face width	9/P<b<13/P	9/P<b<13/P	9/P<b<13/P

Gear Trains



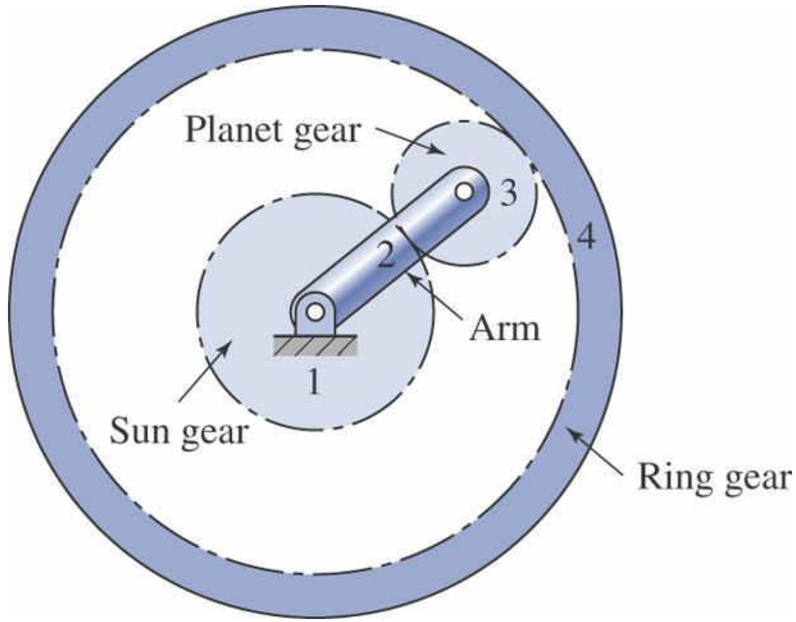
$$\frac{n_5}{n_1} = (-\frac{N_1}{N_2})(-\frac{N_3}{N_4})(-\frac{N_4}{N_5})$$

Planetary Gear train
You can get high torque ratio in a smaller space



There are two inputs to the planetary gears, RPM of sun and Ring, The out put is the speed of the arm.

Example of planetary Gear train



Gear 1, sun , RPM 1200, Number of teeth 20,

Planet Gear , Number of teeth 30

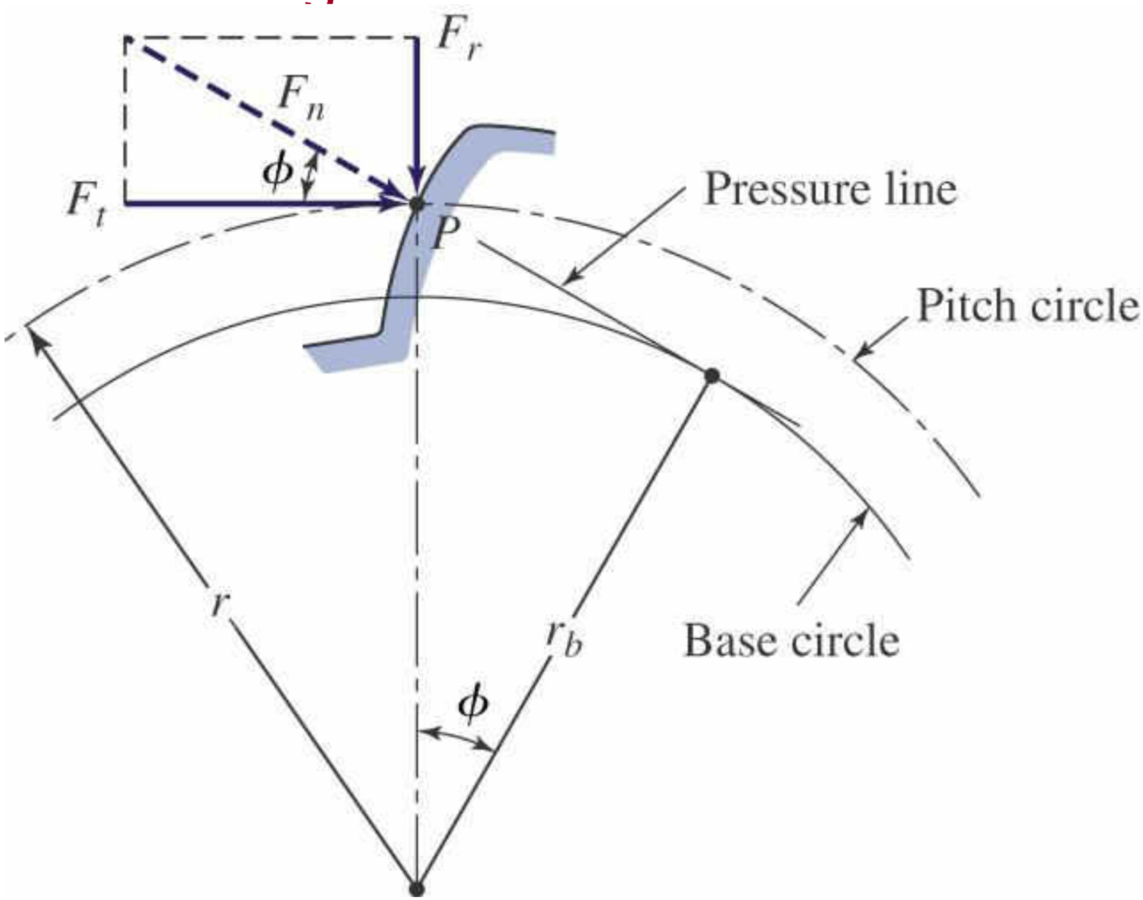
Ring Gear, Rotates RPM 120, and teeth of 80,

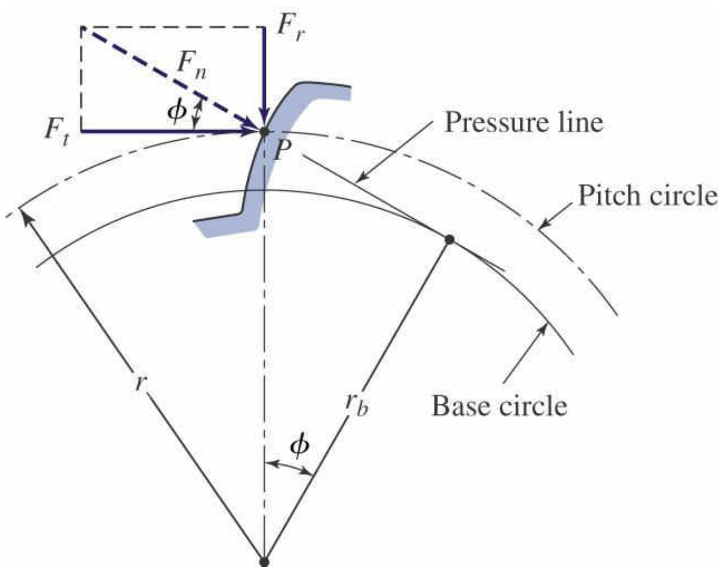
¼ horse power, find the speed of the arm and torque on the ring.

Alternatively you may have Certain Out put Torque requirements

Transmitted Load

- With a pair of gears or gear sets, Power is transmitted by the force developed between contacting Teeth





$$F_t = F_n \cos\phi$$

$$F_r = F_n \sin\phi$$

$$V = d/2\omega = d * \frac{2\pi RPM}{60}$$

d in, RPM rev./min, V in/sec

$$V = \frac{\pi dn}{12}$$

d in, n rpm, V fpm

$$hp = \frac{Tn}{63000}$$

Toque lb-in

$$F_t = \frac{33000hp}{V}$$

V fpm

$$KW = \frac{F_t V}{1000} = \frac{Tn}{9549}$$

T= N.m, V m/s, F Newton

These forces have to be corrected for dynamic effects , we discuss later, considering AGMA factors

References

- Engineering Drawing (book) - Wikipedia
- 41 Best Mechanical Drawing Books of All Time - BookAuthority
- Showing results for mechanical engg ***drawing*** books wikibooks
- https://bookauthority.org/books/best-mechanical-drawing-books

Thank You

Government Polytechnic Nanakpur

DEPARTMENT OF MECHANICAL ENGINEERING

E - NOTES

BASICS OF ELECTRICAL & ELECTRONICS ENGINEERING

HOD/O.I (Mechanical) : Er. **SHALANDER MOR**

Subject Teacher: Er.Gulvender

Semester: 3rd Sem

By: Er. Amit Kumar

Designed, Mechanical Editing & Developmental Editing

LEARNING OUTCOMES

After undergoing this subject, the students will be able to:

- Measure basic electrical quantities.
- Measure and improve power factor in a given circuit.
- Explain the construction, working principle, performance and applications of transformers.
- Identify different wires of distribution system.
- Select and operate single phase and three phase motors.
- Follow electrical safety measures.
- Describe the characteristics and applications of diodes, transistors and thyristor.

1

Application and Advantage of Electricity

Advantages of Electricity

- Inexhaustible fuel source
- Minimal environmental impact
- Viable source--relatively useful levels of energy
- production Can be used throughout the world
- Convenient form. **Electrical energy** is a very useful form of **energy**
- Easy control.
- Greater flexibility
- Cheapness
- Cleanliness.
- High-transmission efficiency.

Applications of Electricity

- Entertainment
- Healthcare
- Engineering
- Transport and Communication
- Outdoors
- Household
- Commercial
- Office
- Fuel
- Space

2

Basic Electrical Quantities

Basic Electrical Quantities

- Basic quantities: current, voltage and power
  - *Current*: time rate of change of electric charge

    $$I = dq/dt$$

    1 Amp = 1 Coulomb/sec
  - *Voltage*: electromotive force or potential, V

    1 Volt = 1 Joule/Coulomb = 1 N·m/coulomb
  - *Power*: $P = I\,V$

    1 Watt = 1 Volt·Amp = 1 Joule/sec

Current, *I*

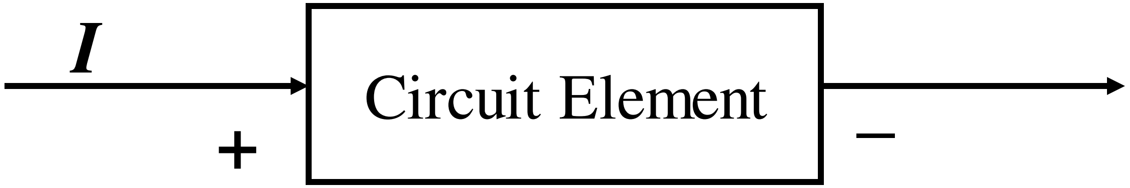
- Normally we talk about the movement of positive charges although we know that, in general, in metallic conductors current results from electron motion (conventionally positive flow)
- The sign of the current indicates the direction of flow
- Types of current:
  - *direct current* (dc): batteries and some special generators
  - *alternating current* (ac): household current which varies with time

Voltage, *V*

- *Voltage* is the difference in energy level of a unit charge located at each of two points in a circuit, and therefore, represents the energy required to move the unit charge from one point to the other

Sign Convention

- *Passive sign convention* : current should enter the positive voltage terminal



- Consequence for $P = I\,V$
  - Positive (+) Power: element absorbs power
  - Negative (-) Power: element supplies power

Electrical Analogies (Physical)

	Electrical	Hydraulic
Base	Charge (q)	Mass (m)
Flow	Current (I)	Fluid flow (G)
Potential	Voltage (V)	Pressure (p)
Power	$P = I\,V$	$P = G\,p$

Class Examples

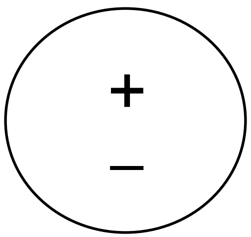
- Learning Extension E1.1
- Learning Extension E1.2

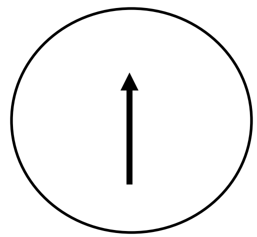
Active vs. Passive Elements

- *Active elements* can generate energy
  - Batteries
  - Voltage and current sources
- *Passive elements* cannot generate energy
  - Resistors
  - Capacitors and Inductors (but CAN store energy)

Independent vs. Dependent Sources

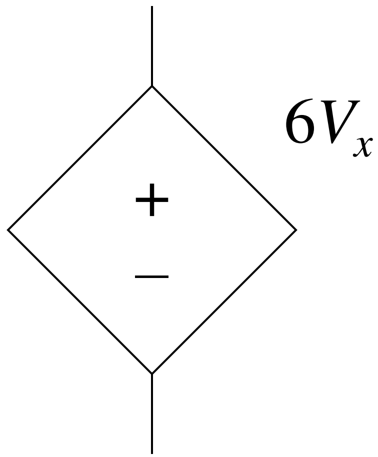
An *independent source* (voltage or current) may be DC (constant) or time-varying, but does not depend on other voltages or currents in the circuit.



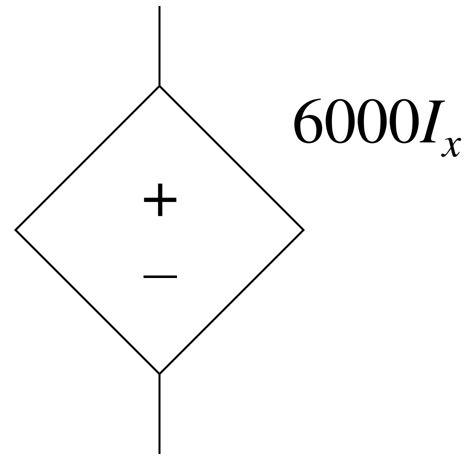


The *dependent source* magnitude is a function of another voltage or current in the circuit.

Dependent Voltage Sources

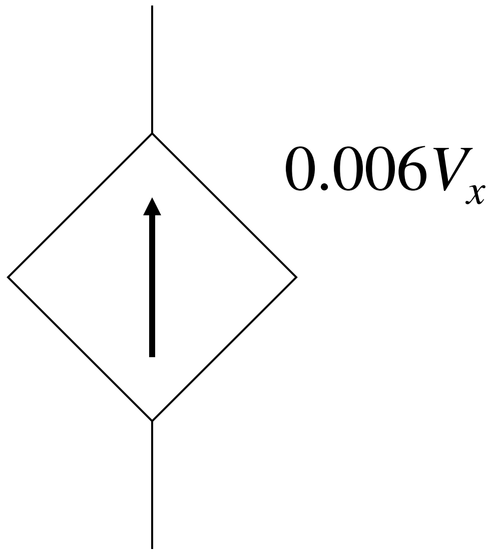


Voltage
Controlled
Voltage Source

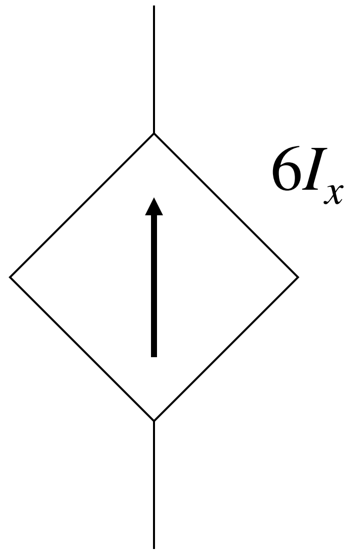


Current
Controlled
Voltage Source

Dependent Current Sources



Voltage
Controlled
Current Source



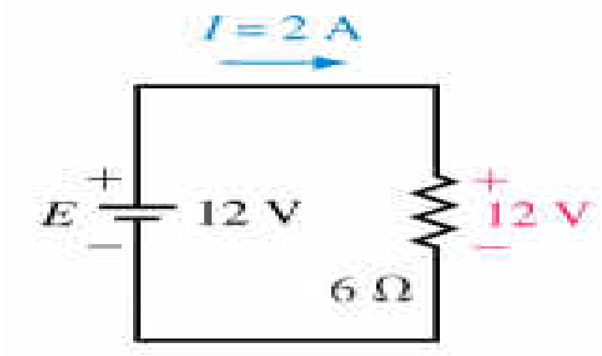
Current
Controlled
Current Source

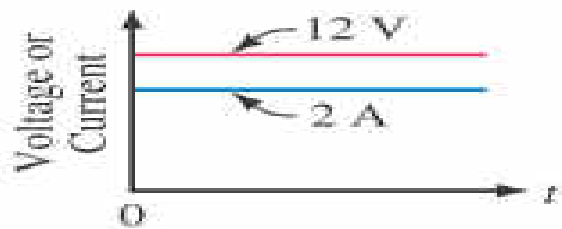
3

AC Fundamentals

AC Fundamentals

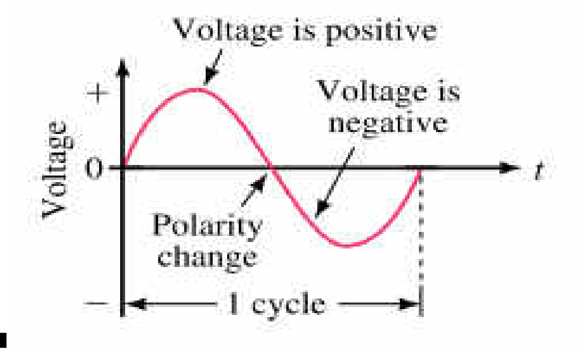
- Previously you learned that DC sources have fixed polarities and constant magnitudes and thus produce currents with constant value and unchanging direction





(b) Voltage and current versus time for dc

- In contrast, the voltages of ac sources alternate in polarity and vary in magnitude and thus produce currents that vary in magnitude and alternate in direction.

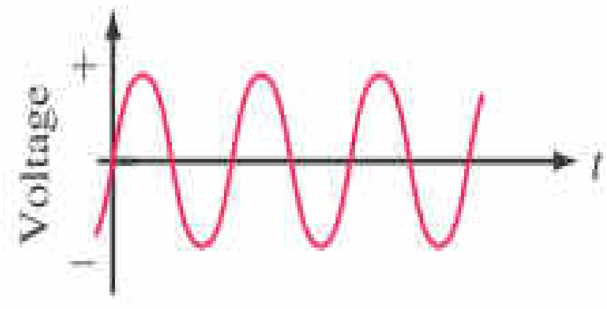


AC Fundamentals

➢ Sinusoidal ac Voltage

One complete variation is referred to as a cycle.

Starting at zero,
the voltage increases to a positive peak amplitude,
decreases to zero,
changes polarity,
increases to a negative peak amplitude,
then returns again to zero.



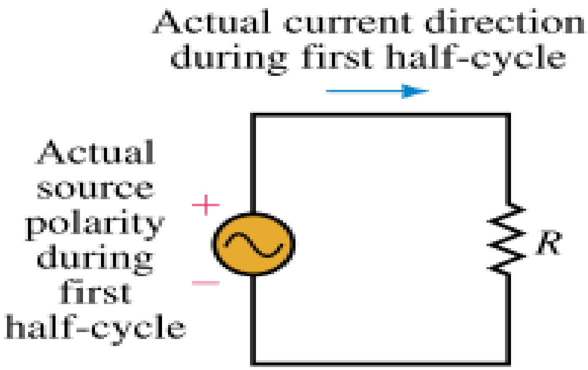
(b) A continuous stream of cycles

➢ Since the waveform repeats itself at regular intervals, it is called a periodic signal.

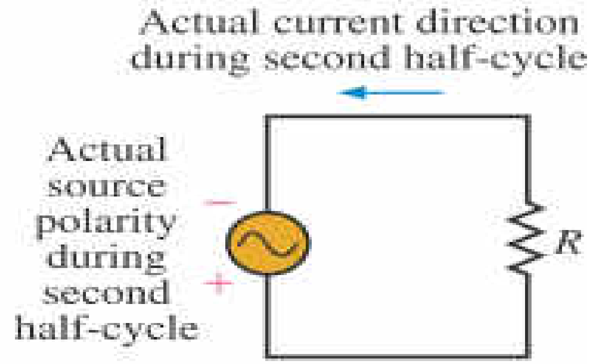
➢ Symbol for an ac Voltage Source

Lowercase letter e is used
to indicate that the voltage varies with time.

Sinusoidal ac Current



(a)



(b)

- During the first half-cycle, the source voltage is positive
- Therefore, the current is in the clockwise direction.

- During the second half-cycle, the voltage polarity reverses
- Therefore, the current is in the counterclockwise direction.

- Since current is proportional to voltage, its shape is also sinusoidal

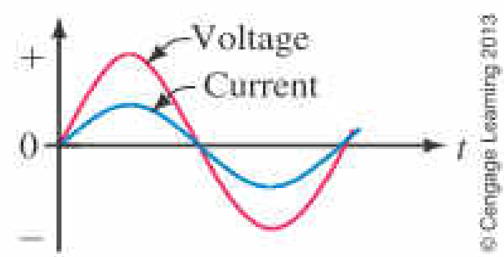
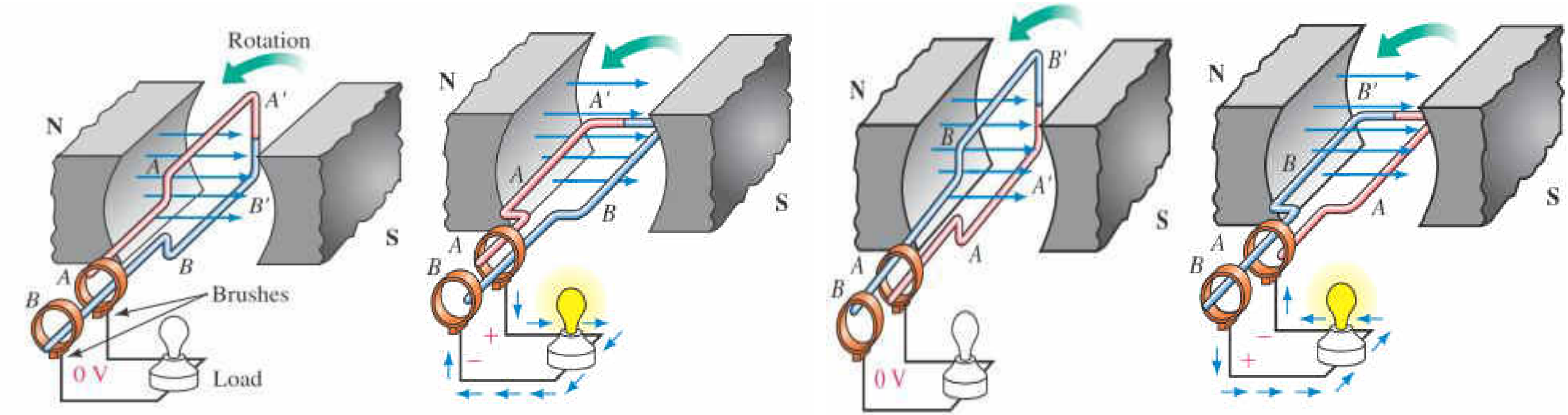


FIGURE 15–5 Current has the same waveshape as voltage.

Generating ac Voltages (Method A)

- One way to generate an ac voltage is to rotate a coil of wire at constant angular velocity in a fixed magnetic field



(a) 0° Position: Coil sides move parallel to flux lines. Since no flux is being cut, induced voltage is zero.

(b) 90° Position: Coil end *A* is positive with respect to *B*. Current direction is out of slip ring *A*.

(c) 180° Position: Coil again cutting no flux. Induced voltage is zero.

(d) 270° Position: Voltage polarity has reversed, therefore, current direction has also reversed.

- The magnitude of the resulting voltage is proportional to the rate at which flux lines are cut

- its polarity is dependent on the direction the coil sides move through the field.

Generating ac Voltages

➢ Since the coil rotates continuously, the voltage produced will be a repetitive,

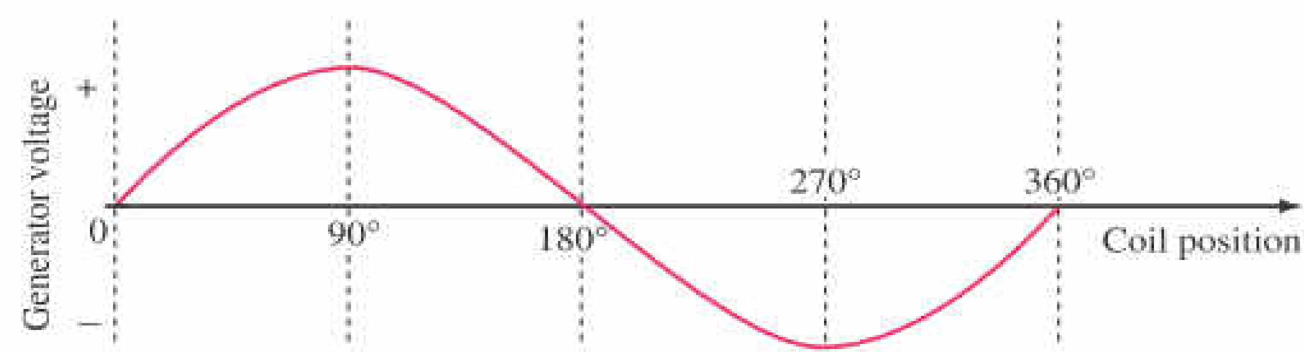


FIGURE 15–7 Coil voltage versus angular position.

➢ Often we need to scale the output voltage in time.

➢ The length of time required to generate one cycle depends on the velocity of rotation.

600 revolutions in 1 minute = 600 rev / 60 s

The time for 1 revolution = one-tenth of a second

= 100 ms

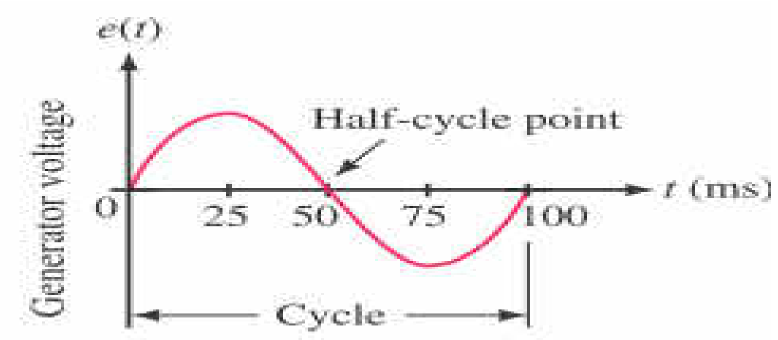


FIGURE 15–8 Cycle scaled in time. At 600 rpm, the cycle length is 100 ms.

Generating ac Voltages (Method B)

- AC waveforms may also be created electronically using function (or signal) generators.
- With function generators, you are not limited to sinusoidal ac. gear.

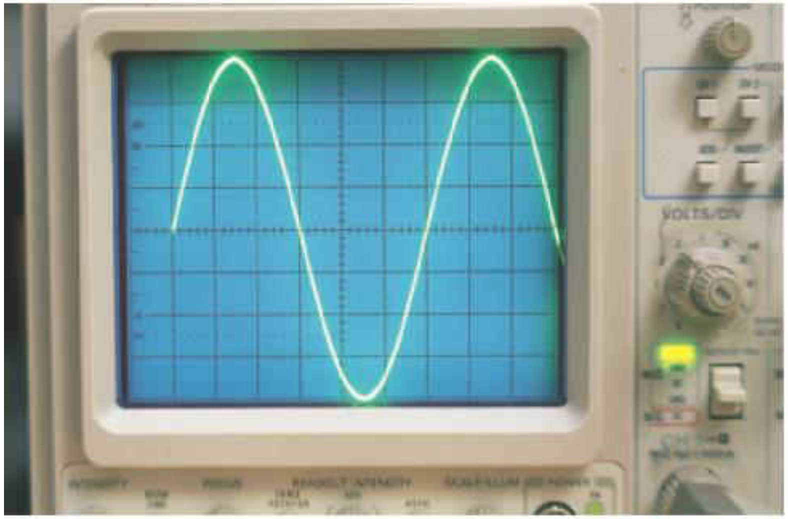
- The unit of Figure can produce a variety of variable-frequency waveforms, including sinusoidal, square wave, triangular, and so on.
- Waveforms such as these are commonly used to test electronic

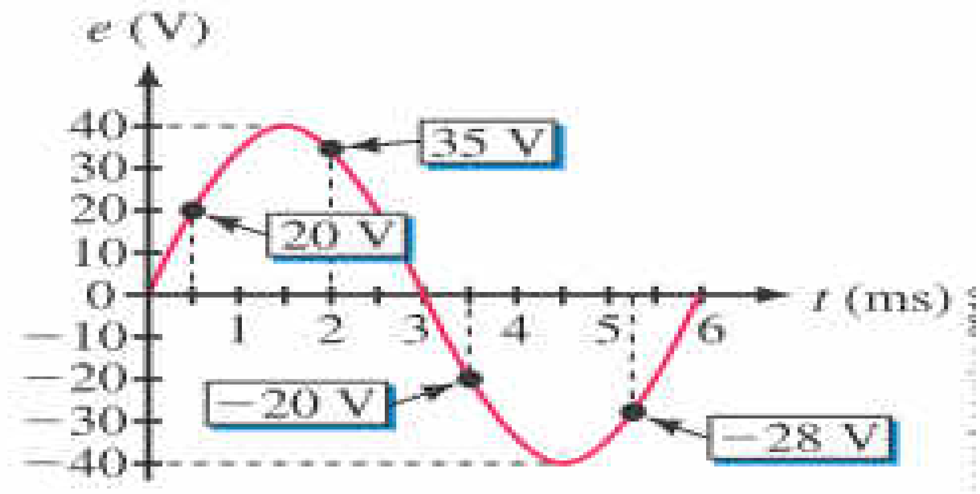
(a) Model 4045 Function Generator (b) Sample waveforms

0 Electronic function generators provide a variety of variable-frequency, variable-amplitude waveforms.

Instantaneous Value

- As the coil voltage changes from instant to instant. The value of voltage at any point on the waveform is referred to as its **instantaneous value.**





(b) Values scaled from the photograph

- The voltage has a peak value of 40 volts
- The cycle time of 6 ms.

✓ at t = 0 ms, the voltage is zero.

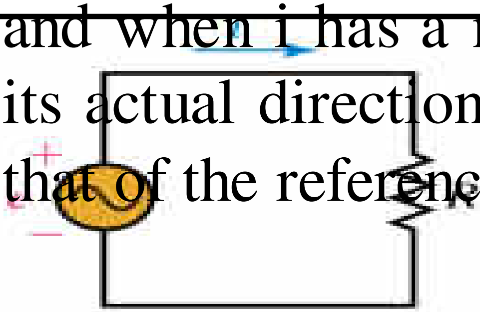
✓ at t=0.5 ms, the voltage is 20V.

Voltage and Current Conventions for ac

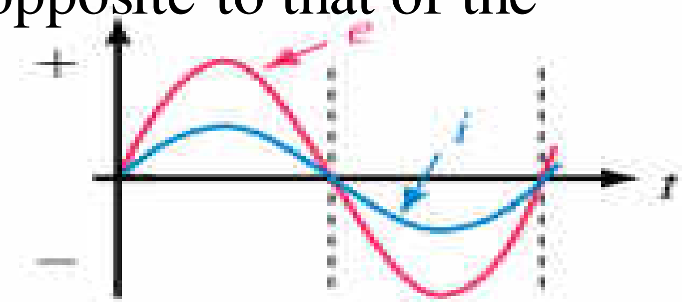
- First, we assign reference polarities for the source and a reference direction for the current.

- We then use the convention that, when e has a positive value, its actual polarity is the same as the reference polarity, and when e has a negative value, its actual polarity is opposite to that of the reference.

- For current, we use the convention that when i has a positive value, its actual direction is the same as the reference arrow,
- and when i has a negative value, its actual direction is opposite to that of the reference.

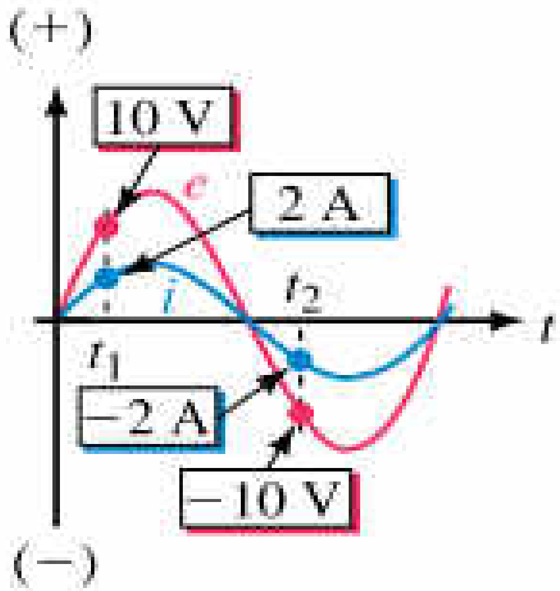


(a) References for voltage and current.

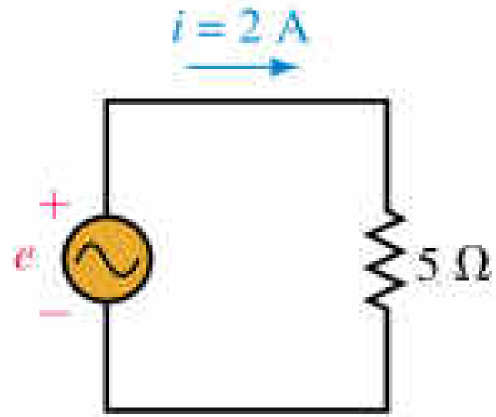


(b) During the first half-cycle, voltage polarity and current direction are as shown in (a). Therefore, *e* and *i* are positive. During the second half-cycle, voltage polarity and current direction are opposite to that shown in (a). Therefore, *e* and *i* are negative.

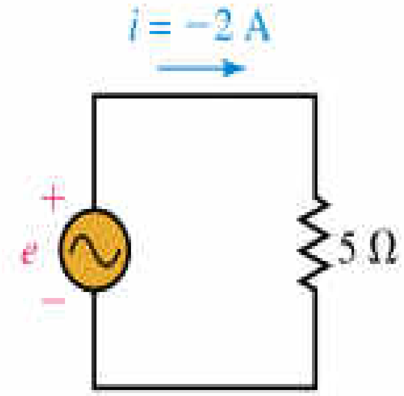
Voltage and Current Conventions for ac



(a)



(b) Time t_1: $e = 10$ V and $i = 2$ A. Thus voltage and current have the polarity and direction indicated

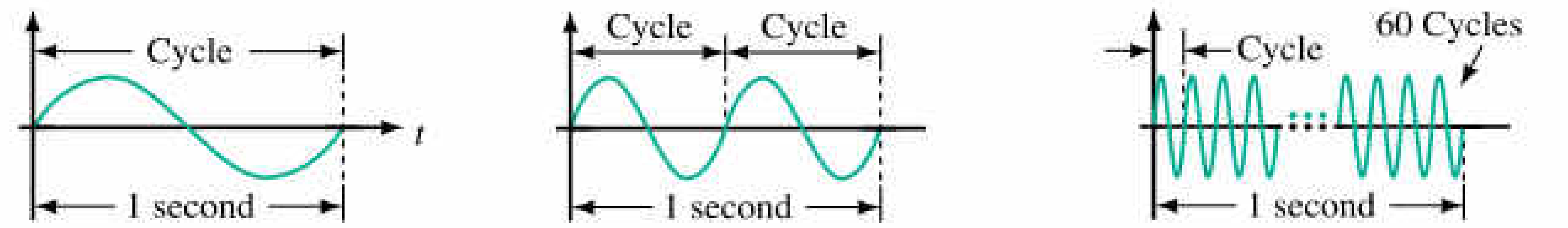


(c) Time t_2: $e = -10$ V and $i = -2$ A. Thus, voltage polarity is opposite to that indicated and current direction is opposite to the arrow direction.

Attributes of Periodic Waveforms

- Periodic waveforms (i.e., waveforms that repeat at regular intervals), regardless of their wave shape, may be described by a group of attributes such as:
  - ✓ Frequency, Period, Amplitude, Peak value.

Frequency: The number of cycles per second of a waveform is defined



(a) 1 cycle per second = 1 Hz (b) 2 cycles per second = 2 Hz (c) 60 cycles per second = 60 Hz

- Frequency is denoted by the lower-case letter f.
- In the SI system, its unit is the hertz (Hz, named in honor of pioneer researcher Heinrich Hertz, 1857–1894).

1 Hz = 1 cycle per second

Attributes of Periodic Waveforms

Frequency Ranges:

- The range of frequencies is huge.
  - ✓ Power line frequencies, for example, are 60 Hz in North America and 50 Hz in many other parts of the world.
  - ✓ Audible sound frequencies range from about 20 Hz to about 20 kHz.
  - ✓ The standard AM radio band occupies from 550 kHz to 1.6 MHz
  - ✓ The FM band extends from 88 MHz to 108 MHz.
  - ✓ TV transmissions occupy several bands in the 54-MHz to 890-MHz range.
  - ✓ Above 300 GHz are optical and X-ray frequencies.

Attributes of Periodic Waveforms

- **Period:**

- The period, T, of a waveform, is the duration of one cycle.

- It is the inverse of frequency.

$$T = \frac{1}{f} \quad (\mathrm{s})$$

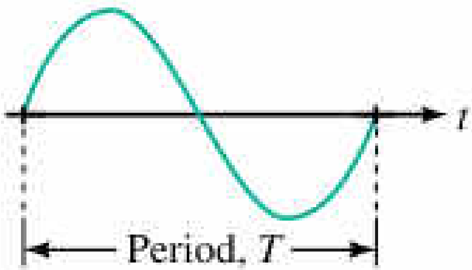


FIGURE 15–15 Period T is the duration of one cycle, measured in seconds.

- The period of a waveform can be measured between any two corresponding points (Often it is measured between zero points because they are easy to establish on an oscilloscope trace).

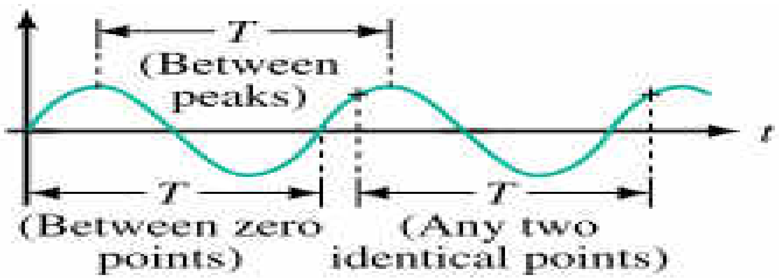


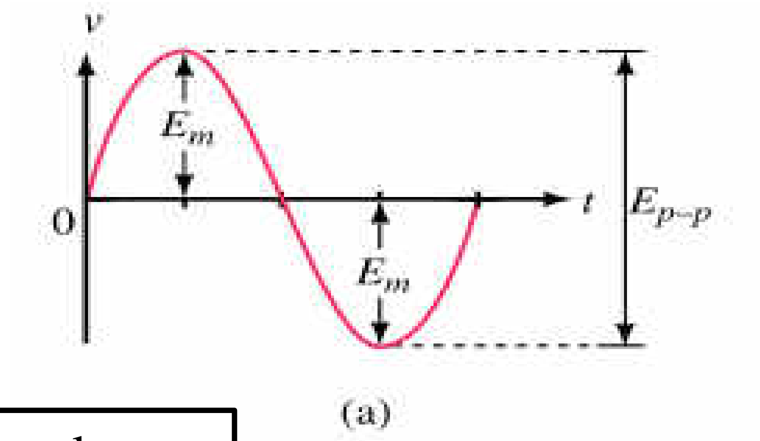
FIGURE 15–17 Period may be measured between any two corresponding points.

Attributes of Periodic Waveforms

Amplitude , Peak-Value, and Peak-to-Peak Value

Amplitude (Em):

The amplitude of a sine wave is the distance from its average to its peak.



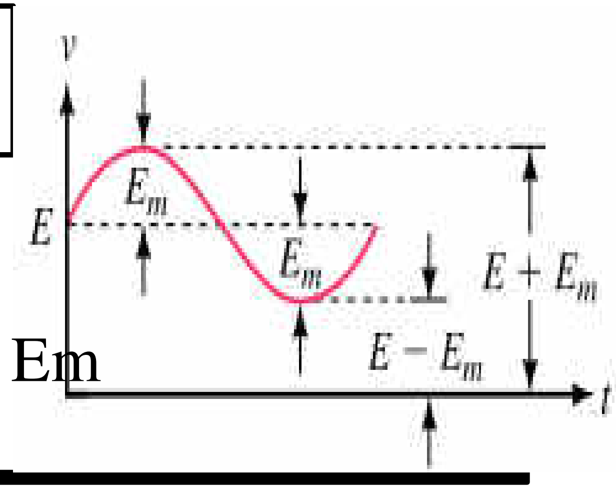
(a)

Peak-to-Peak Value (Ep-p):

It is measured between minimum and maximum peaks.

Peak Value

The peak value of a voltage or current is its maximum value with respect to zero.



In this figure : Peak voltage = E + Em

The Basic Sine Wave Equation

The voltage produced by the previously described generator is:

$$e = E_m \sin \alpha \quad (V)$$

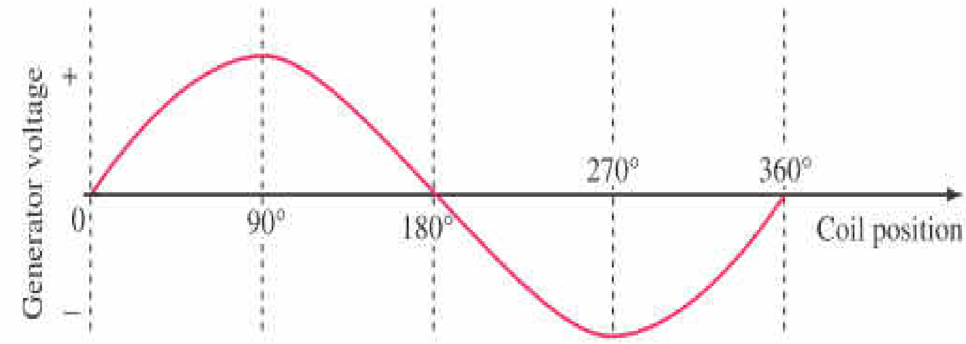


FIGURE 15–7 Coil voltage versus angular position.

- Em: the maximum coil voltage and
- α : the instantaneous angular position of the coil.

- For a given generator and rotational velocity, Em is constant.)
- Note that a 0° represents the horizontal position of the coil and that one complete cycle corresponds to 360°.

Angular Velocity (ω)

The rate at which the generator coil rotates is called its angular velocity

If the coil rotates through an angle of 30° in one second, its angular velocity is 30° per second.

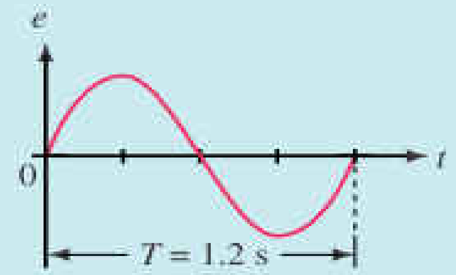
- When you know the angular velocity of a coil and the length of time that it has rotated, you can compute the angle through which it has turned using:

$$\alpha = \omega t$$

EXAMPLE 15–6 If the coil of Figure 15–23 rotates at $\omega = 300°/\text{s}$, how long does it take to complete one revolution?

Solution One revolution is 360°. Thus,

$$t = \frac{\alpha}{\omega} = \frac{360 \text{ degrees}}{300 \dfrac{\text{degrees}}{\text{s}}} = 1.2 \text{ s}$$

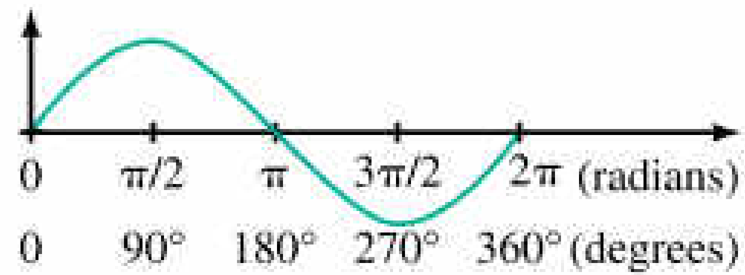


Since this is one period, we should use the symbol T. Thus, $T = 1.2$ s, as in Figure 15–25.

Radian Measure

- In practice, q is usually expressed in radians per second,
- Radians and degrees are related by :

$$2\pi \text{ radians} = 360^\circ$$



(b) Cycle length scaled in degrees and radians

For Conversion:

$$\alpha_{\text{radians}} = \frac{\pi}{180^\circ} \times \alpha_{\text{degrees}}$$

$$\alpha_{\text{degrees}} = \frac{180^\circ}{\pi} \times \alpha_{\text{radians}}$$

Relationship between ω, T, and f

- Earlier you learned that one cycle of sine wave may be represented as either:

$$\alpha = 2\pi \text{ rads} \qquad t = T \text{ s}$$

- Substituting these into: $\alpha = \omega t$

$$\omega T = 2\pi \quad \text{(rad)}$$

$$\omega = \frac{2\pi}{T} \quad \text{(rad/s)}$$

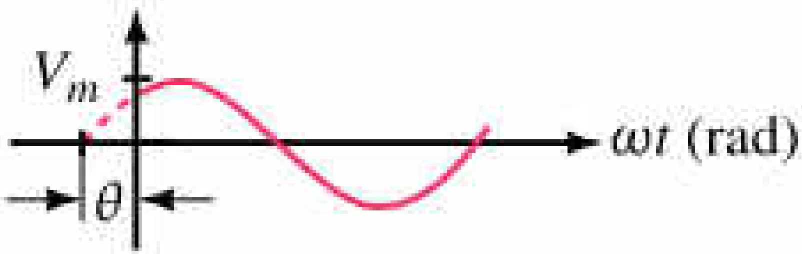
$$\omega = 2\pi f \quad \text{(rad/s)}$$

Sinusoidal Voltages and Currents as Functions of Time:

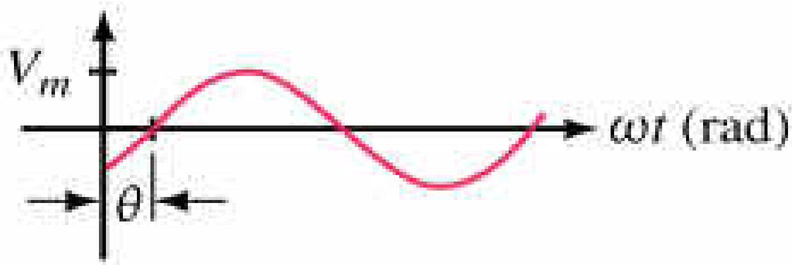
- We could replace the angle α as: $e = E_m \sin \omega t$

Voltages and Currents with Phase Shifts

- If a sine wave does not pass through zero at t =0 s, it has a phase shift.
- Waveforms may be shifted to the left or to the right



(a) $v = V_m \sin(\omega t + \theta)$



(b) $v = V_m \sin(\omega t - \theta)$

Introduction to Phasors

- A phasor is a rotating line whose projection on a vertical axis can be used to represent sinusoidally varying quantities.

- To get at the idea, consider the red line of length **Vm** shown in Figure :

The vertical projection of this line (indicated in dotted red) is :

$$v = V_m \sin \alpha.$$

- By assuming that the phasor rotates at angular velocity of ω rad/s in the counterclockwise direction

$$v = V_m \sin \omega t$$

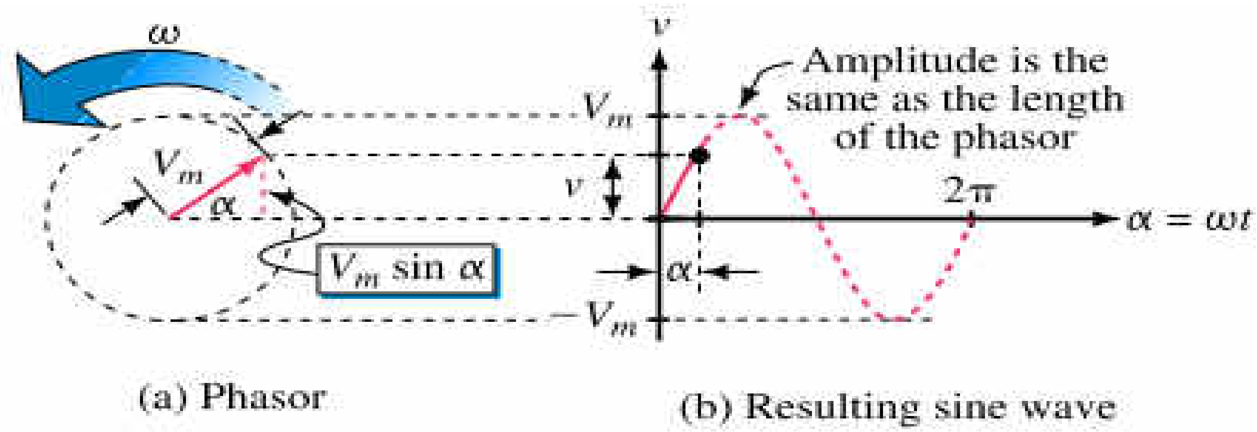
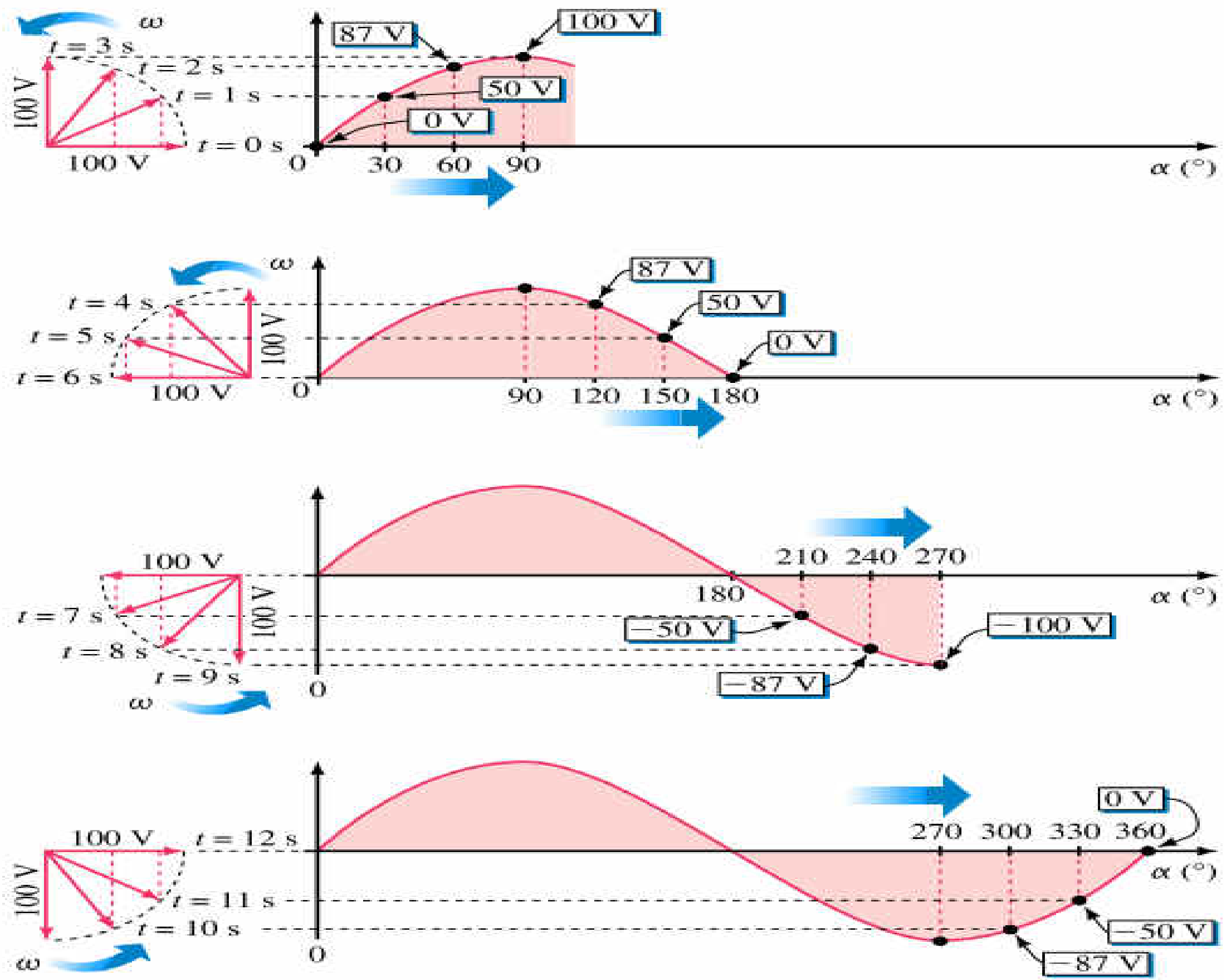


FIGURE 15–34 As the phasor rotates about the origin, its vertical projection creates a sine wave. (Figure 15–35 illustrates the process.)

Introduction to Phasors



Introduction to Phasors

EXAMPLE 15–15 Draw the phasor and waveform for current $i = 25 \sin \omega t$ mA for $f = 100$ Hz.

Solution The phasor has a length of 25 mA and is drawn at its $t = 0$ position, which is zero degrees as indicated in Figure 15–36. Since $f = 100$ Hz, the period is $T = 1/f = 10$ ms.

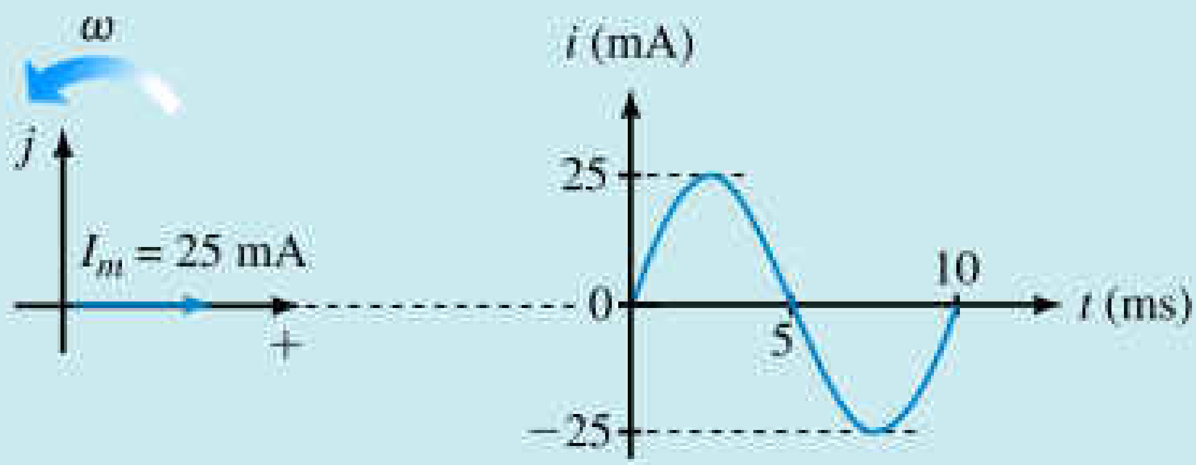


FIGURE 15–36 The reference position of the phasor is its $t = 0$ position.

Shifted Sine Waves Phasor Representation

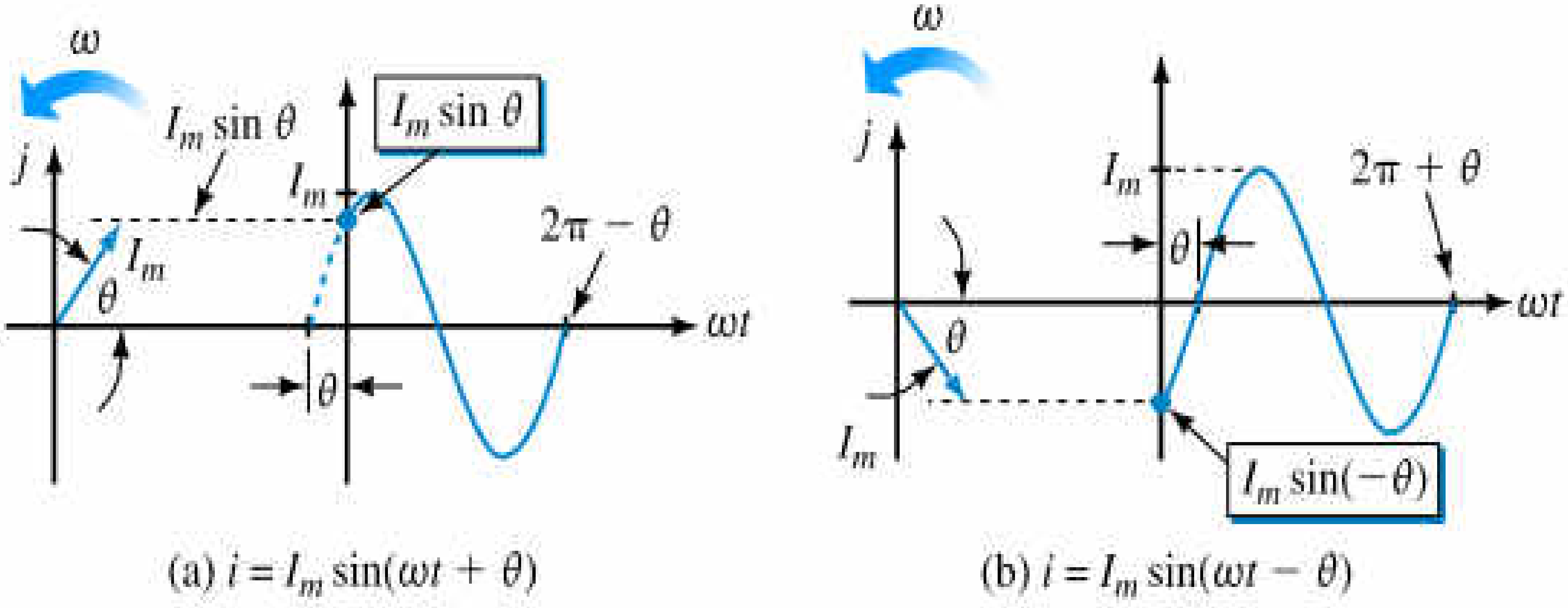
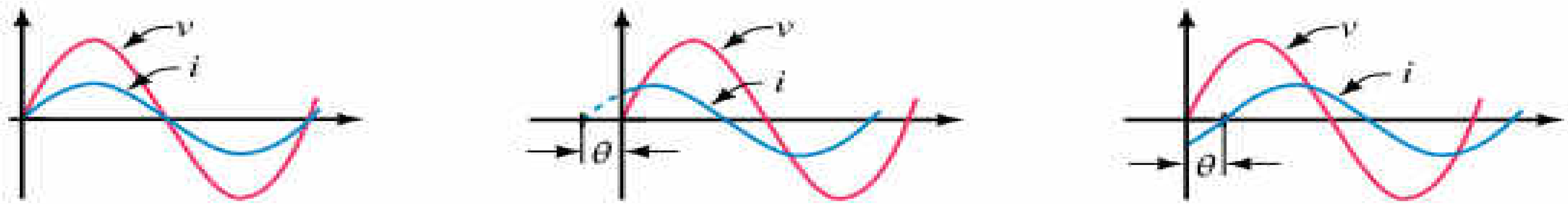


FIGURE 15–37 Phasors for shifted waveforms. Angle θ is the position of the phasor at $t = 0$ s.

Phasor Difference

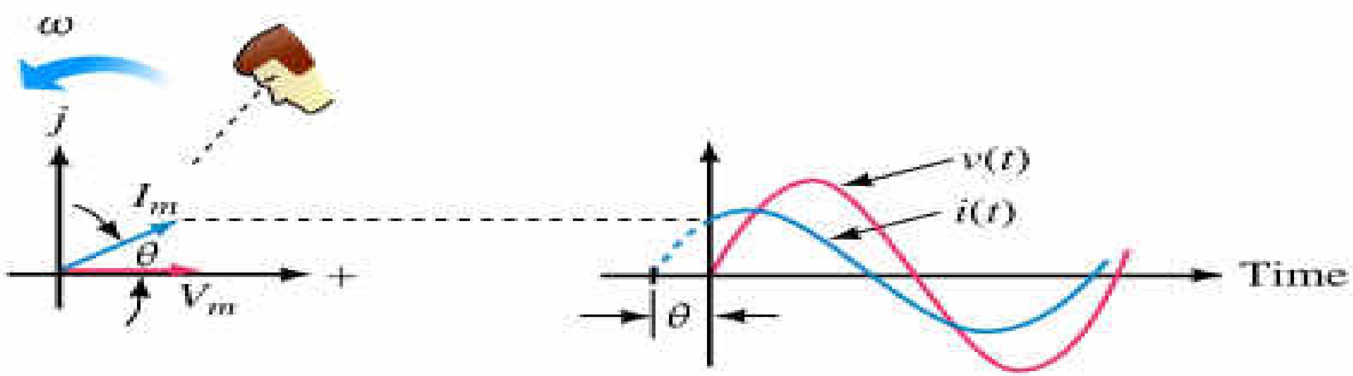
- Phase difference refers to the angular displacement between different waveforms of the same frequency.



(a) In phase (b) Current leads (c) Current lags

FIGURE 15–40 Illustrating phase difference. In these examples, voltage is taken as reference.

- The terms lead and lag can be understood in terms of phasors. If you observe phasors rotating as in Figure, the one that you see passing first is leading and the other is lagging.



(a) I_m leads V_m (b) Therefore, $i(t)$ leads $v(t)$

AC Waveforms and Average Value

- Since ac quantities constantly change its value, we need one single numerical value that truly represents a waveform over its complete cycle.

Average Values:

- To find the average of a set of marks for example, you add them, then divide by the number of items summed.
- For waveforms, the process is conceptually the same. You can sum the instantaneous values over a full cycle, then divide by the number of points used.
- The trouble with this approach is that waveforms do not consist of discrete values.

Average in Terms of the Area Under a Curve:

$$\text{average} = \frac{\text{area under curve}}{\text{length of base}}$$

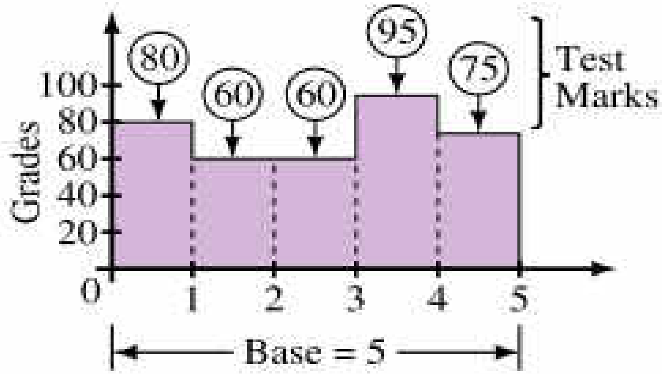


FIGURE 15–50 Determining average by area.

$$\text{average} = (80 + 60 + 60 + 95 + 75)/5 = 74$$

Or use area

$$\frac{(80 \times 1) + (60 \times 2) + (95 \times 1) + (75 \times 1)}{5} = 74$$

AC Waveforms and Average Value

- To find the average value of a waveform, divide the area under the waveform by the length of its base.
- Areas above the axis are counted as positive, while areas below the axis are counted as negative.
- This approach is valid regardless of waveshape.

- Average values are also called **dc values**, because dc meters indicate average values rather than instantaneous values.

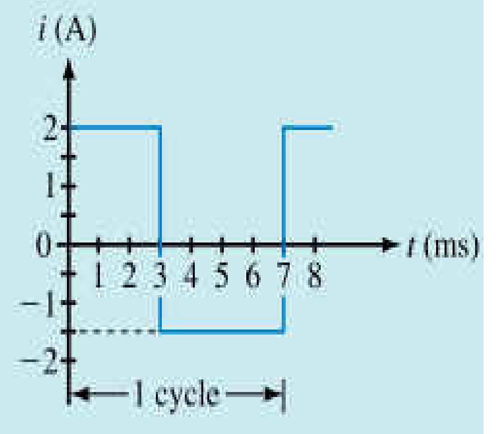
AC Waveforms and Average Value

EXAMPLE 15–23

a. Compute the average for the current waveform of Figure 15–51.

b. If the negative portion of Figure 15–51 is −3 A instead of −1.5 A, what is the average?

c. If the current is measured by a dc ammeter, what will the ammeter indicate?



Solution

a. The waveform repeats itself after 7 ms. Thus, $T = 7$ ms and the average is

$$I_{avg} = \frac{(2\text{ A} \times 3\text{ ms}) - (1.5\text{ A} \times 4\text{ ms})}{7\text{ ms}} = \frac{6-6}{7} = 0\text{ A}$$

b. $$I_{avg} = \frac{(2\text{ A} \times 3\text{ ms}) - (3\text{ A} \times 4\text{ ms})}{7\text{ ms}} = \frac{-6\text{ A}}{7} = -0.857\text{ A}$$

c. A dc ammeter measuring (a) will indicate zero, while for (b) it will indicate −0.857 A.

4

Transformers

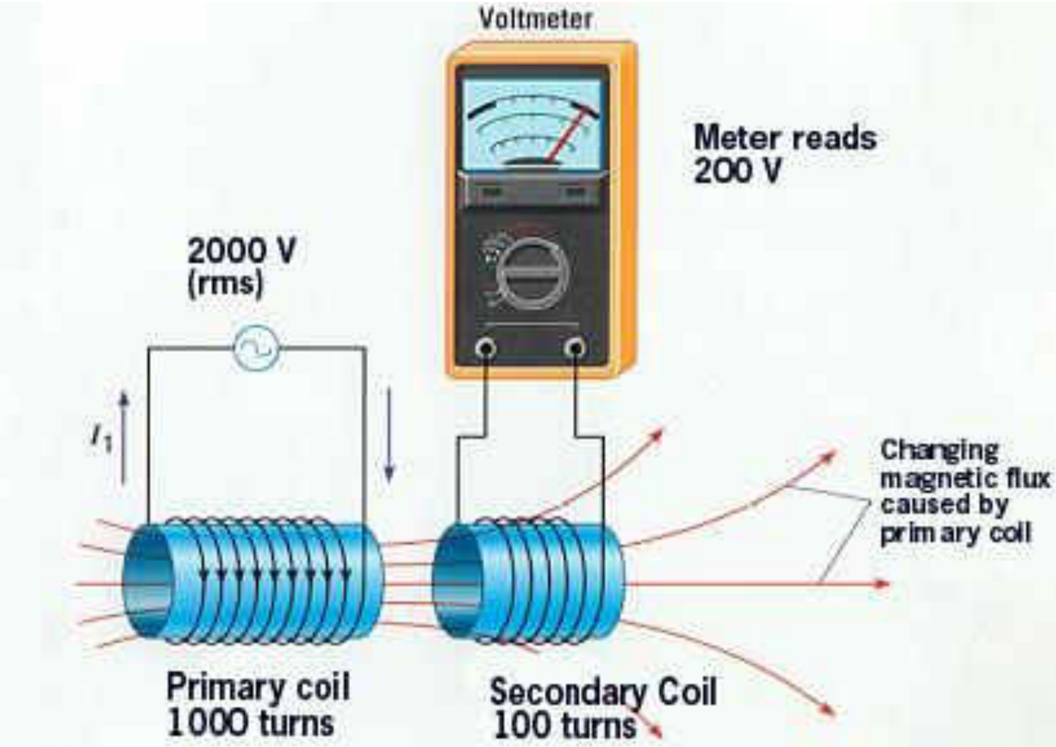
Transformer

An A.C. device used to change high voltage low current A.C. into low voltage high current A.C. and vice-versa without changing the frequency

In brief,

1. Transfers electric power from one circuit to another
2. It does so without a change of frequency
3. It accomplishes this by electromagnetic induction
4. Where the two electric circuits are in mutual inductive influence of each other.

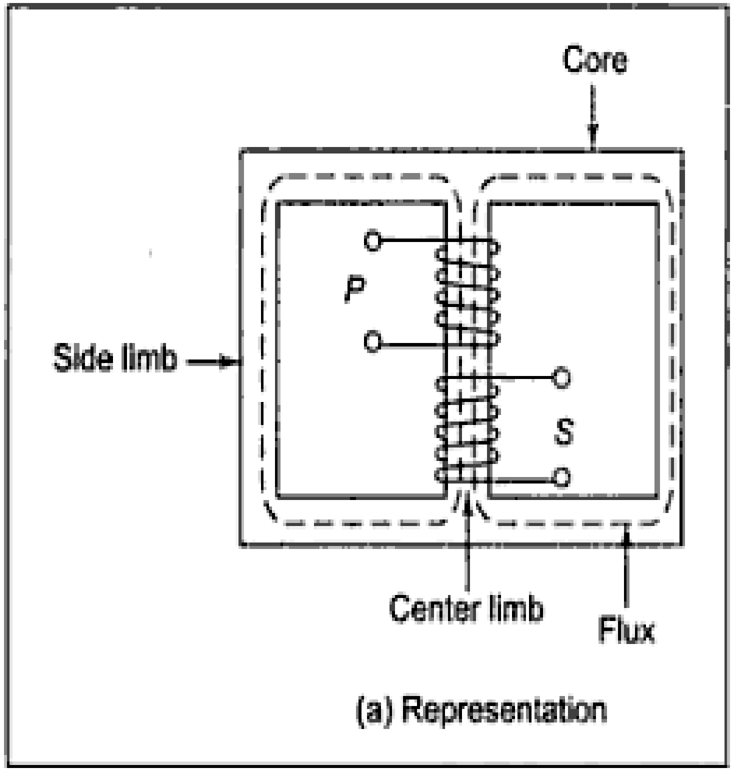
Principle of operation

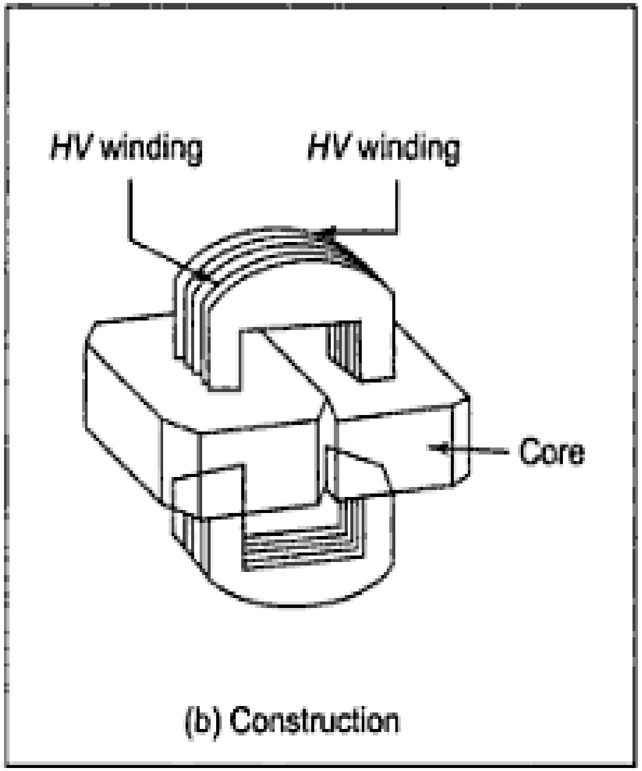


It is based on principle of MUTUAL INDUCTION.

According to which an e.m.f. is induced in a coil when current in the neighbouring coil changes.

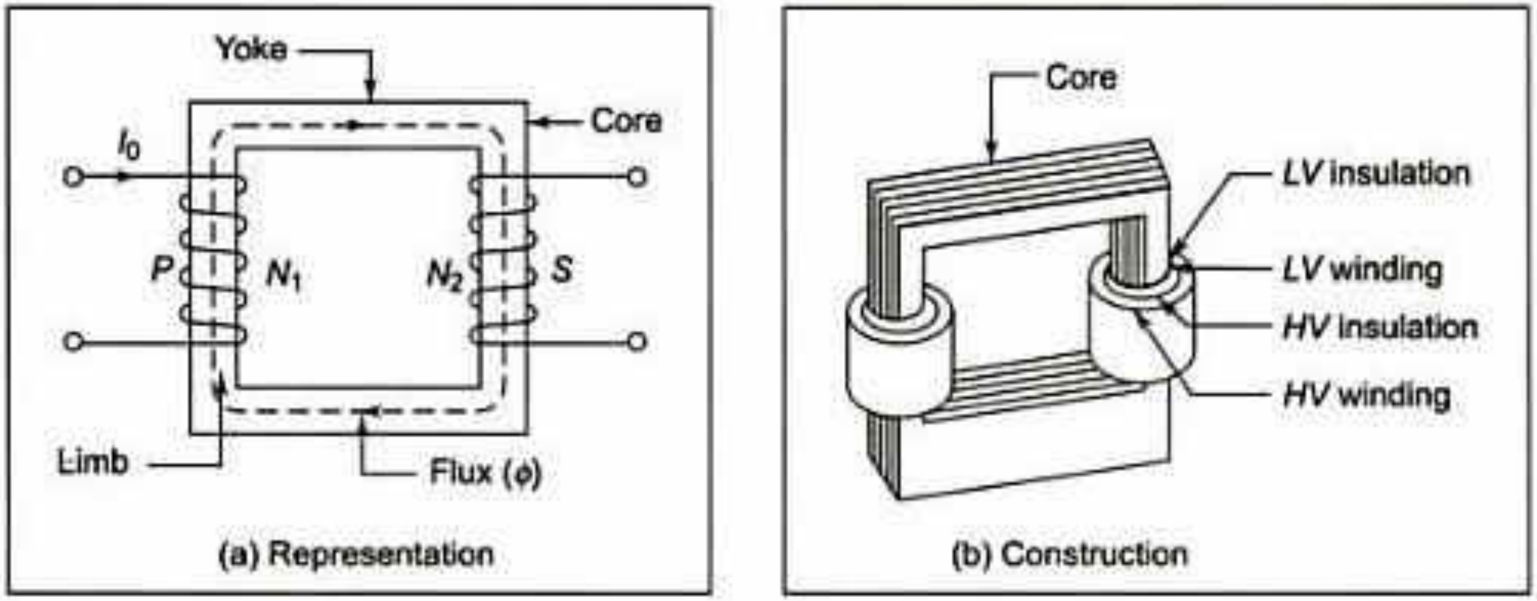
Constructional detail : Shell type





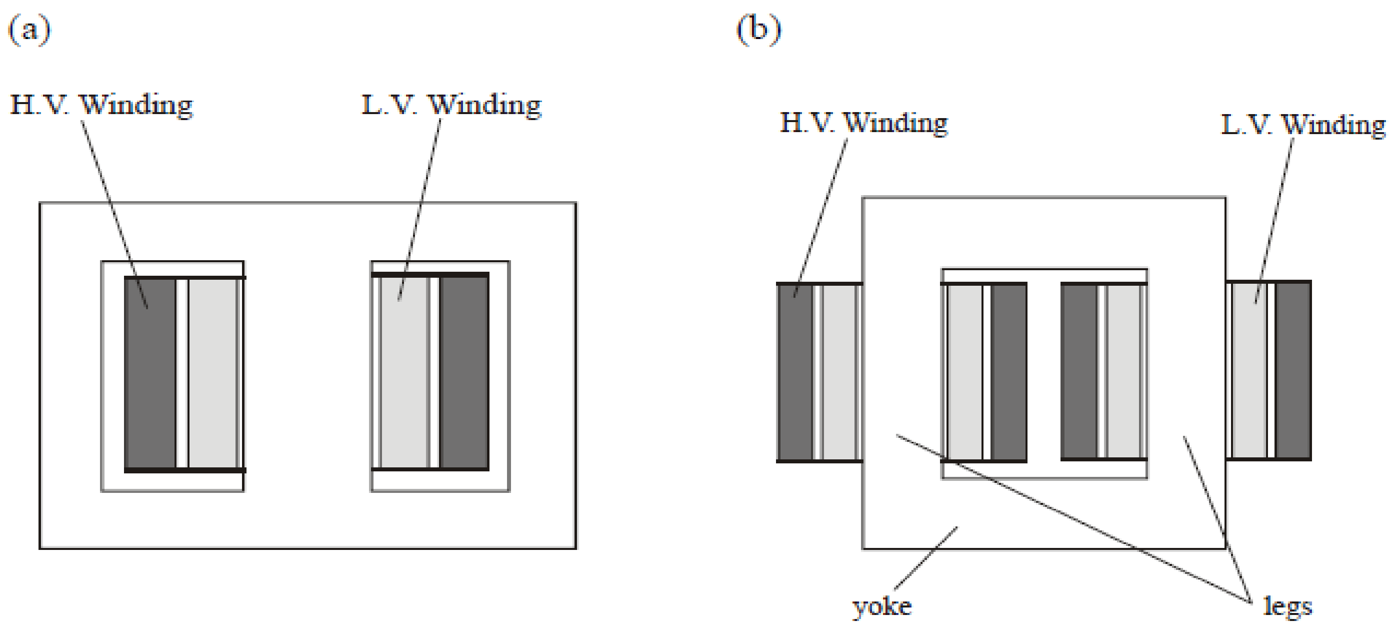
- Windings are wrapped around the center leg of a laminated core.

Core type



- Windings are wrapped around two sides of a laminated square core.

Sectional view of transformers

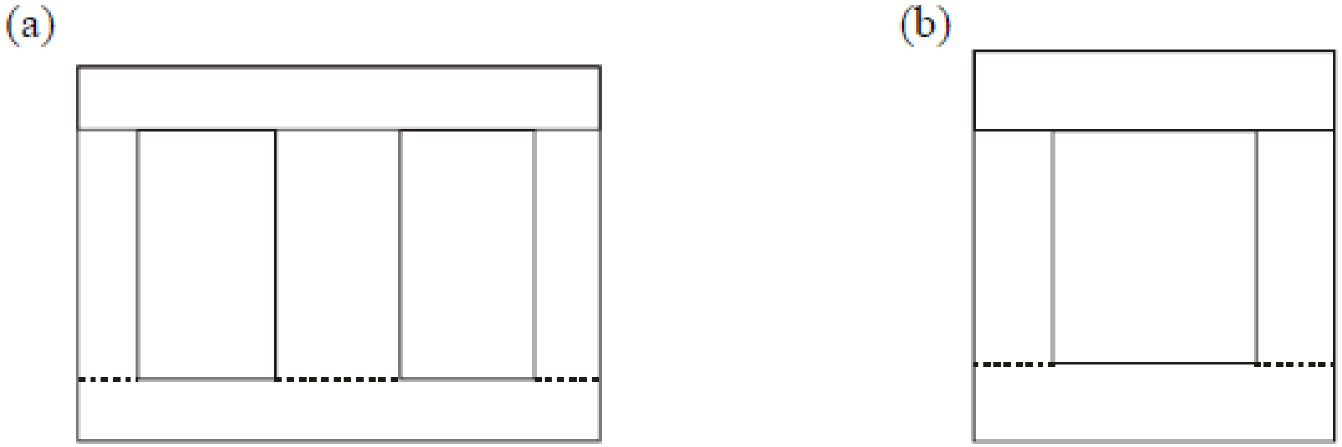


(a) Shell-type transformer, (b) core-type transformer

Note:

High voltage conductors are smaller cross section conductors than the low voltage coils

Construction of transformer from stampings



(a) Shell-type transformer, (b) core-type transformer

Core type

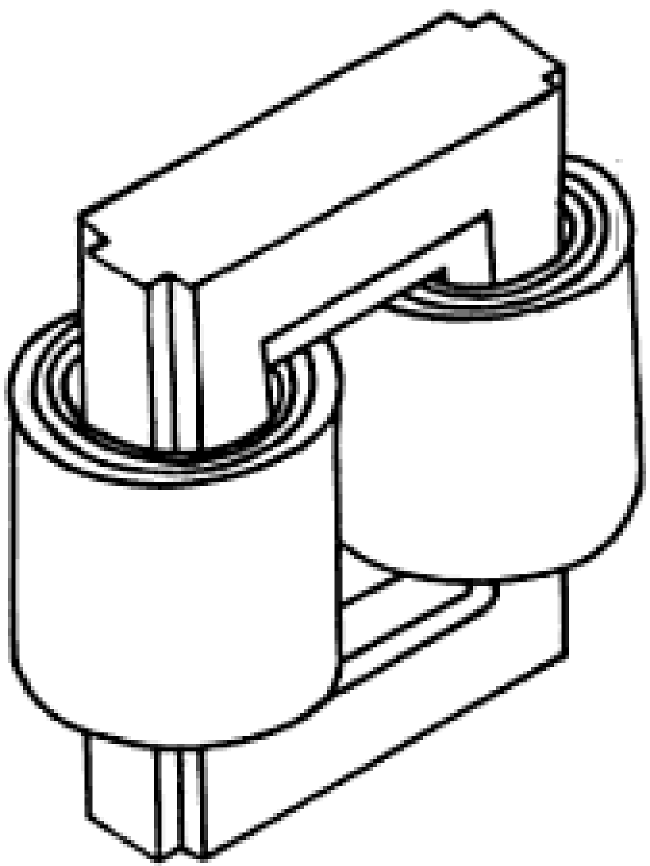


Fig1: Coil and laminations of core type transformer

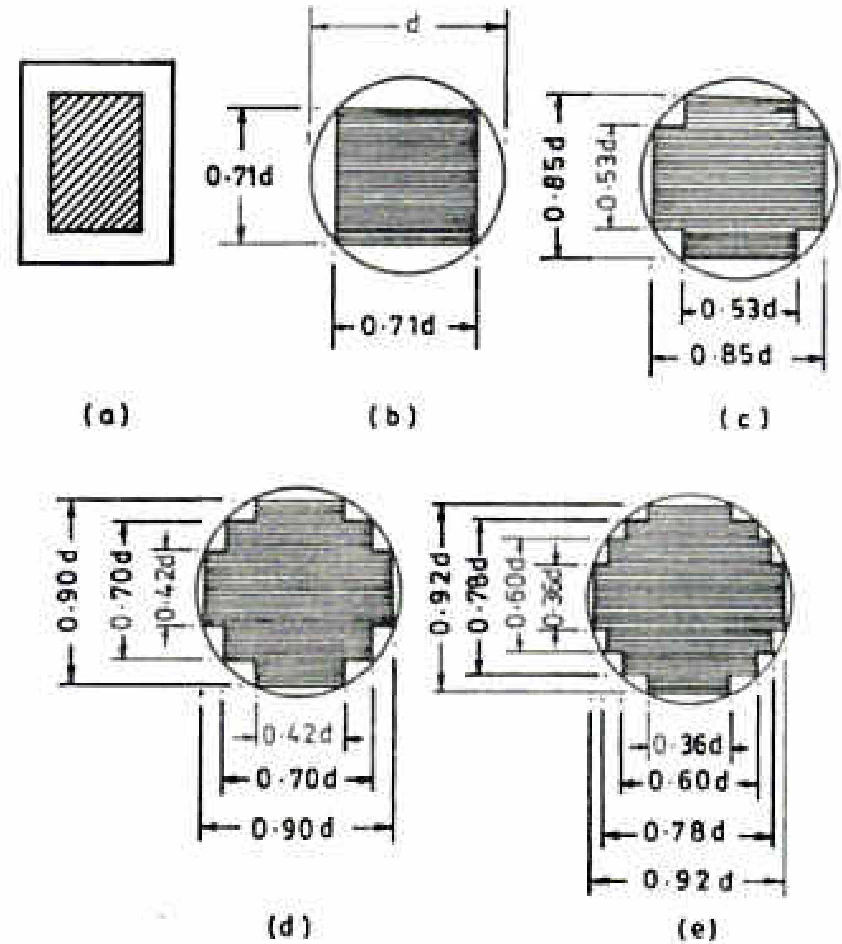


Fig2: Various types of cores

Shell type

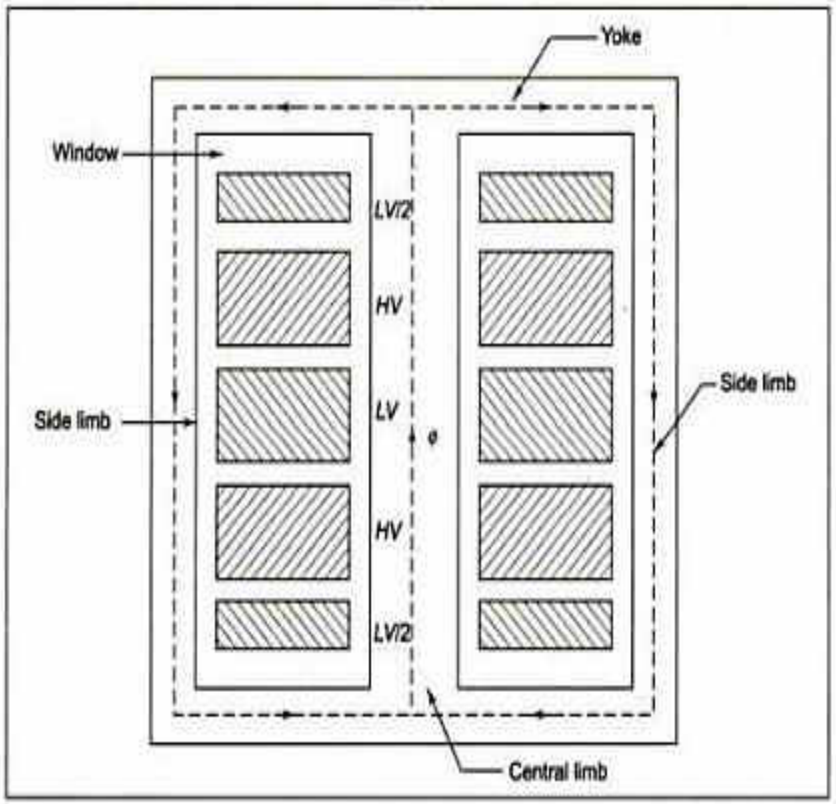
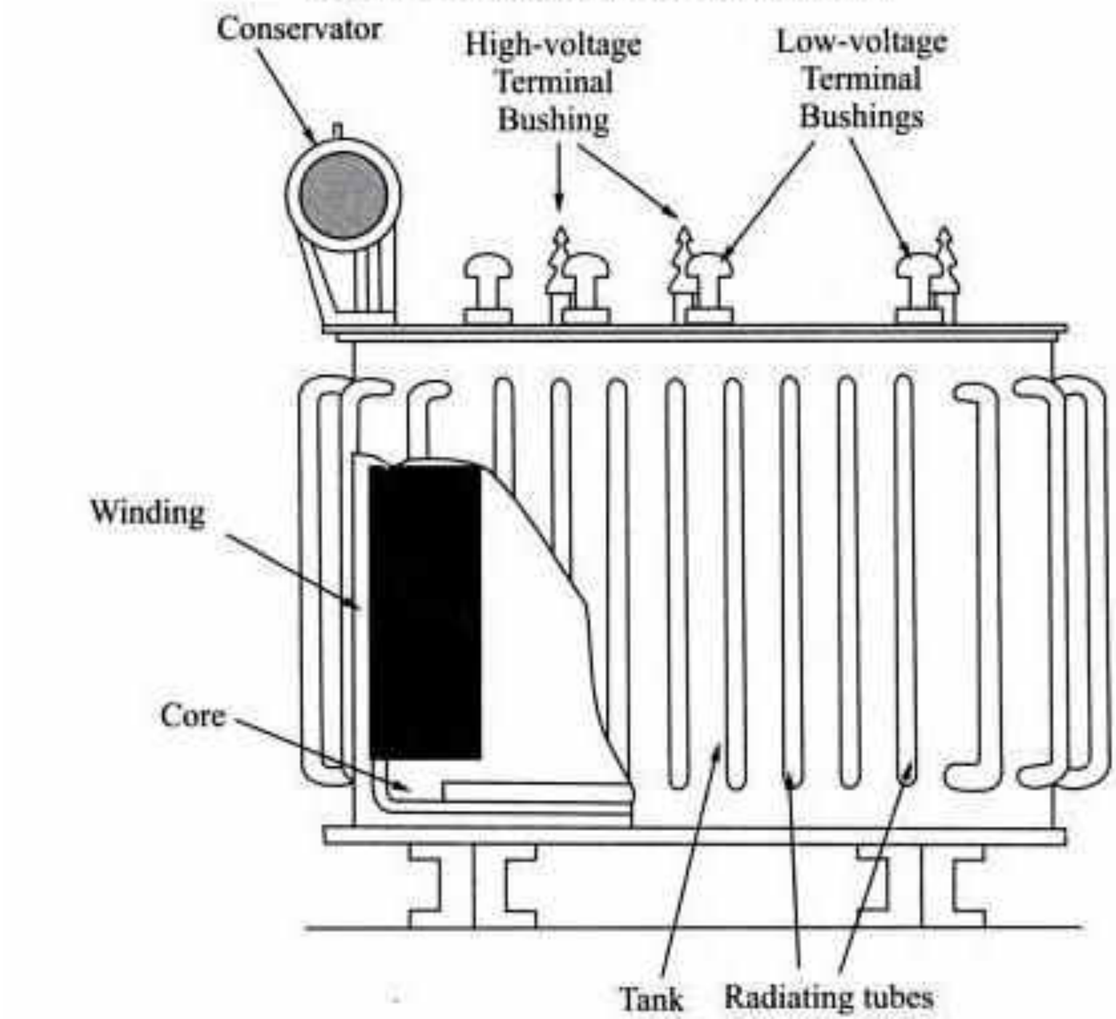


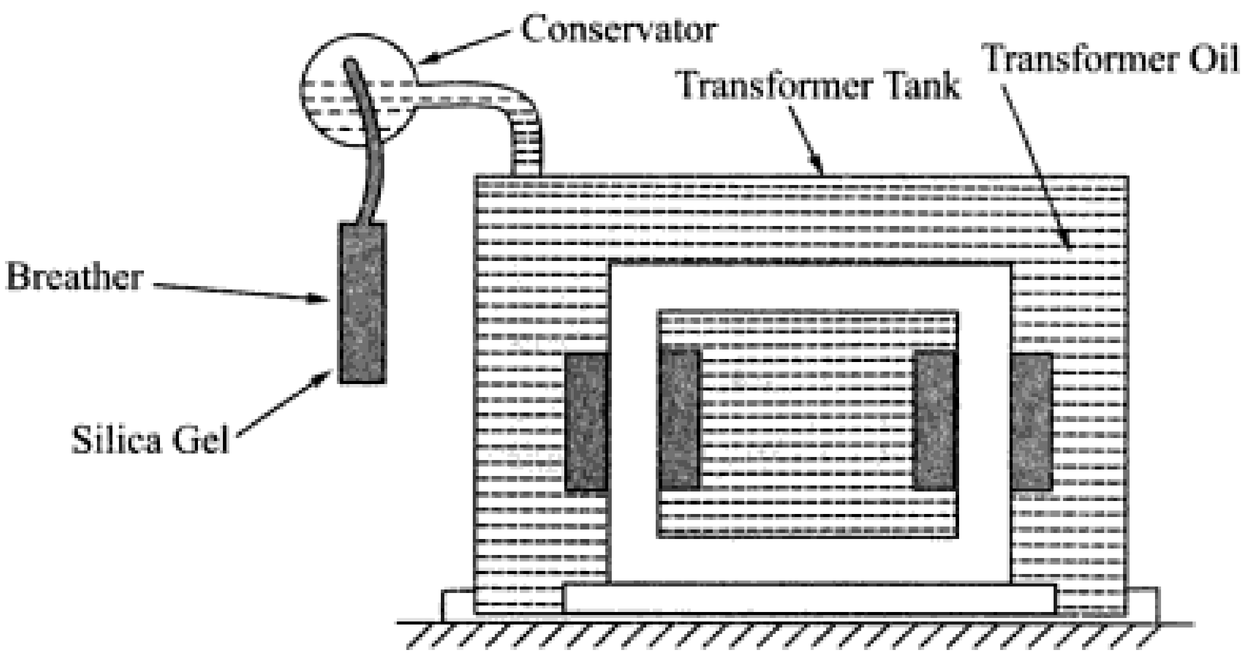
Fig: Sandwich windings

- The HV and LV windings are split into no. of sections
- Where HV winding lies between two LV windings
- In sandwich coils leakage can be controlled

Cut view of transformer

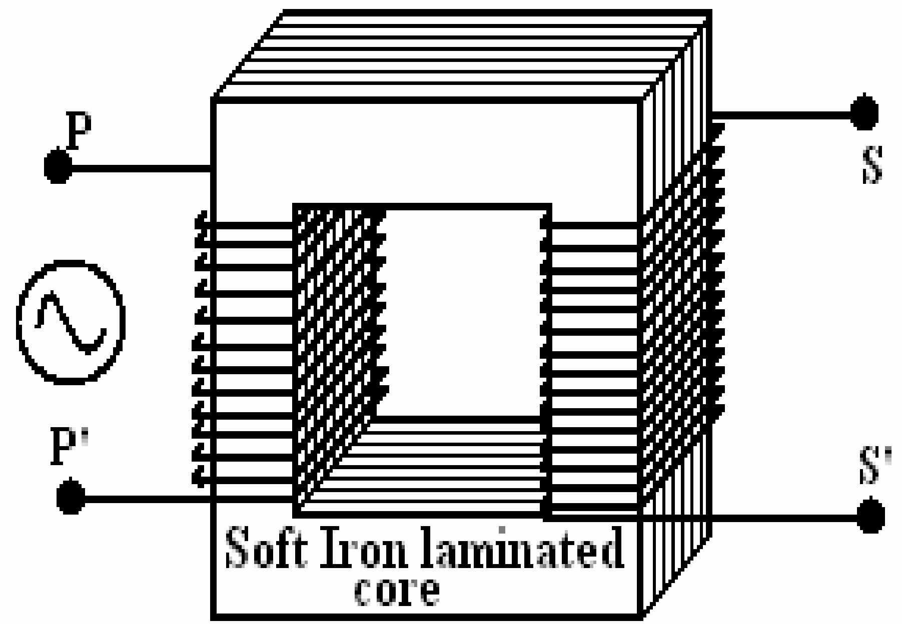


Transformer with conservator and breather



Working of a transformer

1. When current in the primary coil changes being alternating in nature, a changing magnetic field is produced
2. This changing magnetic field gets associated with the secondary through the soft iron core
3. Hence magnetic flux linked with the secondary coil changes.
4. Which induces e.m.f. in the secondary.



Single Phase Transformer

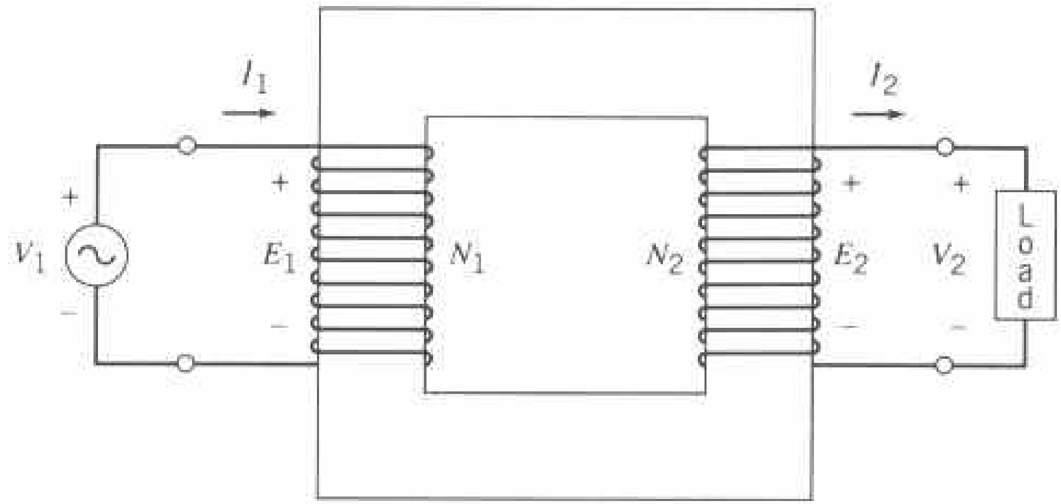


FIGURE 4.8 A transformer circuit.

- A single phase transformer
  - Two or more winding, coupled by a common magnetic core

Ideal Transformers

- **Zero leakage flux:**

  -Fluxes produced by the primary and secondary currents are confined within the core

- **The windings have no resistance:**

  - Induced voltages equal applied voltages

- **The core has infinite permeability**

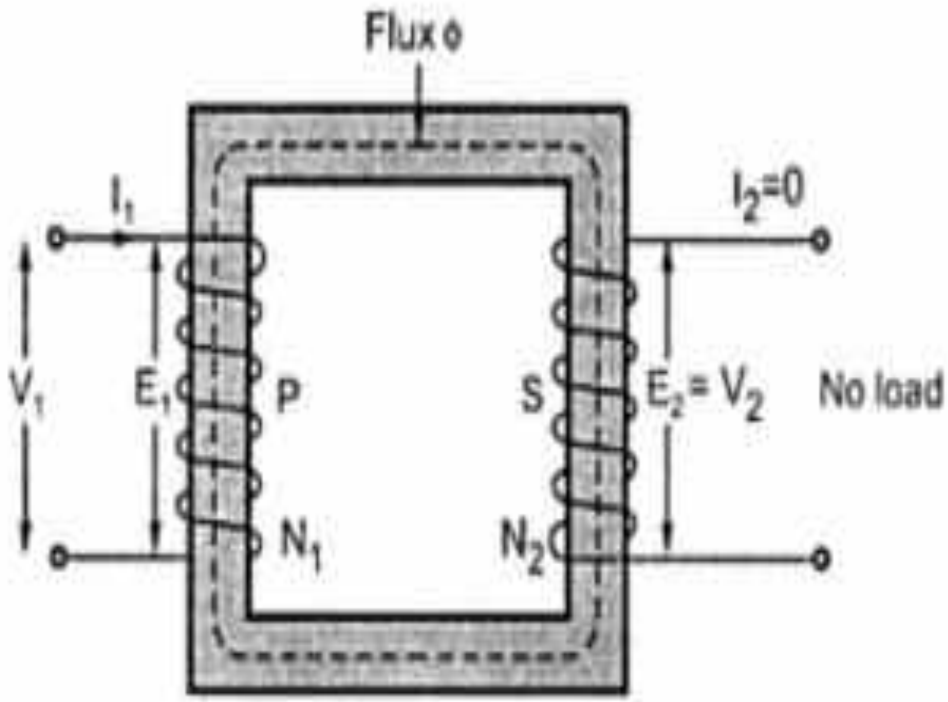
  - Reluctance of the core is zero

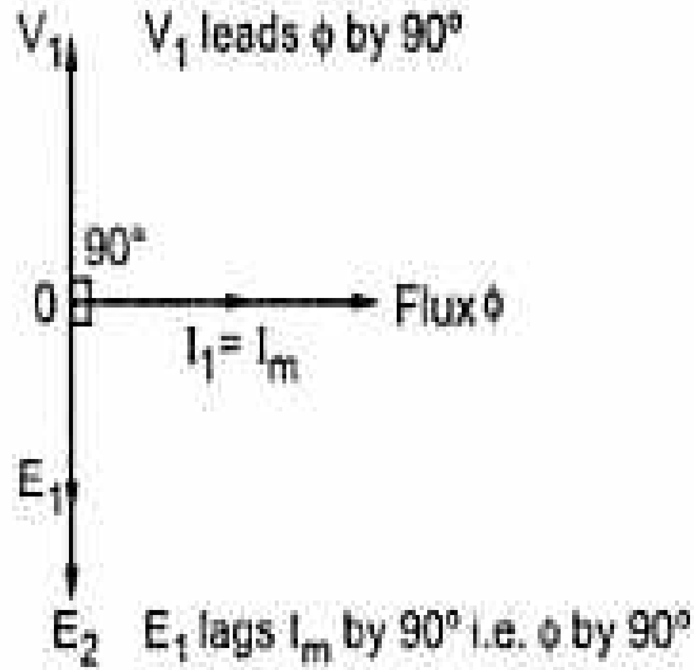
  - Negligible current is required to establish magnetic flux

- **Loss-less magnetic core**

  - No hysteresis or eddy currents

Ideal transformer



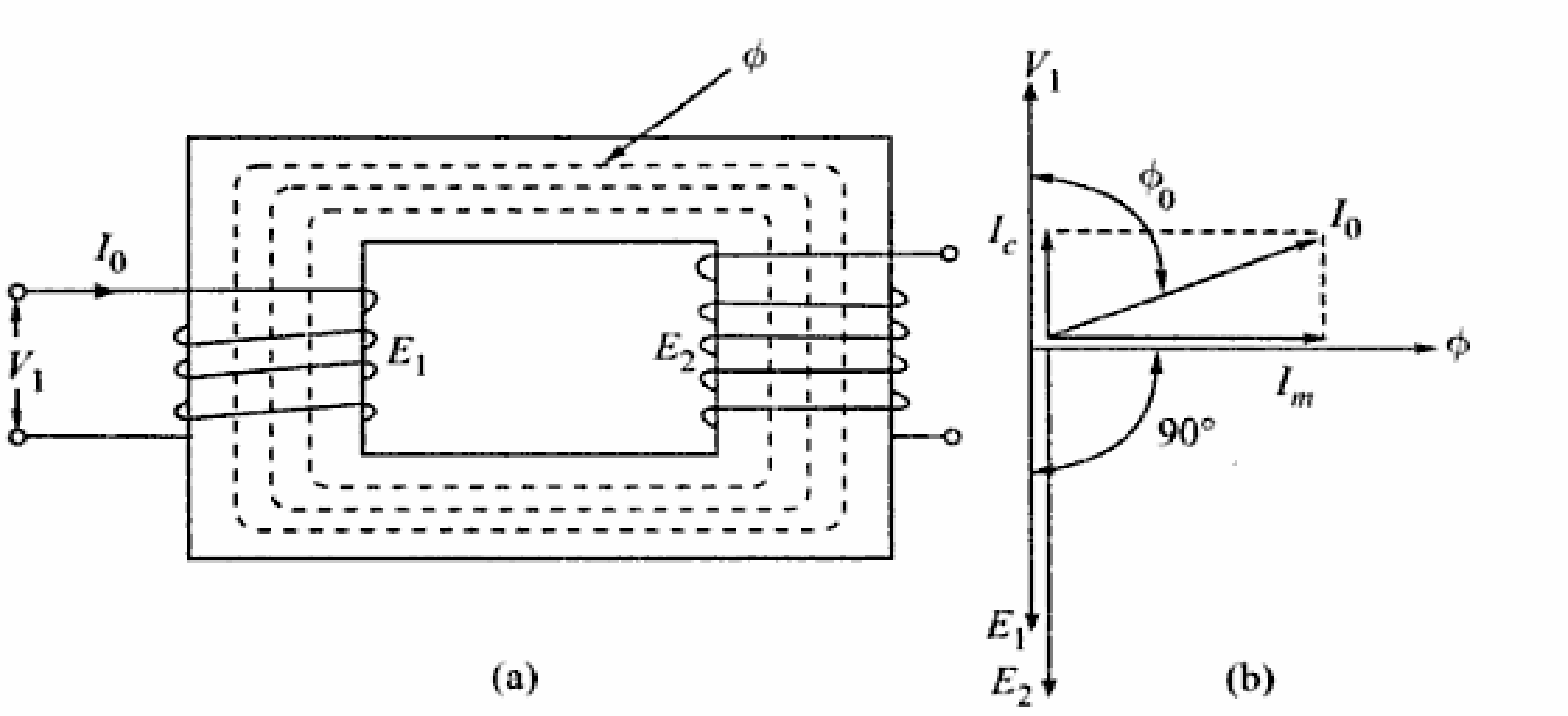


V_1 – supply voltage ; I_1- noload input current ;
V_{2-} output voltgae; I_2- output current
I_m- magnetising current;
E_1-self induced emf ; E_2- mutually induced emf

EMF equation of a transformer

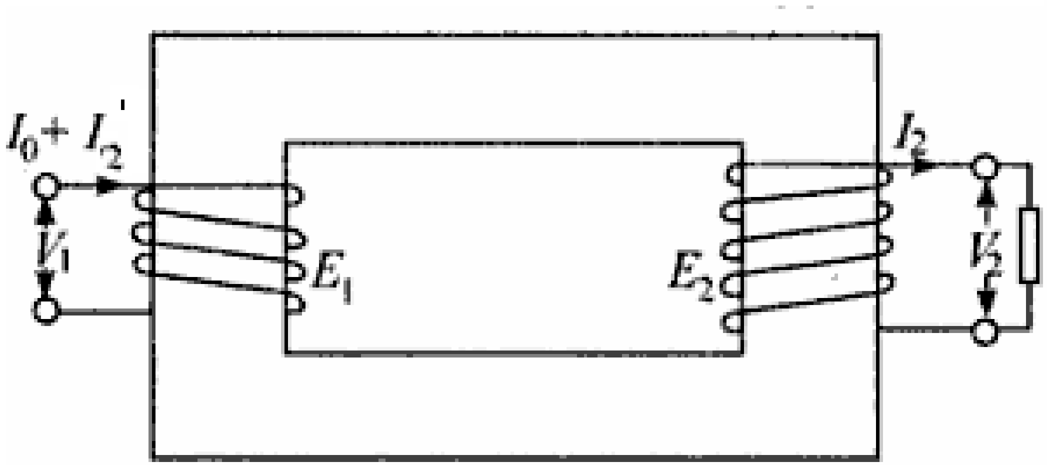
- Worked out on board /
- Refer pdf file: emf-equation-of-tranformer

Phasor diagram: Transformer on No-load



(a) Transformer on no-load (b) Phasor diagram of a transformer on no-load

Transformer on load assuming no voltage drop in the winding



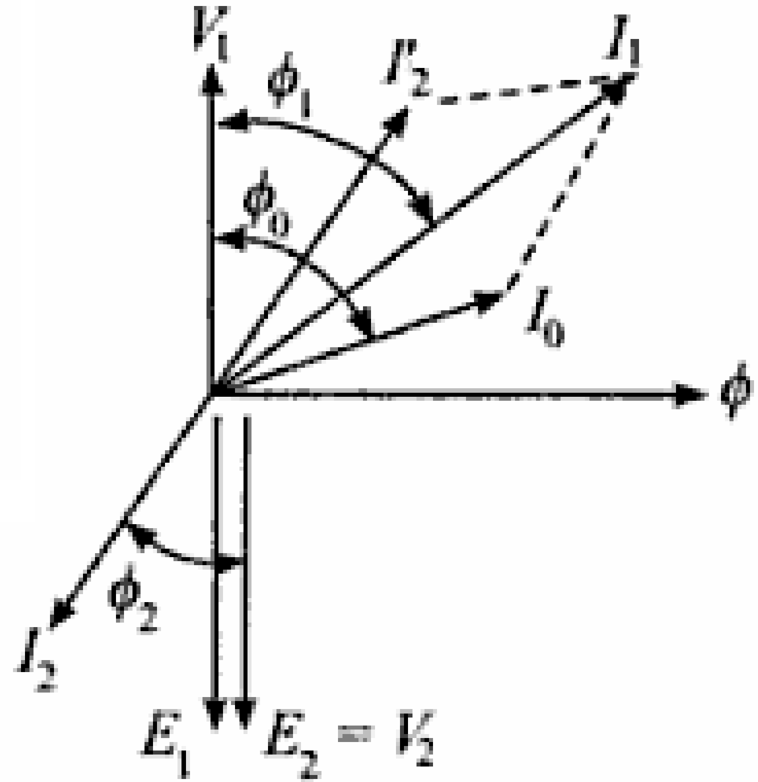


Fig shows the Phasor diagram of a transformer on load by assuming

1. No voltage drop in the winding
2. Equal no. of primary and secondary turns

Transformer on load

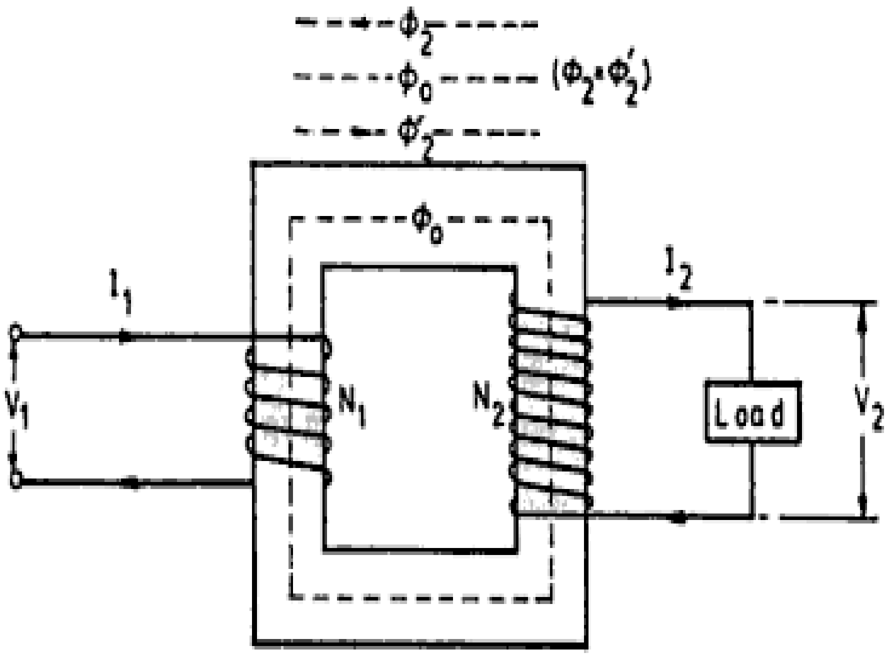


Fig. a: Ideal transformer on load

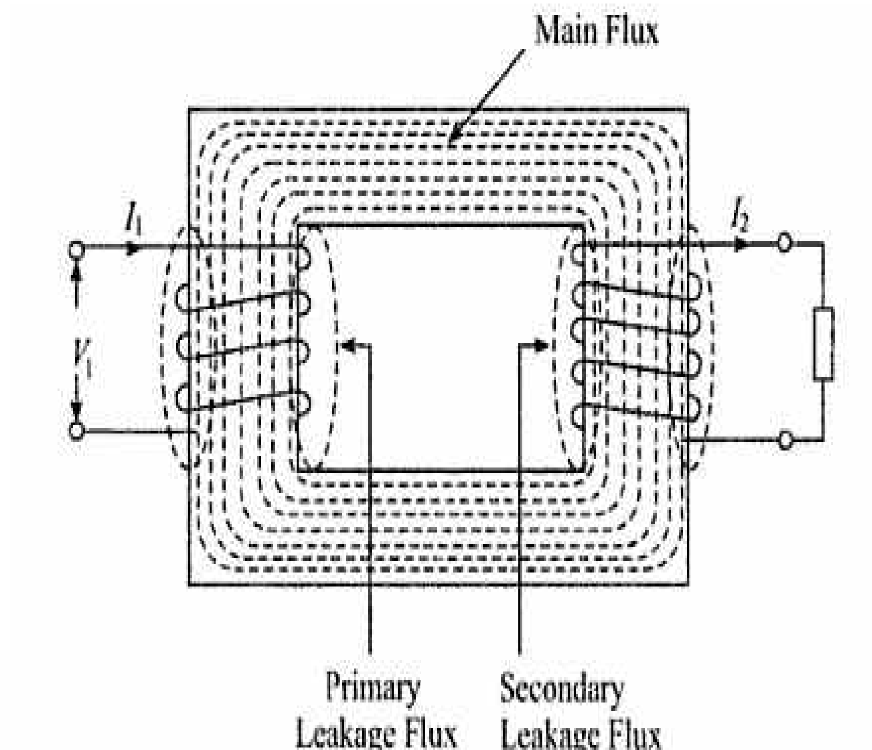
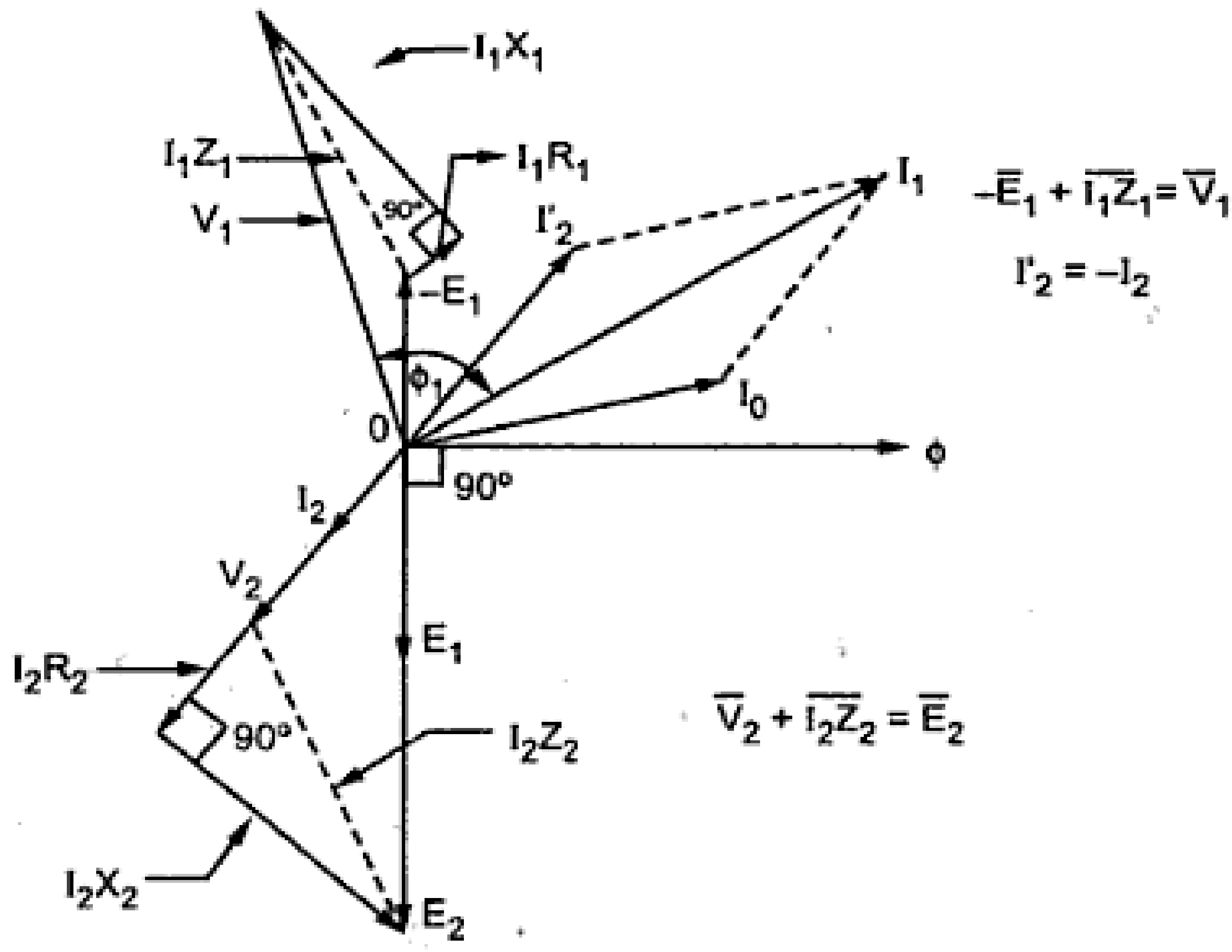
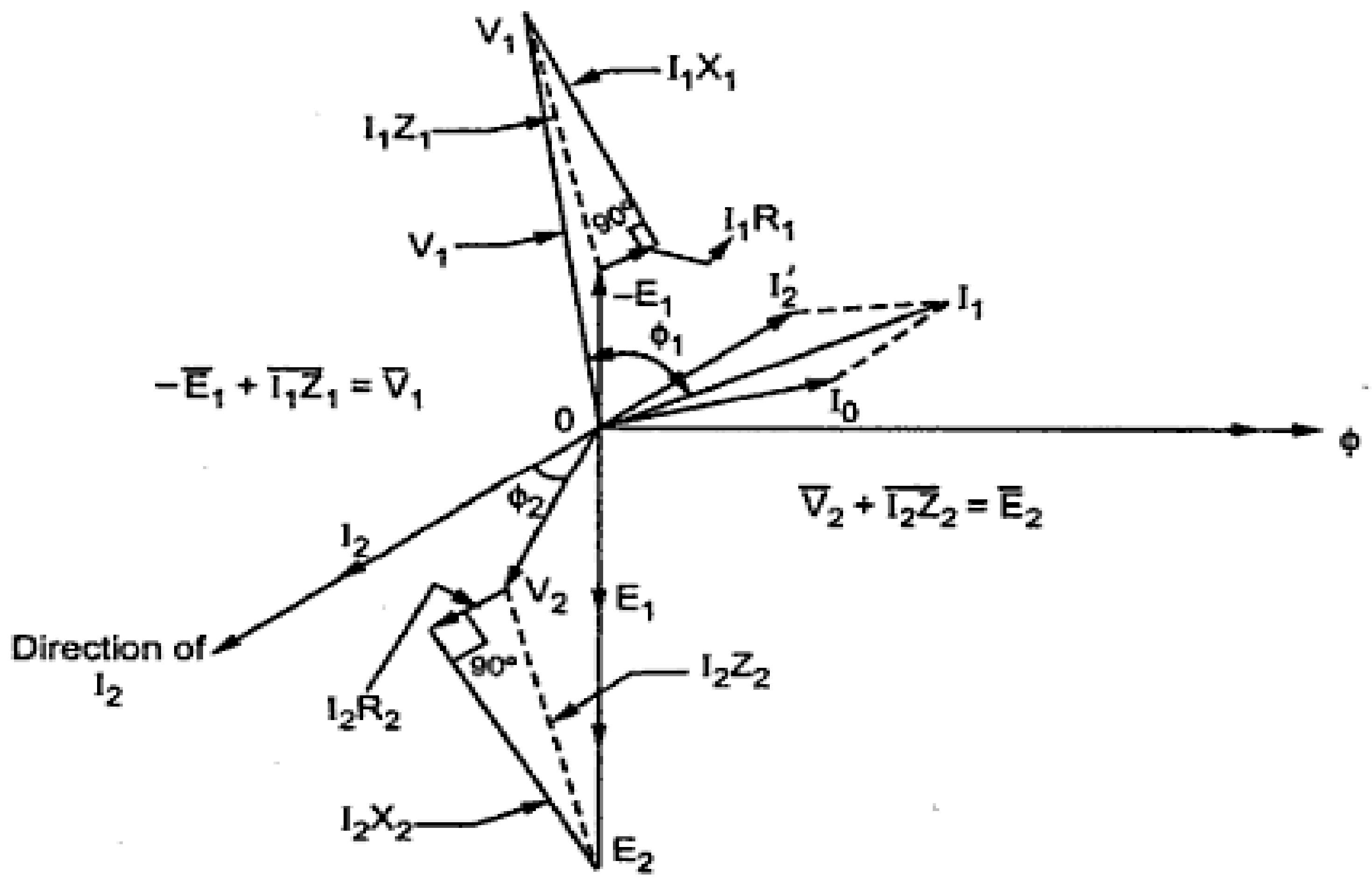


Fig. b: Main flux and leakage flux in a transformer

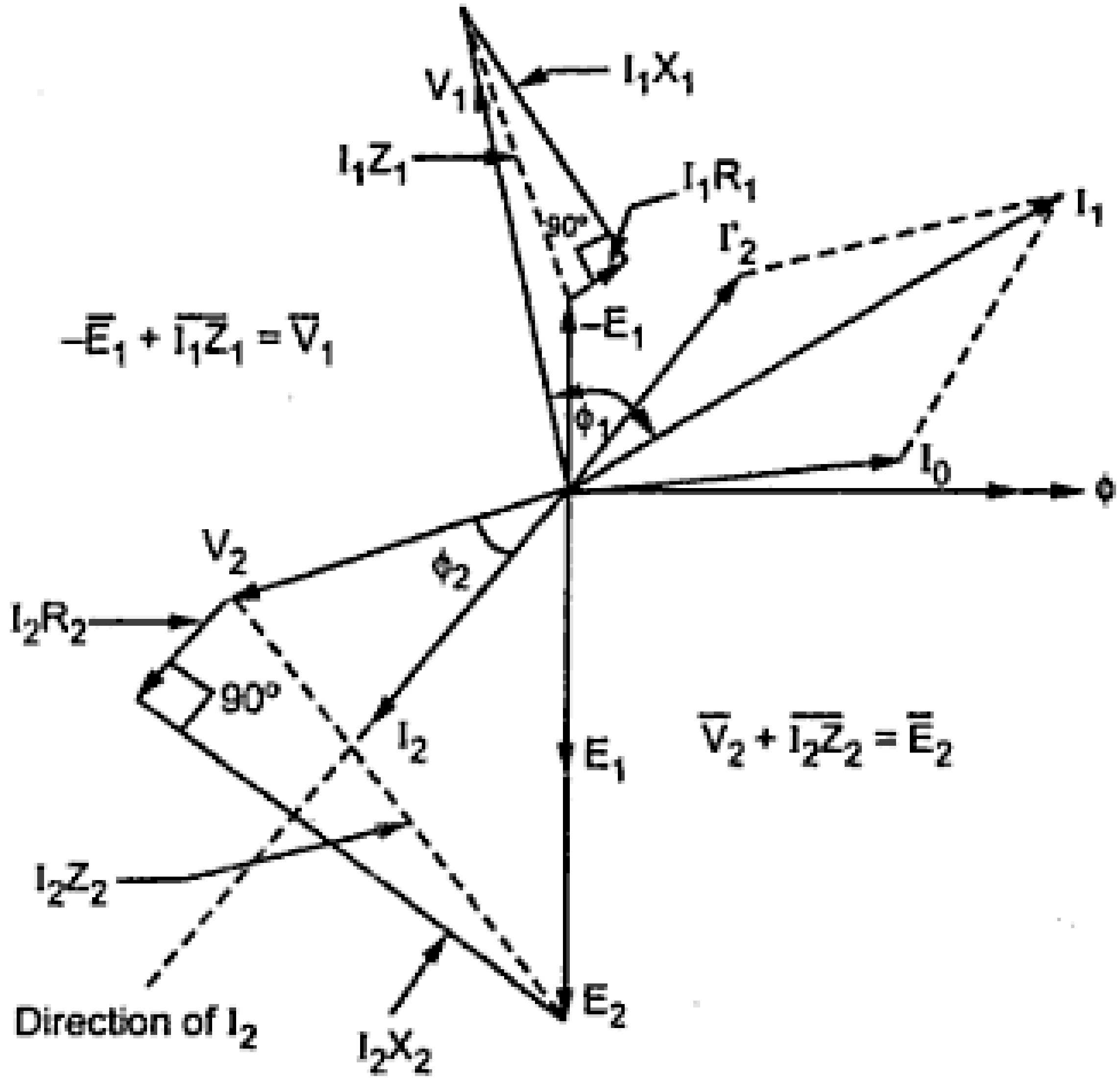
Phasor diagram of transformer with UPF load



Phasor diagram of transformer with lagging p.f load

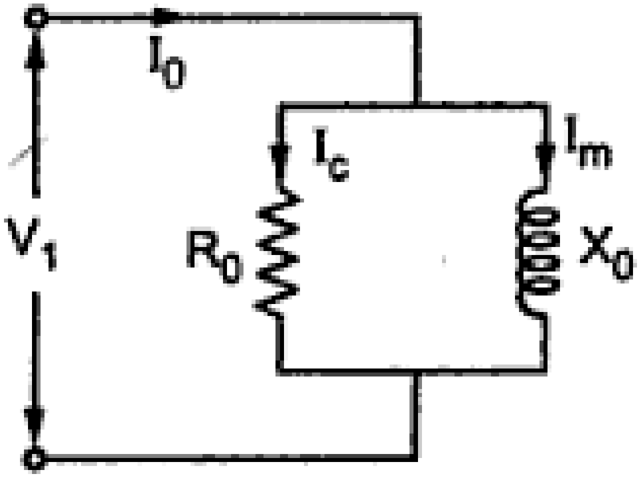


Phasor diagram of transformer with



Equivalent circuit of a transformer

No load equivalent circuit:



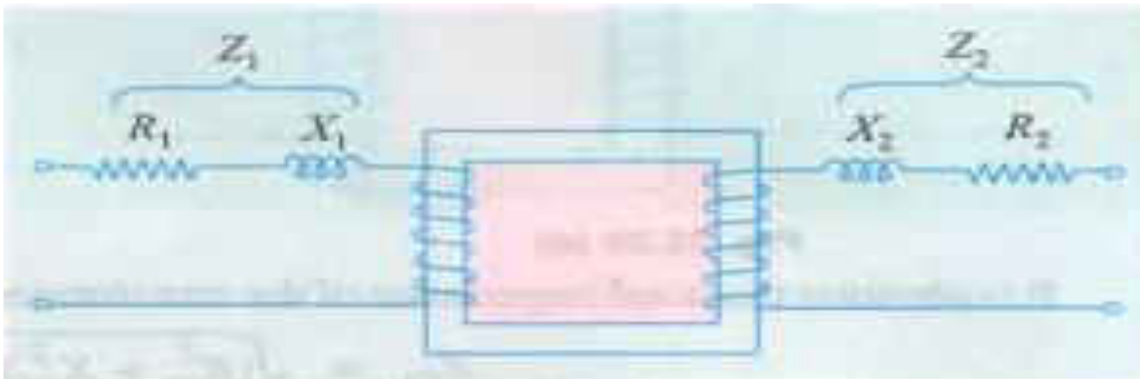
$$R_0 = \frac{V_1}{I_c}$$

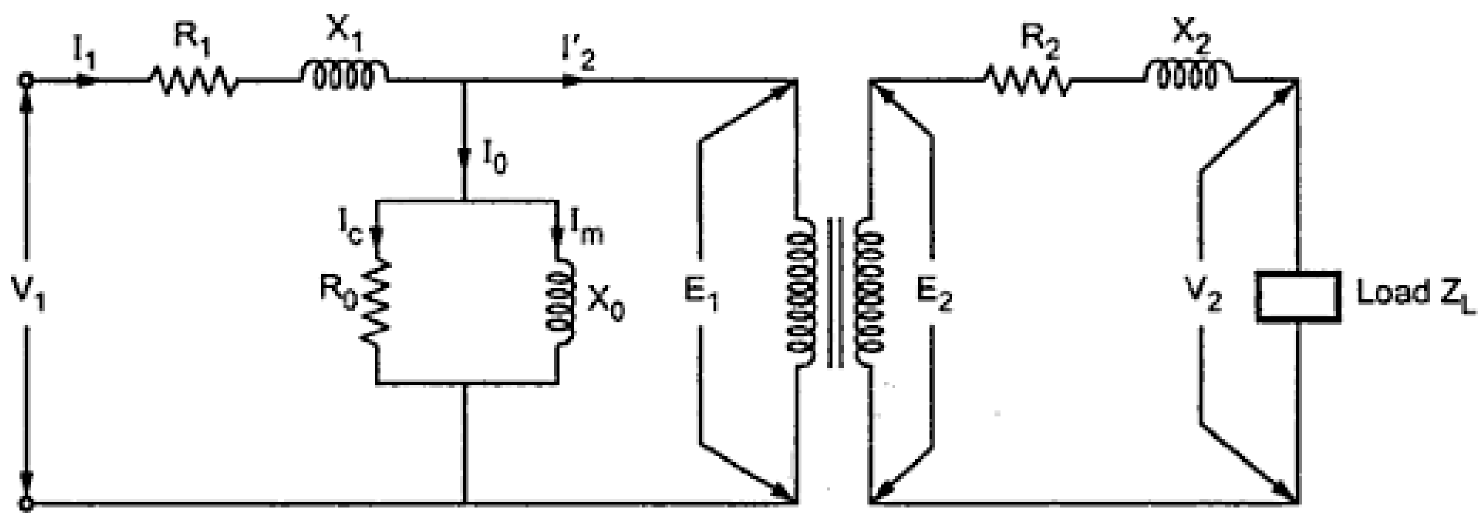
$$X_0 = \frac{V_1}{I_m}$$

$I_m = I_0 \sin\phi_0$ = Magnetising component

$I_c = I_0 \cos\phi_0$ = Active component

Equivalent circuit parameters referred to primary and secondary sides respectively





Contd.,

- The effect of circuit parameters shouldn't be changed while transferring the parameters from one side to another side
- It can be proved that a resistance of R_2 in sec. is equivalent to R_2/k^2 will be denoted as R_2'(ie. Equivalent sec. resistance w.r.t primary) which would have caused the same loss as R_2 in secondary,

$$I_1^2 R_2' = I_2^2 R_2$$

$$R_2' = \left(\frac{I_2}{I_1}\right)^2 R_2$$

$$= \frac{R_2}{k^2}$$

Voltage regulation of a transformer

$$\text{Voltage regulation} = \frac{\text{no-load voltage} - \text{full-load voltage}}{\text{no-load voltage}}$$

recall $\dfrac{V_s}{V_p} = \dfrac{N_s}{N_p}$

Secondary voltage on no-load $V_2 = V_1\left(\dfrac{N_2}{N_1}\right)$

V_2 is a secondary terminal voltage on full load

Substitute we have

$$\text{Voltage regulation} = \frac{V_1\left(\frac{N_2}{N_1}\right) - V_2}{V_1\left(\frac{N_2}{N_1}\right)}$$

Losses in a transformer

Core or Iron loss:

Hysteresis loss $W_h = \eta B^{1.6}{}_{max} f V$ watt;

eddy current loss $W_e = \eta B^2{}_{max} f^2 t^2$ watt

Copper loss:

Total Cu loss $= I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_{01} + I_2^2 R_{02}$.

Condition for maximum efficiency

$$\text{Cu loss} = I_1^2 R_{01} \quad \text{or} \quad I_2^2 R_{02} = W_{cu}$$

$$\text{Iron loss} = \text{Hysteresis loss} + \text{Eddy current loss} = W_h + W_e = W_i$$

Considering primary side,

$$\text{Primary input} = V_1 I_1 \cos\phi_1$$

$$\eta = \frac{V_1 I_1 \cos\phi_1 - \text{losses}}{V_1 I_1 \cos\phi_1} = \frac{V_1 I_1 \cos\phi_1 - I_1^2 R_{01} - W_i}{V_1 I_1 \cos\phi_1}$$

$$= 1 - \frac{I_1 R_{01}}{V_1 \cos\phi_1} - \frac{W_i}{V_1 I_1 \cos\phi_1}$$

Differentiating both sides with respect to I_1, we get

$$\frac{d\eta}{dI_1} = 0 - \frac{R_{01}}{V_1 \cos\phi_1} + \frac{W_i}{V_1 I_1^2 \cos\phi_1}$$

For η to be maximum, $\frac{d\eta}{dI_1} = 0$. Hence, the above equation becomes

$$\frac{R_{01}}{V_1 \cos\phi_1} = \frac{W_i}{V_1 I_1^2 \cos\phi_1} \quad \text{or} \quad W_i = I_1^2 R_{01} \quad \text{or} \quad I_2^2 R_{02}$$

or **Cu loss = Iron loss**

All day efficiency

$$\text{ordinary commercial efficiency} = \frac{\text{out put in watts}}{\text{input in watts}}$$

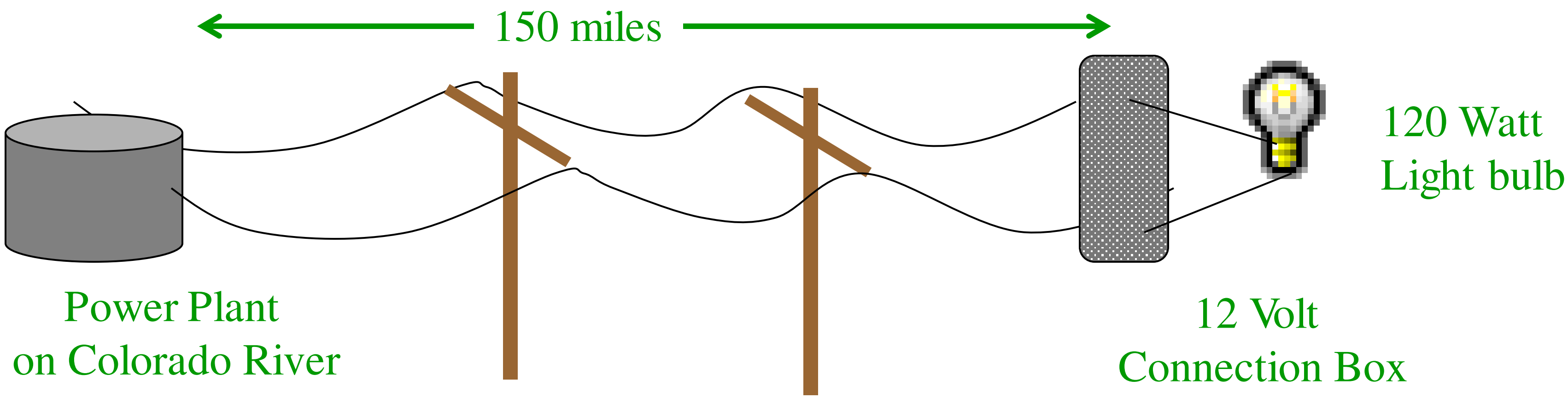
$$\eta_{all\ \mathrm{day}} = \frac{\text{output in kWh}}{\text{Input in kWh}}(\mathit{for}\ 24\ \text{hours})$$

- All day efficiency is always less than the commercial efficiency

5

Distribution System

Power Dissipated in an Electricity Distribution System



- Estimate resistance of power lines: say 0.001 Ohms per meter, times 200 km = 0.001 Ω/m $\times$ 2×10^5 m = 20 Ohms
- We can figure out the current required by a single bulb using $P = VI$ so $I = P/V$ = 120 Watts/12 Volts = 10 Amps (!)
- Power in transmission line is $P = I^2R = 10^2 \times 20$ = 2,000 Watts!!
- "Efficiency" is ε = 120 Watts/4120 Watts = 0.3%!!!
- What could we change in order to do better?

The Tradeoff

- The thing that kills us most is the high current through the (fixed resistance) transmission lines
- Need less current
  - it's that square in I^2R that has the most dramatic effect
- But our appliance needs a certain amount of power
  - $P = VI$ so less current demands higher voltage
- Solution is high voltage transmission
  - Repeating the above calculation with 12,000 Volts delivered to the house draws only

    I = 120 Watts/12 kV = 0.01 Amps for one bulb, giving

    $P = I^2R = (0.01)^2 20 = 20\times10^{-4}$ Watts, so

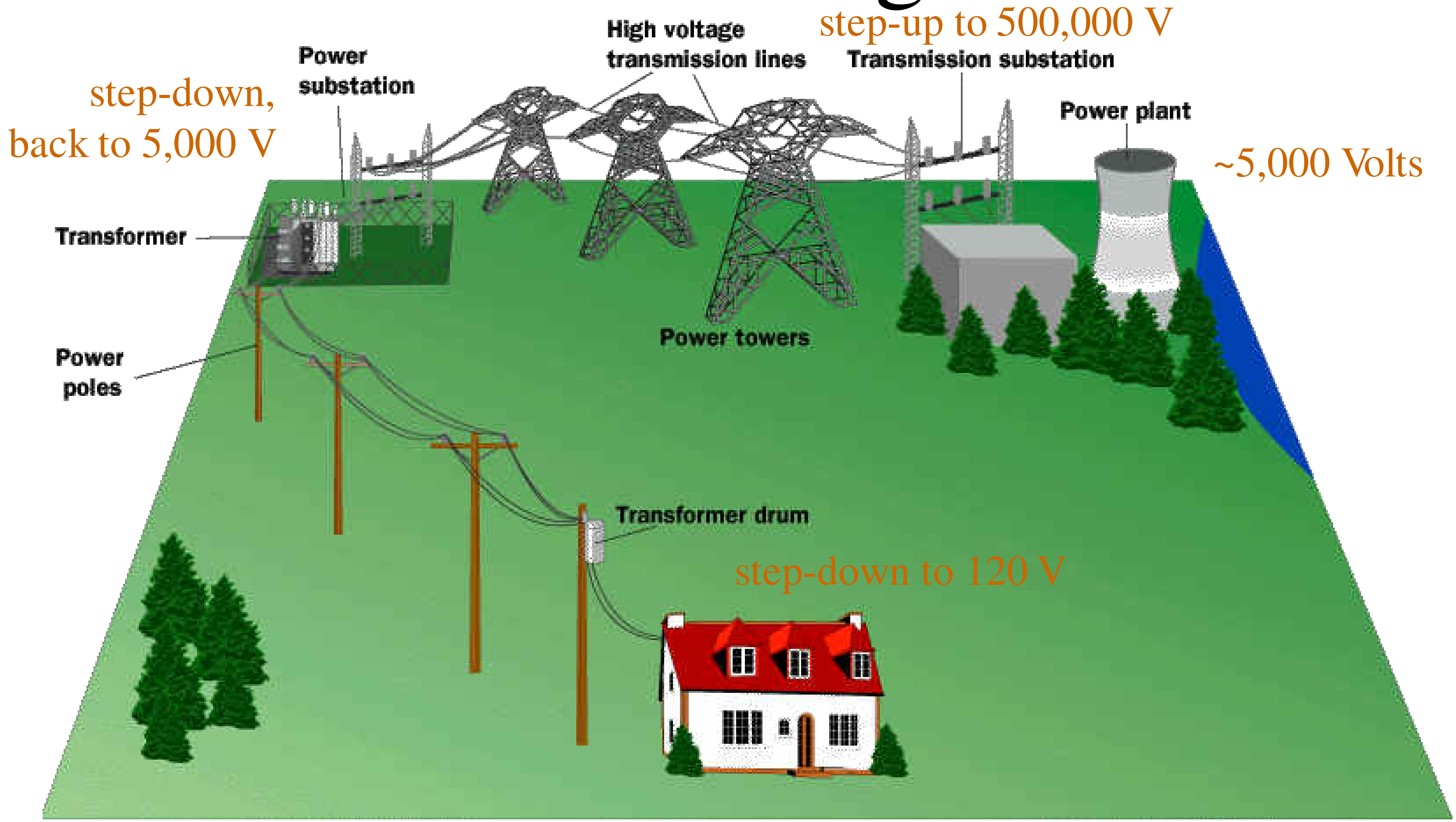
    P = 0.002 Watts of power dissipated in transmission line

  Efficiency in this case is ε = 120 Watts/120.004 = 99.996%

DANGER!

- But having high voltage in each household is a recipe for disaster
  - sparks every time you plug something in
  - risk of fire
  - not cat-friendly
- Need a way to step-up/step-down voltage at will
  - can’t do this with DC, so go to AC

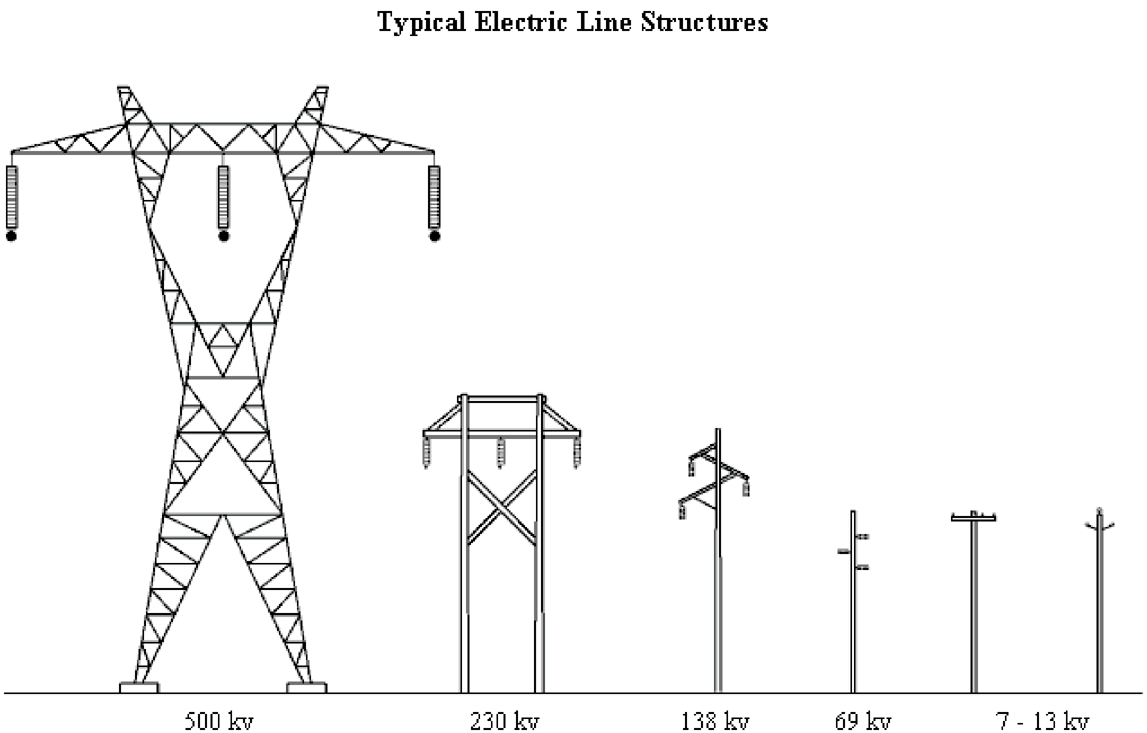
A way to provide high efficiency, safe low voltage:



High Voltage **Transmission Lines**

Low Voltage to Consumers

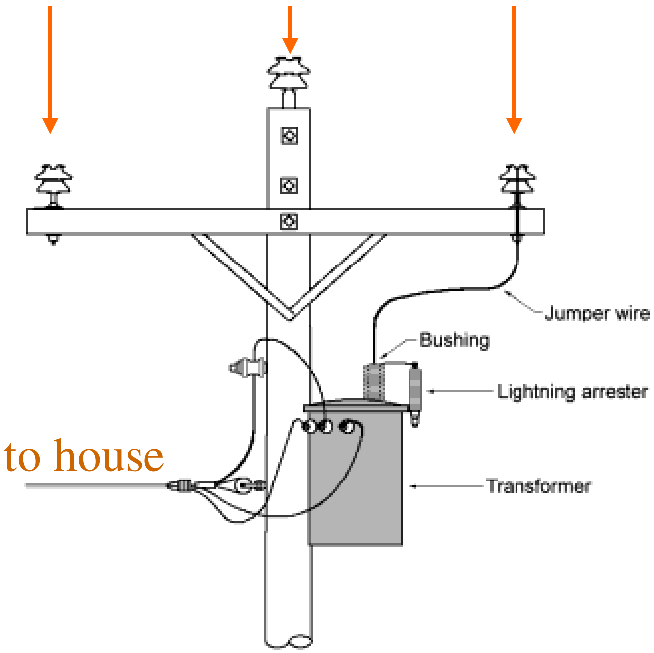
Transmission structures



500,000 230,000 138,000 69,000 7–13,000

long-distance → neighborhood

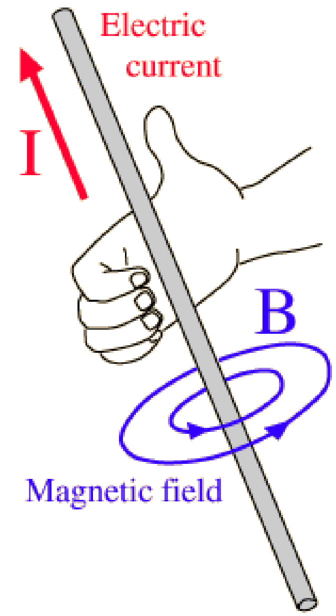
three-phase “live” wires



to house

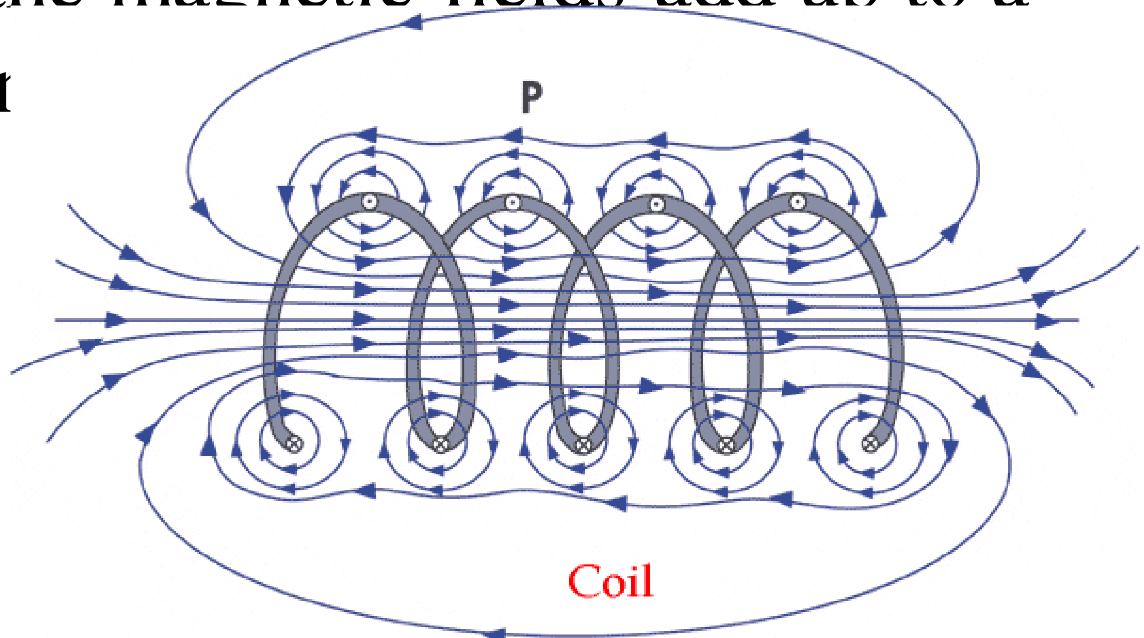
Why is AC the solution?

- AC, or *alternating current*, is necessary to carry out the transformation
- To understand why, we need to know something about the relationship between electric current and magnetic fields
- Any current-carrying wire has a circulating magnetic field around it:

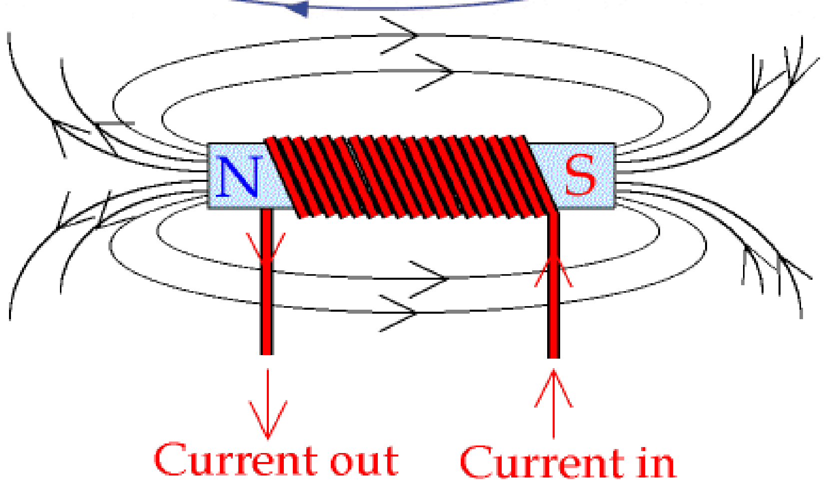


Electromagnet Coil

- By arranging wire into a loop, you can make the magnetic fields add up to a substa

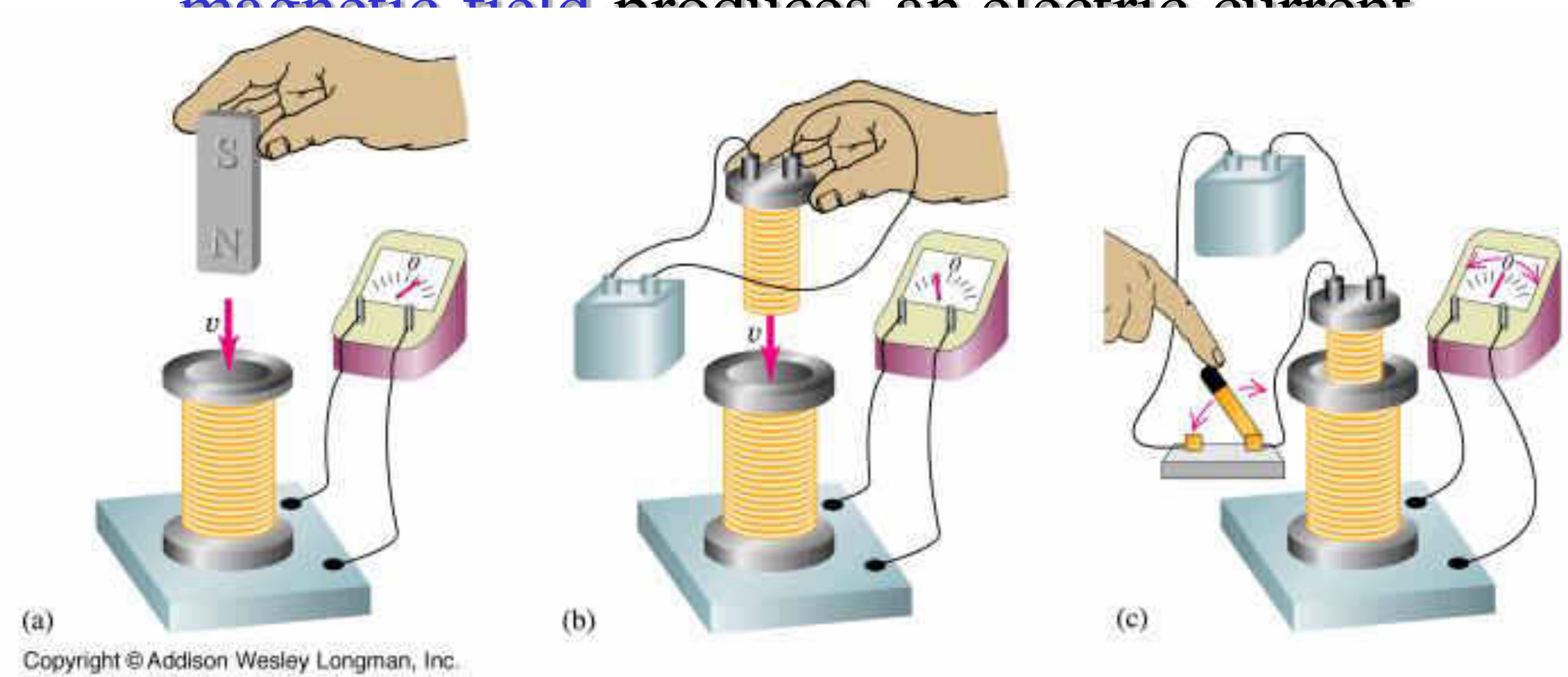


looks just like a magnet

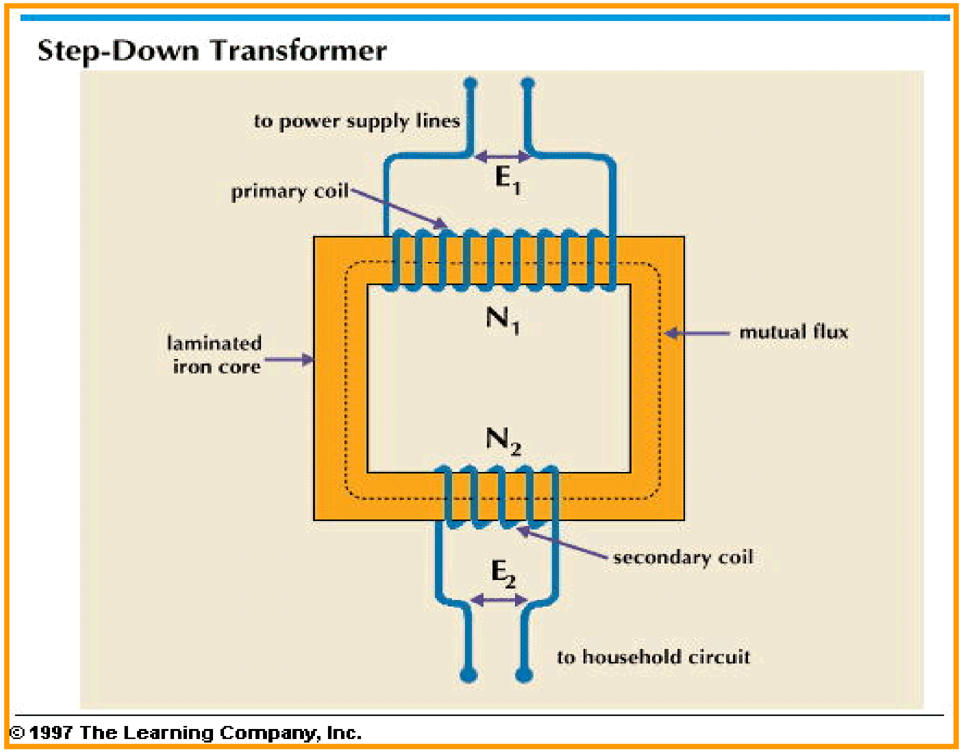


Induced Current

- The next part of the story is that a *changing* magnetic field produces an electric current



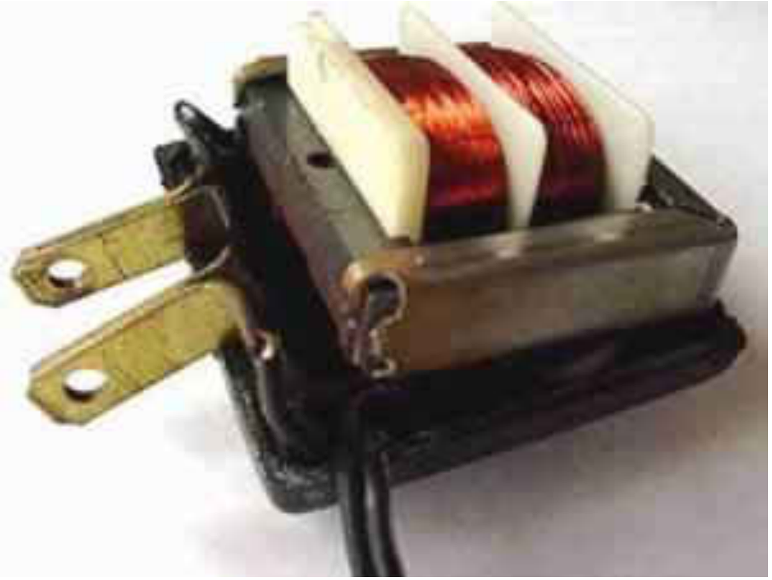
Transformer is just wire coiled around metal

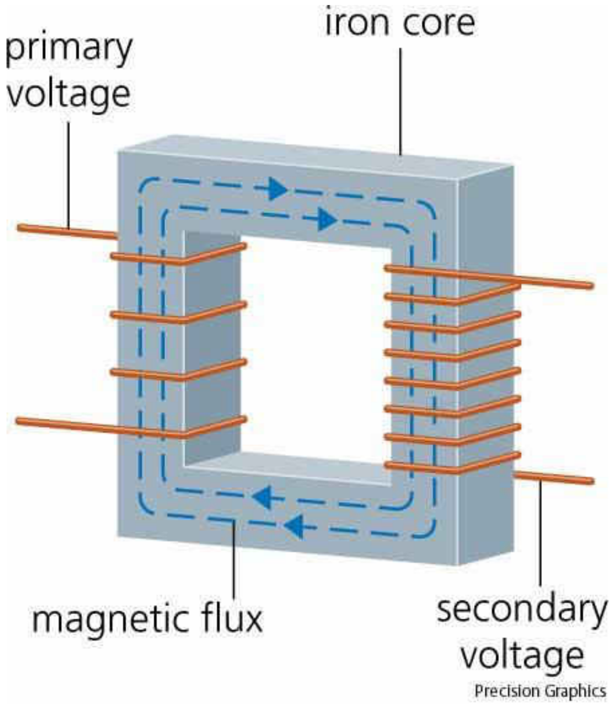


- Magnetic field is generated by current in primary coil
- Iron core channels magnetic field through secondary coil
- Secondary Voltage is $V_2 = (N_2/N_1)\ V_1$
- Secondary Current is $I_2 = (N_1/N_2)\ I_1$
- But Power in = Power out
  - negligible power lost in transformer
- Works only for AC, not DC

If the primary wires and secondary wires don't actually connect, how does the energy get from the primary circuit to the secondary circuit?!

Typical Transformers



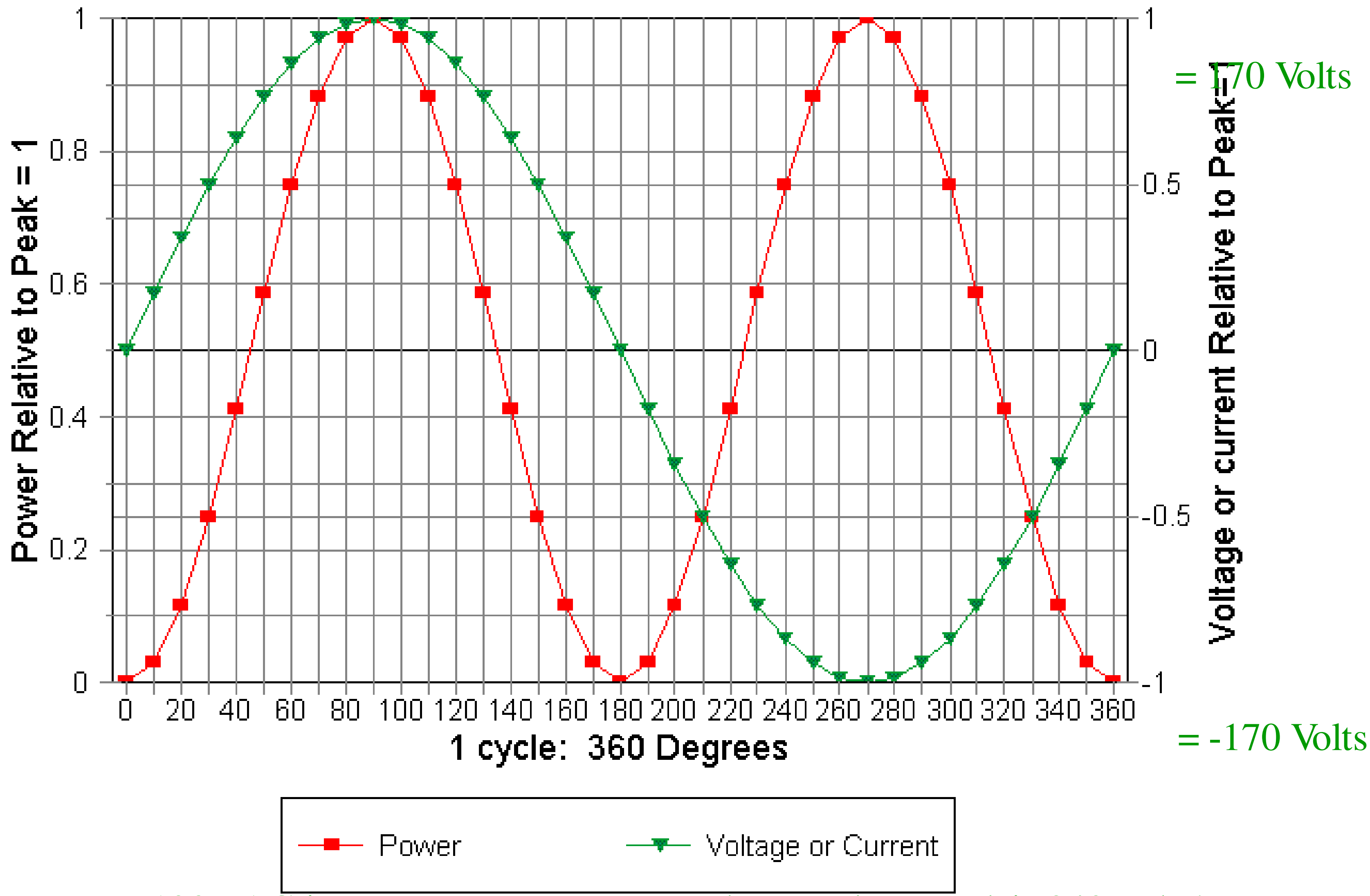


transformers usually heavy due to iron core

Alternating Current (AC) vs. Direct Current (DC)

- AC is like a battery where the terminals exchange sign periodically!
- AC sloshes back and forth in the wires
- Recall when we hooked up a bulb to a battery, the direction of current flow didn’t affect its brightness
- Although *net* electron flow over one cycle is zero, can still do useful work!

AC Voltage or Current and AC Power



120 VAC is a root-mean-square number: peak-to-peak is 340 Volts!

AC Receptacle

- Receptacles have three holes each
- Lower (rounded) hole is earth ground
  - connected to pipes, usually
  - green wire
- Larger slot is “neutral”
  - for current “return”
  - never far from ground
  - white wire
  - if wired correctly
- Smaller slot is “hot”
  - swings to +170 and –170
  - black wire
  - dangerous one

6

Electric Motor

Electric Motors

- Introduction
- A Simple AC Generator
- A Simple DC Generator
- DC Generators or Dynamos
- AC Generators or Alternators
- DC Motors
- AC Motors
- Universal Motors
- Electrical Machines – A Summary

Introduction

- In this lecture we consider various forms of rotating **electrical machines**
- These can be divided into:
  - **generators** – which convert mechanical energy into electrical energy
  - **motors** – which convert electrical energy into mechanical energy
- Both types operate through the interaction between a *magnetic field* and a set of *windings*

A Simple AC Generator

- We noted earlier that Faraday's law dictates that if a coil of N turns experiences a change in magnetic flux, then the induced voltage V is given by

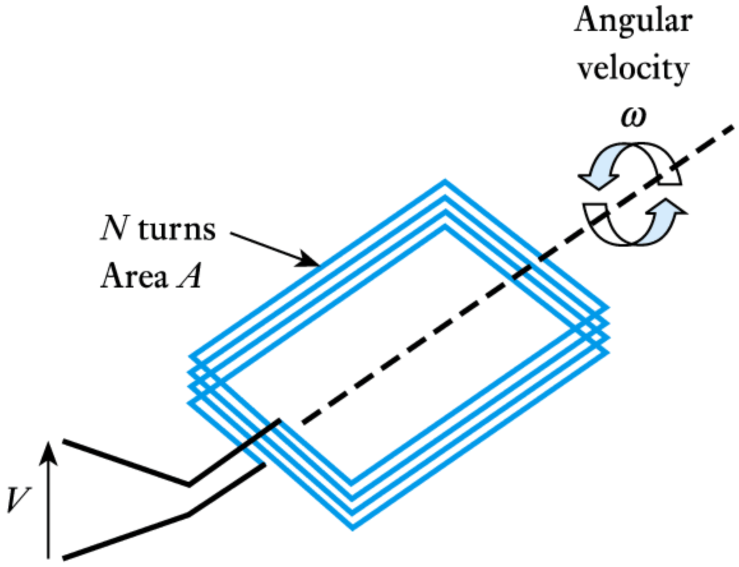
$$V = N\frac{\mathrm{d}\Phi}{\mathrm{d}t}$$

- If a coil of area A rotates with respect to a field B, and if at a particular time it is at an angle θ to the field, then the flux linking the coil is $BA\cos\theta$, and the rate of change of flux is given by

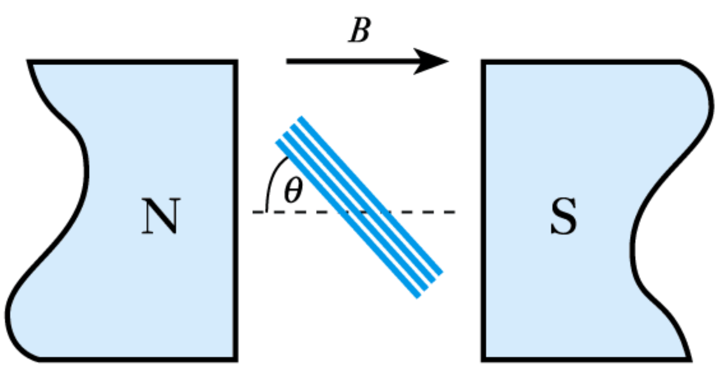
$$\frac{d\Phi}{dt} = BA\frac{\mathrm{d}(\sin\theta)}{\mathrm{d}t} = \frac{\mathrm{d}\theta}{\mathrm{d}t}\cos\theta = \omega\cos\theta$$

- Thus for the arrangement shown below

$$V = N\frac{\mathrm{d}\Phi}{\mathrm{d}t} = NBA\frac{\mathrm{d}(\sin\theta)}{\mathrm{d}t} = NBA\omega\cos\theta$$

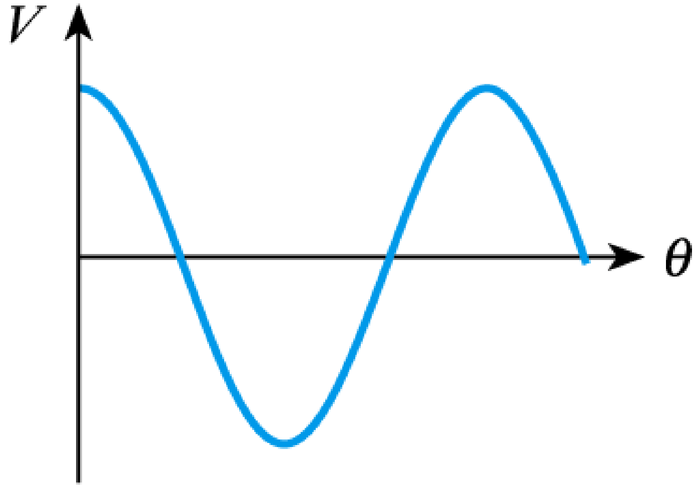


(a) Coil arrangement

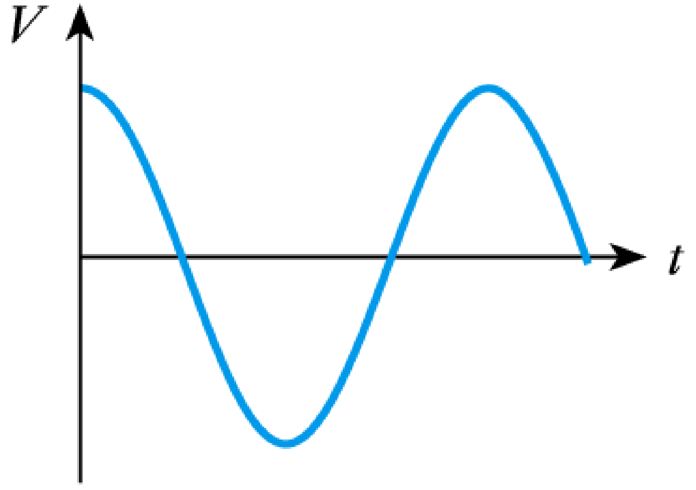


(b) Relationship between coil and field

- Therefore this arrangement produces a sinusoidal output as shown below

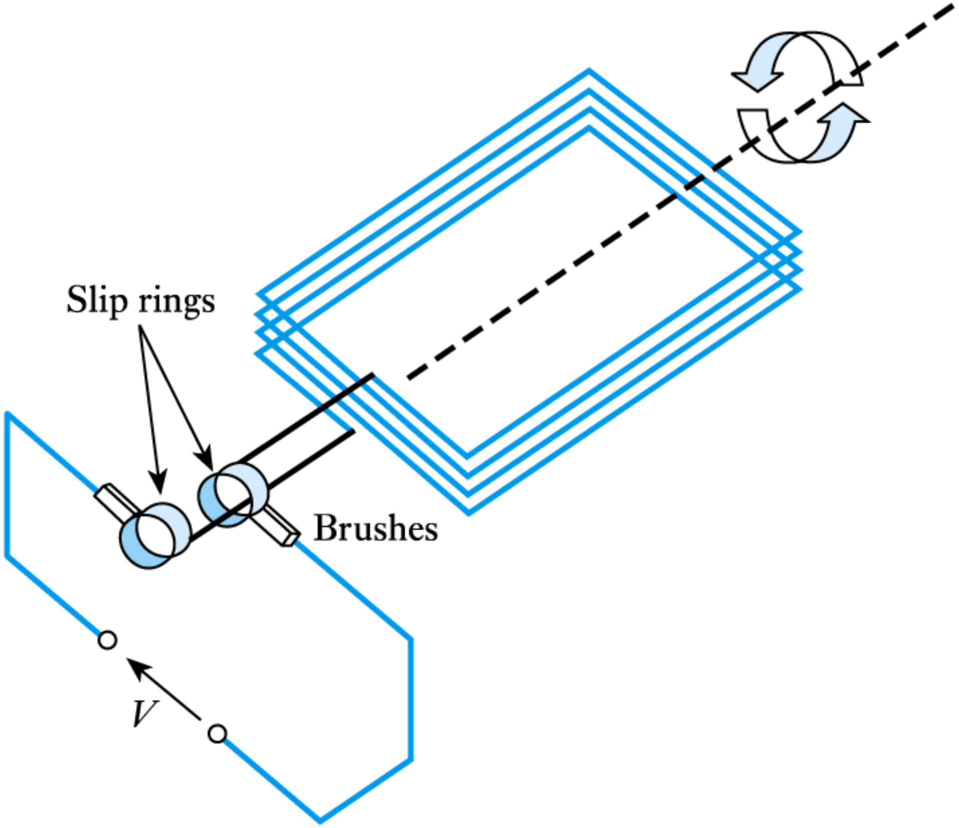


(a)



(b)

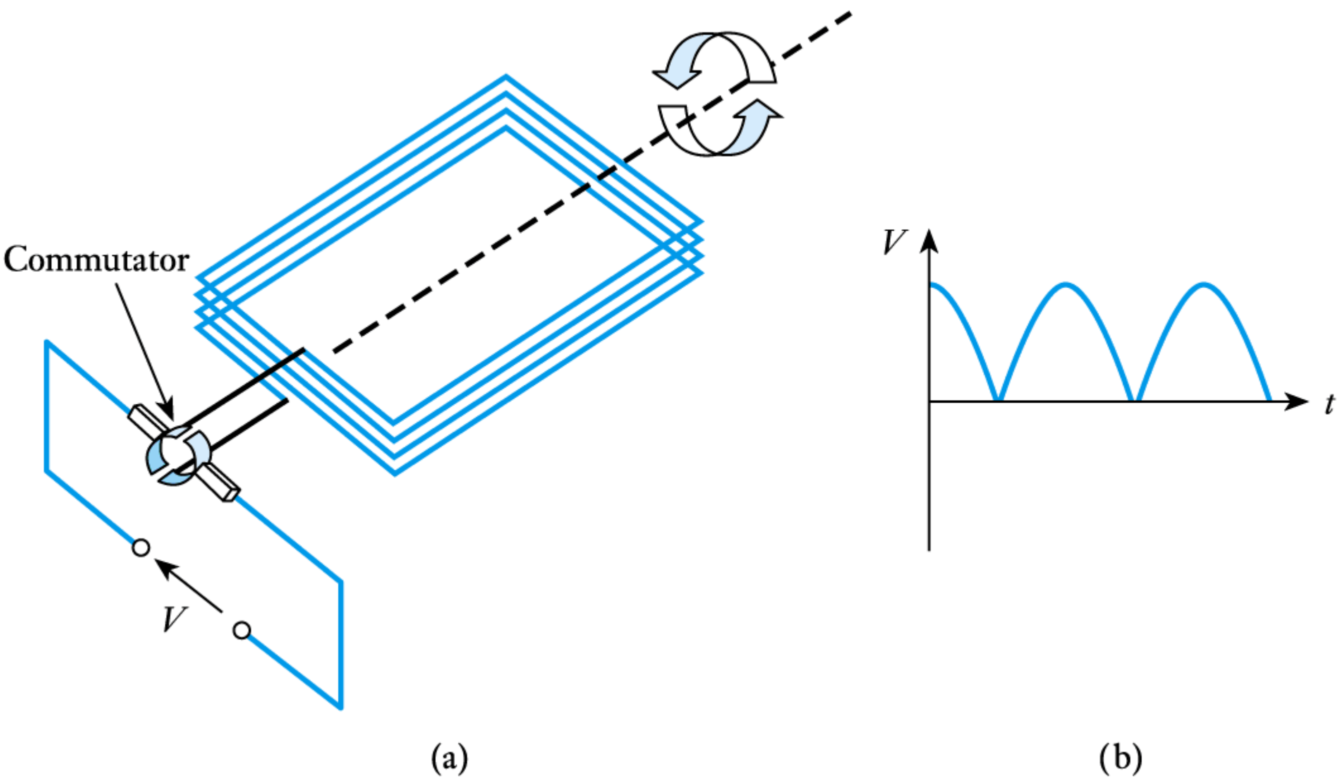
- Wires connected to the rotating coil would get twisted
- Therefore we use circular **slip rings** with sliding contacts called **brushes**



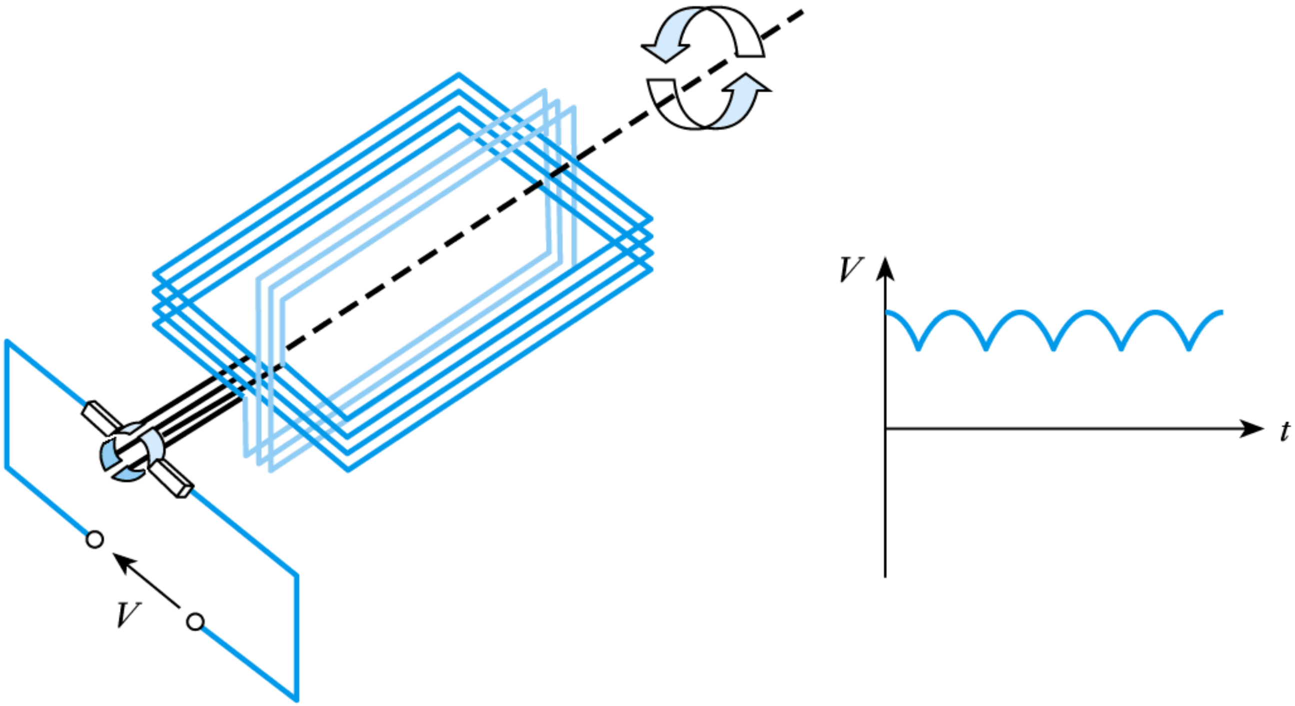
A Simple DC Generator

- The alternating signal from the earlier AC generator *could* be converted to DC using a rectifier
- A more efficient approach is to replace the two slip rings with a single split slip ring called a **commutator**
  - this is arranged so that connections to the coil are reversed as the voltage from the coil changes polarity
  - hence the voltage across the brushes is of a single polarity
  - adding additional coils produces a more constant output

- **Use of a commutator**

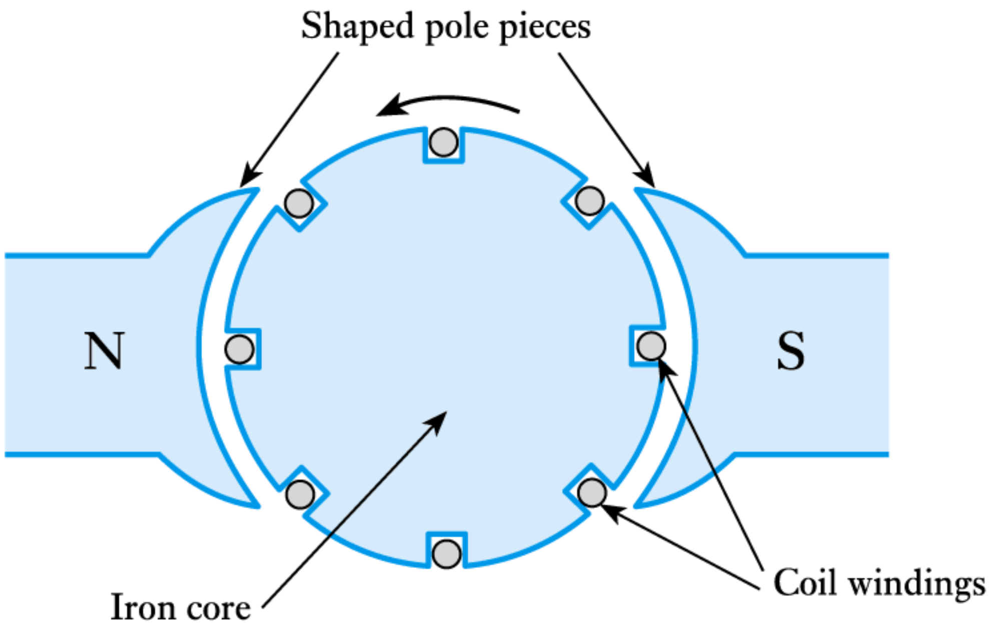


- **A simple generator with two coils**



(a) (b)

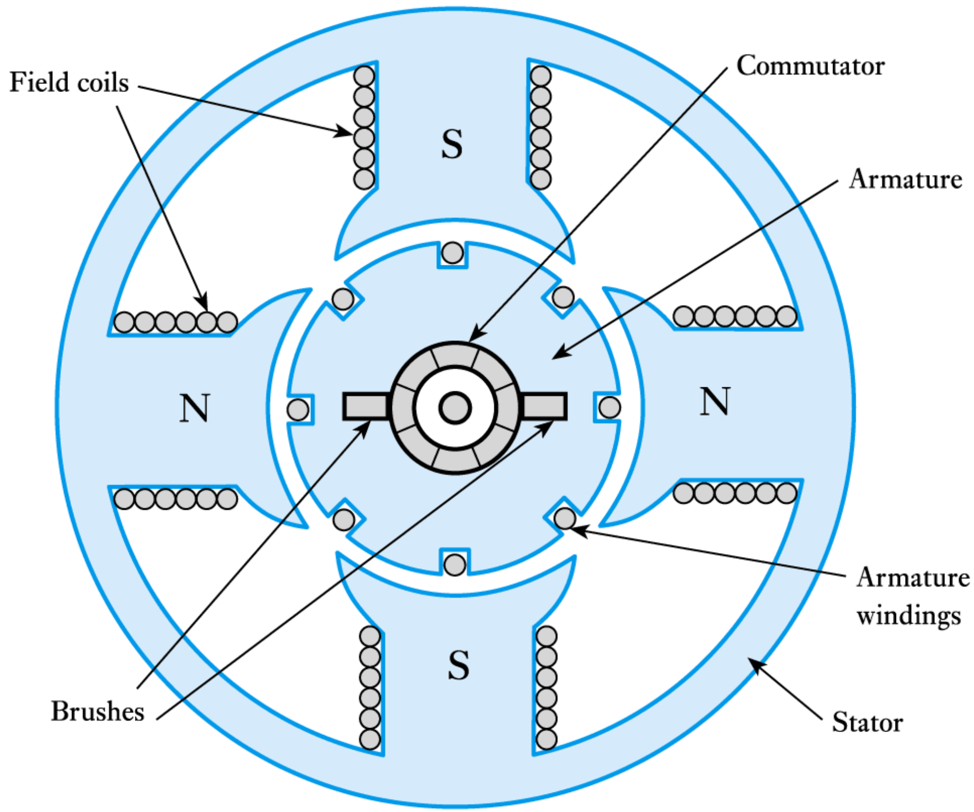
- The ripple can be further reduced by the use of a cylindrical iron core and by shaping the pole pieces
  - this produces an approximately uniform field in t narrow air gap
  - the arrangement of coils and core is known as the **armature**



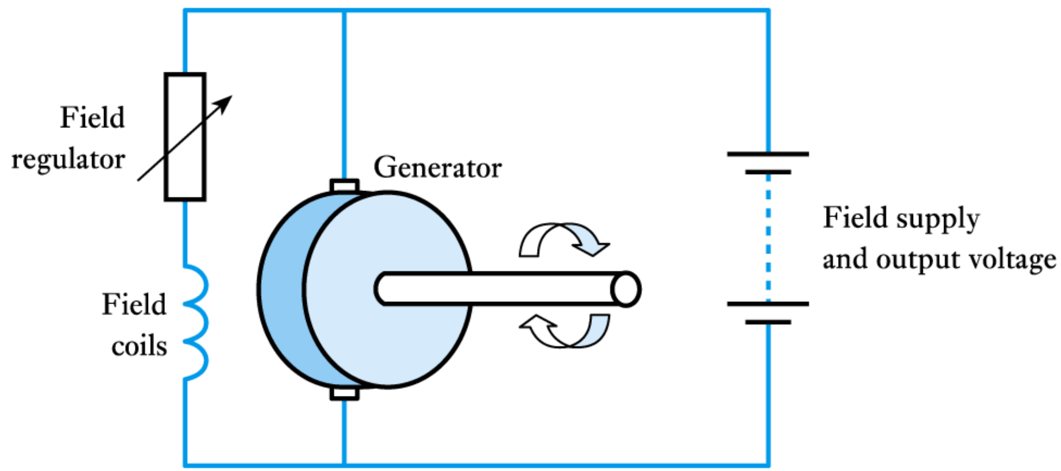
DC Generators or Dynamos

- Practical **DC generators** or **dynamos** can take a number of forms depending on how the magnetic field is produced
  - can use a **permanent magnet**
  - more often it is generated electrically using **field coils**
    - current in the field coils can come from an external supply
      - this is known as a **separately excited generator**
    - but usually the field coils are driven from the generator output
      - this is called a **self-excited generator**
  - often use multiple poles held in place by a steel tube called the **stator**

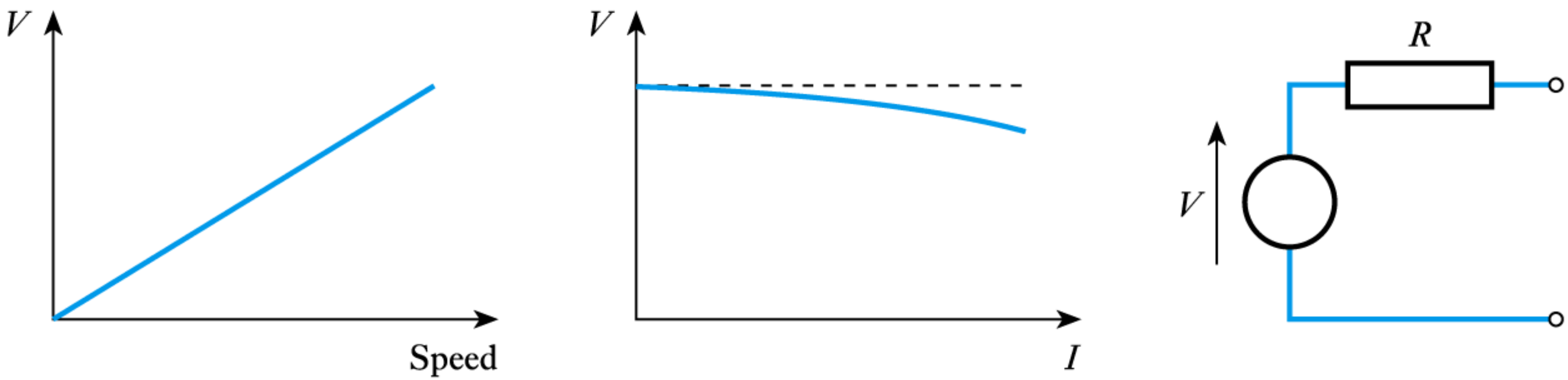
- **A four-pole DC generator**



- **Field coil excitation**
  - sometimes the field coils are connected in **series** with the armature, sometimes in parallel (**shunt**) and sometimes a combination of the two (**compound**)
  - these different forms produce slightly different characteristics
  - diagram here shows a **shunt-wound generator**



- **DC generator characteristics**
  - vary slightly between forms
  - examples shown here are for a shunt-wound



(a) Speed–voltage characteristic

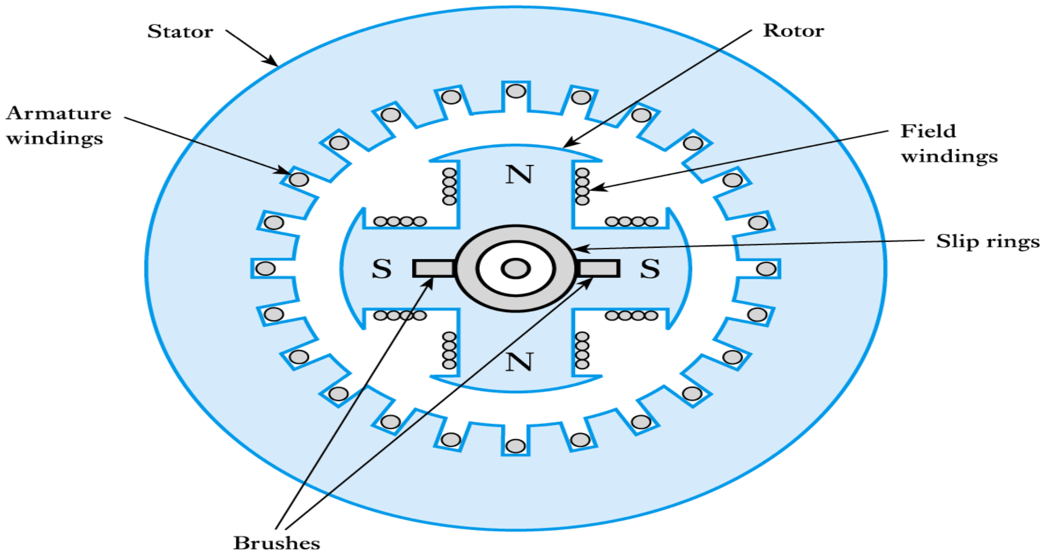
(b) Current–voltage characteristic

(c) Approximate equivalent circuit

AC Generators or Alternators

- Alternators do not require commutation
  - this allows a simpler construction
  - the *field coils* are made to rotate while the *armature* windings are stationary
    - *Note*: the armature windings are those that produce the output
  - thus the large heavy **armature windings** are in the **stator**
  - the lighter **field coils** are mounted on the **rotor** and direct current is fed to these by a set of slip rings

- **A four-pole alternator**

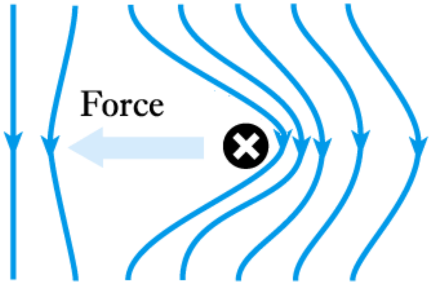


- As with DC generators multiple poles and sets of windings are used to improve efficiency
  - sometimes three sets of armature windings are spaced 120° apart around the stator to form a **three-phase generator**
- The e.m.f. produced is in sync with rotation of the rotor so this is a **synchronous generator**
  - if the generator has a single set of poles the output frequency is equal to the rotation frequency
  - if additional pole-pairs are used the frequency is increased accordingly

DC Motors

- When current flows in a conductor it produces a magnetic field about it - as shown in (a) below
  - when the current-carrying conductor is within an externally generated magnetic field, the fields interact and a force is exerted on the conductor - as in (b)

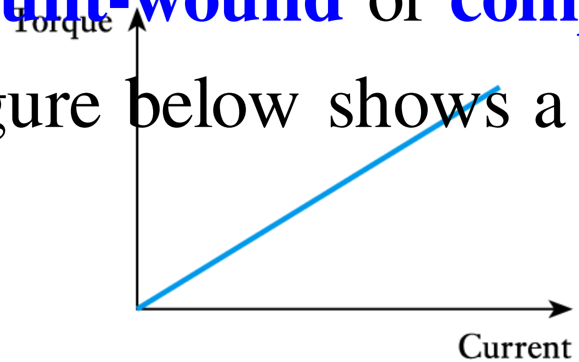
(a) The magnetic field about a current flowing into the page



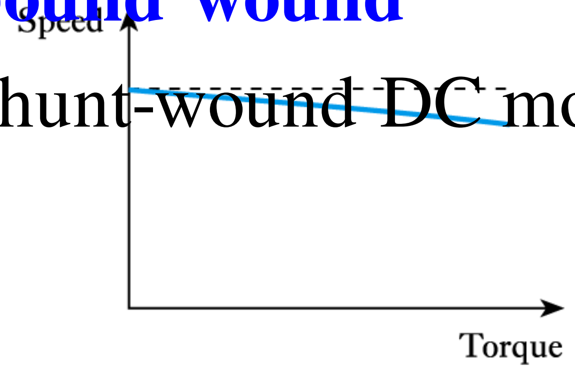
(b) The effects of an external magnetic field

- Therefore if a conductor lies within a magnetic field:
  - *motion* of the conductor produces an electric *current*
  - an electric *current* in the conductor will generate *motion*
- The reciprocal nature of this relationship means that, for example, the DC generator above will function as a DC motor
  - although machines designed as motors are more efficient in this role
- Thus the four-pole DC generator shown earlier could equally well be a four-pole DC motor

- **DC motor characteristics**
  - many forms – each with slightly different characteristics
  - again can be **permanent magnet**, or **series-wound**, **shunt-wound** or **compound wound**
  - figure below shows a shunt-wound DC motor



(a) Torque–current characteristic



(b) Speed–torque characteristic with a constant applied voltage

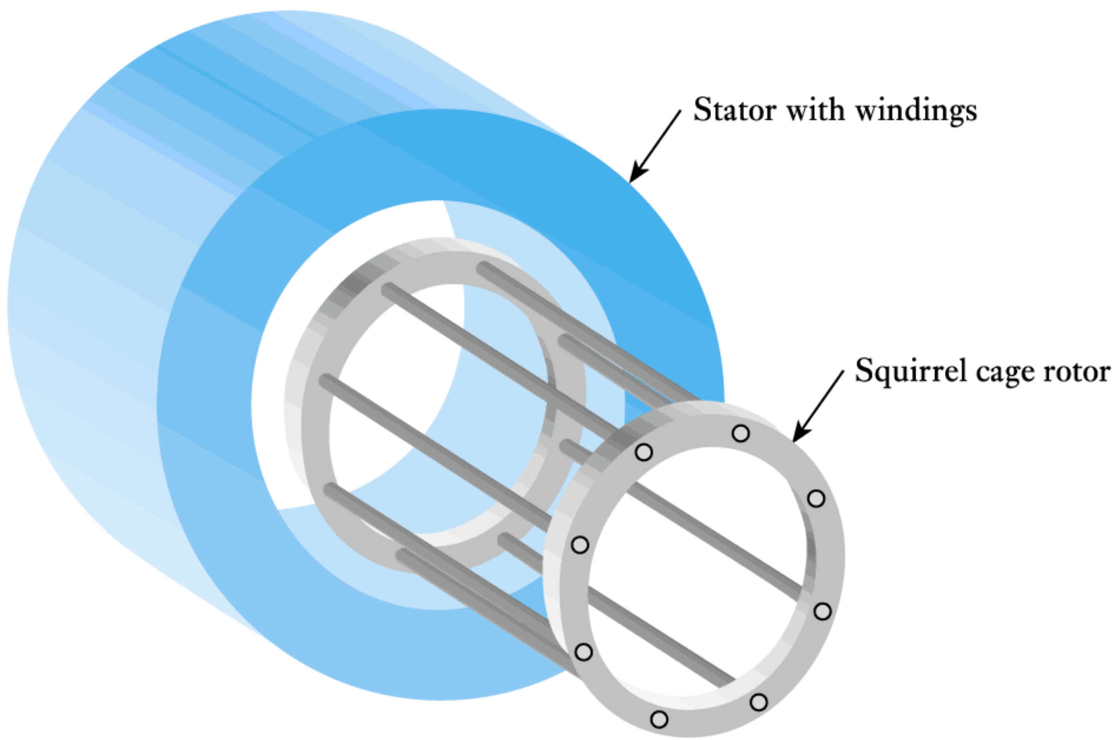
AC Motors

- AC motors can be divided into two main forms:
  - **synchronous motors**
  - **induction motors**
- High-power versions of either type invariably operate from a **three-phase supply**, but single-phase versions of each are also widely used – particularly in a domestic setting

- **Synchronous motors**
  - just as a DC generator can be used as a DC motor, so AC generators (or alternators) can be used as **synchronous AC motors**
  - **three phase motors** use three sets of stator coils
    - the rotating magnetic field drags the rotor around with it
  - **single phase motors** require some starting mechanism
  - torque is only produced when the rotor is in sync with the rotating magnetic field
    - **not self-starting** – may be configured as an induction motor until its gets up to speed, then becomes a synchronous motor

- **Induction motors**
  - these are perhaps the most important form of AC motor
  - rather than use slip rings to pass current to the field coils in the rotor, current is *induced* in the rotor by transformer action
  - the stator is similar to that in a synchronous motor
  - the rotor is simply a set of parallel conductors shorted together at either end by two conducting rings

- **A squirrel-cage induction motor**



- In a **three-phase induction motor** the three phases produce a rotating magnetic field (as in a three-phase synchronous motor)
  - a stationary conductor will see a varying magnetic field and this will induce a current
  - current is induced in the field coils in the same way that current is induced in the secondary of a transformer
  - this current turns the rotor into an electromagnet which is dragged around by the rotating magnetic field
  - the rotor always goes slightly slower than the magnetic field – this is the **slip** of the motor

- In **single-phase induction motors** other techniques must be used to produce the rotating magnetic field
  - various techniques are used leading to various forms of motor such as
    - capacitor motors
    - shaded-pole motors
  - such motors are inexpensive and are widely used in domestic applications

Universal Motors

- While most motors operate from either AC or DC, some can operate from either
- These are **universal motors** and resemble series-wound DC motors, but are designed for both AC and DC operation
  - typically operate at high speed (usually > 10,000 rpm)
  - offer high power-to-weight ratio
  - ideal for portable equipment such as hand drills and vacuum cleaners

Electrical Machines – A Summary

- **Power generation is dominated by AC machines**
  - range from automotive alternators to the synchronous generators used in power stations
  - efficiency increases with size (up to 98%)
- **Both DC and AC motors are used**
  - high-power motors are usually AC, three-phase
  - domestic applications often use single-phase induction motors
  - DC motors are useful in control applications

7

Domestic InstallationSystem

Amps, Volts, Watts

- Amperes: measure of the rate of flow of electricity in a conductor
- Volts: measure of electrical pressure
- Watts: measure of the amount of energy or work that can be done
- Ohms: measure of electrical resistance to flow

Ohm's Law

- Ohm = R
- Volts = E
- Amps = I
- Ohm's Law: $E = IR$

  $I=E/R$

  $R=E/I$

Electrical Safety

- Shock and Fire
- Never disconnect any safety device
- Don’t touch electrical items with wet hands or feet
- Don’t remove ground plug prong
- Use GFI in wet areas
- Discontinue use of extension cord that feels warm
- Don’t put extension cords under carpet

Electrical Safety

- Install wiring according to NEC
- Blown fuse or breaker, determine cause
- Don’t replace fuse with larger fuse
- Don’t leave heat producing appliances unattended
- Heaters & lamps away from combustibles
- Don’t remove back of TV (30,000v when off)
- Electric motors lubricated, free of grease etc.

Electrical Safety

- Keep appliances dry
- Don’t use damaged switches, outlets, fixtures, extension cords
- Follow manufacturer’s instructions for installation and use of electrical equipment

Service Entrance

- Power from from power company
- Transformer: drops volts from 25,000 volts to 240 volts
- Service drop: wires etc from transformer to house
- Entrance head: weather-proof at house
- Meter: $$$
- Service Entrance Panel (SEP): box with fuses or breakers

Electric Meter

- Kilowatthours: how electricity is sold
- Kilo = 1000
- Watthour = use of 1 watt for one hour
  - 100 watt light bulb for 1 hour - 100 watthours
- Kilowatthour = 1000 watts for one hour

Branch Circuits

- usually begin at SEP
- branch out into a variety of places
- only 1 motor or;
- series of outlets or;
- series of lights
- use correct size wire and fuse or breaker

Types of Cable

- Nonmetallic sheathed cable: copper or alluminum wire covered with paper, rubber, or vinyl for insulation
- Armored cable: flexible metal sheath with individual wires inside. Wires are insulated
- Conduit: tubing with individually insulated wires

Wire Type and Size

- copper
- No 14 (14 gauge) = 15 amp circuits
- No 12 = 20 amps
- No 10 = 30 amps
- aluminum use one size larger
- lower gauge number = larger wire
- No 8 and larger use bundles of wires
- current travels on outer surface of wire, so a bundle of smaller wires can carry more

Voltage Drop

- loss of voltage as it travels along a wire
- lights dim, motors overheat
- larger wires have less voltage drop for a given amount of current
- longer wire = greater problem
- must increase wire size as distance increases

Wire Identification

- Type of outer covering, individual wire covering, cable construction, number of wires
- Wire type stamped on outer surface

Wire Types

- Type T - dry locations
- Type TW - dry or wet
- THHN - dry, high temps
- THW and THWN - wet, high temps
- XHHW - high moisture & heat resistance
- UF - direct burial in soil but not concrete

Wire Identification

- Color coded: black, red, & blue = positive or hot wires which carry current to appliances
- White = neutral wires carry current from appliance back to source
- Green or Bare = ground all metal boxes and appliances

Wire Identification

- Wire Size: 12-2 has two strands of No. 12 wire (black & white)
- 12-2 w/g same, with one green or bare
- 12-3 has three strands of No. 12 (black, red, white)
- 12-3 w/g same, with green or bare

8

Electrical Safety

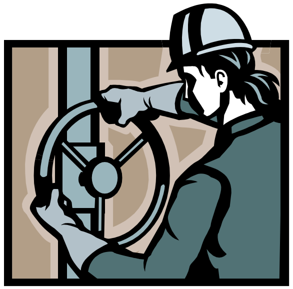
Electrical Safety

Recognizing and Mitigating Specific Hazards in the Work Place Encountered by the Non-Electrical Skilled Worker

Module 6

Non-Electrical Skilled Worker

- This training provides additional electrical safety training for electrical hazards non-electrical skilled workers are exposed to in the work place.
- It is developed as an add-on module to the basic electrical safety training module for non-electrical workers.



Review of Basic Electrical Safety Hazard Awareness for the Non-Electrical Worker

- You should have taken as a prerequisite for this training “Basic Electrical Safety Hazard Awareness for Non-Electrical Personnel”.
- This training covered the hazards associated with electrical energy – Shock, Arc and Blast.
  - These hazards can cause disability or death.
  - You were taught how to recognize electrical hazards.

Review of Basic Electrical Safety Hazard Awareness for the Non-Electrical Worker (continued)

- **You were taught basic electrical safety that included:**
  - **Ground-fault Circuit Interrupters (GFCIs)**
  - **Basic electrical cord safety**
  - **Resetting Breakers**
  - **Conductive Apparel**
  - **Wall Penetrations**
  - **Safe Work Practices for Equipment Applications**
  - **Only qualified electrical workers can perform electrical work**

Review of Basic Electrical Safety Hazard Awareness for the Non-Electrical Worker (cont.)

- **You were taught basic electrical safety that included:**
  - **What to do in case of an electrical emergency.**
  - **To inspect your work area for unsafe electrical conditions.**
  - **To use equipment per its Listing and Labeling instructions i.e. no daisy chaining, no overloading of circuits, etc.**
  - **What to do if you identify an electrical hazard.**
  - **To contact your Site Electrical Safety Officer or Safety Engineer for specific electrical safety items.**

- Non-Electrical Skilled workers are:
  - Exposed to specific electrical hazards
  - Expected to work safely around electrical energy
  - To use electrical tools safely
  - To follow electrical safety requirements
  - To help keep other workers safe from electrical hazards.
  - Obey all postings and barriers protecting exposed energized electrical hazards.

Requirements (cont.)

- 70E requirements for energized work apply if an **exposed energized** condition exists.
- 70E requires that an electrically safe work condition (Lockout/Tagout – LO/TO) must be established unless work around energized equipment with exposed electrical components is permitted with all the required safety precautions established.

70E Requirements (cont.)

- If an exposed energized condition exists, there will be a Flash Protection Boundary and a Shock Protection Boundary that will have specific PPE and access requirements.
- These boundaries are established to protect you from the heat energy of an arc and from getting shocked.
- 70E requires proper barriers, posting, and/or attendants to inform unqualified workers of existing hazards.
- The work control document should address these boundaries and your work task relationship to them.
- Do not cross these boundaries unless you are qualified and authorized or are escorted by a qualified electrical worker.
  - You must have the PPE required by 70E for the boundary to be crossed.

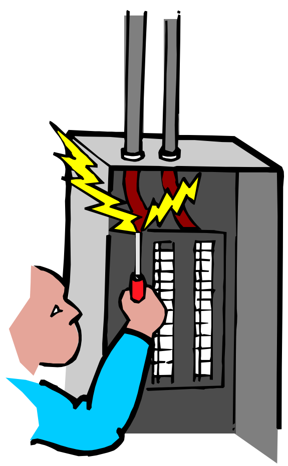
70 E Requirements (cont.)

- Generally for systems under 600V the Flash Protection Boundary (FPB) is 4 ft. unless calculated under engineering supervision. The FPB is established to protect you from the heat energy of an ARC.
- The shock protection boundaries are based on a table in 70E. The non- electrical worker can be escorted by a qualified electrical worker inside the Limited Approach Boundary, but can approach no closer to exposed energized components. See following slide for copy of table listing approach distances from 70E.

The following slides will discuss specific electrical hazards the non-electrical skilled worker is exposed to in the work place and the methods used to mitigate the hazards.

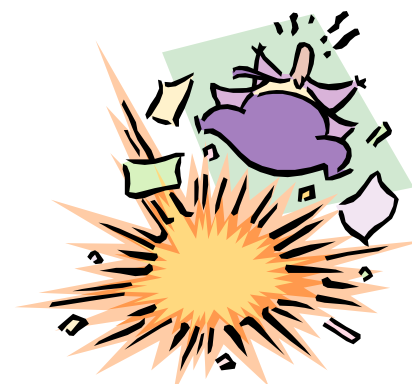
Hazards of Electricity

Shock

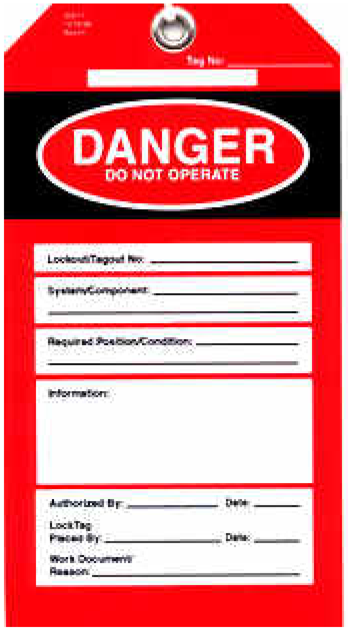


Arc

Blast



The most effective way to protect against electrical hazards is to use LO/TO.

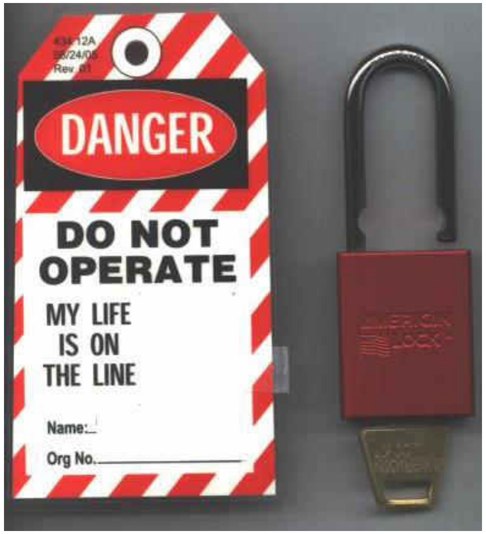


- Your facility will have specific requirements for LO/TO. **Always comply with the requirements.**
- Only LO/TO qualified employees may work under the protection of a LO/TO.
- LO/TO accomplishes a zero energy state and there is no electrical hazard.
- You are required to comply with all LO/TO requirements.
- ***Failure to comply can result in injury or death!***

DO NOT OPERATE!

Lockout/Tagout

- Your personal lock and personal danger tag is what protects you from systems being re-energized while you are working on them.
- You are the only person authorized to remove them except under specially controlled conditions.
- **If you don't install them, you are not protected!**

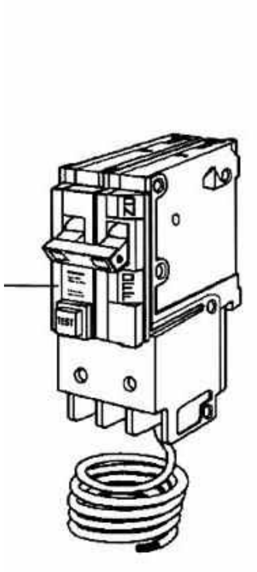


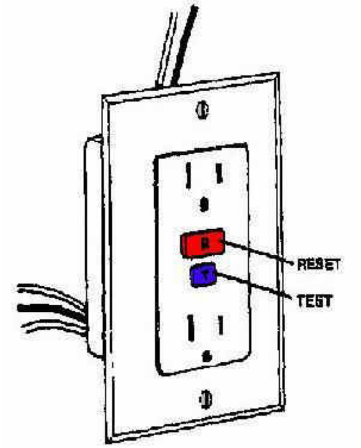
A Ground Fault Circuit Interrupter (GFCI) is a very effective device to protect employees in the work place from electrical shock.

- The number of deaths from electrical shock in the work place has been cut in half since GFCIs have been introduced.
- GFCIs are required for all maintenance activities.
- GFCIs protect you from electrical shock by tripping on current leakage to ground, which may be through you.
- Plugging one GFCI into another one does not create a hazard. The most sensitive one will trip first.
- GFCIs may be permanently installed in the facility or a portable device. – Use them.

Use of Specific Safety-Related Equipment and Work Practices

GFCIs

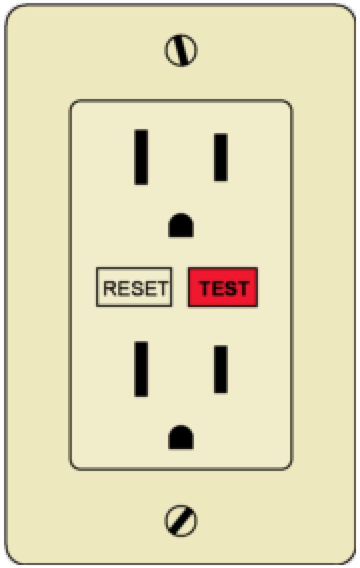




RECEPTACLE TYPE GFCI

- **Ground Fault Circuit Interrupters (GFCIs) are required for all 125-volt, single phase, 15 and 20-ampere receptacle outlets used for temporary electric power, or as an extension to the power supply cord.**
- **Test Before Use.** **Push the test button and verify the GFCI has shut off by plugging a safe device into it (i.e. portable lamp or tool). If it doesn't shut off, don't use it. Reset it. If it turns on, it is safe to use.**
- **Report a malfunctioning GFCI to the designated facility organization.**

GFCIs



- Most facilities will allow resetting the GFCI one time. Verify with your facility.
- If it trips a second time, have it evaluated by a qualified electrical worker.
- It may have tripped to save your life!
- Repeated resetting is not allowed.

Circuit Breaker Tripping

Anytime a circuit has been de-energized by the operation of an over current protective device (such as a fuse or circuit breaker) by a short circuit or ground-fault, the circuit must be checked by a qualified person to determine if it can be reenergized safely.

The repetitive manual re-closing of circuit breakers or reenergizing circuits through replacing fuses is prohibited.

Circuit Breaker Tripping

- If you are allowed to reset circuit breakers or other electrical switches, position yourself in the safest location possible.
- Never stand directly in front of or reach across the device.

- Some facilities have specific requirements for who is allowed to operate breakers and disconnects. Make sure you know the requirement before performing these actions.

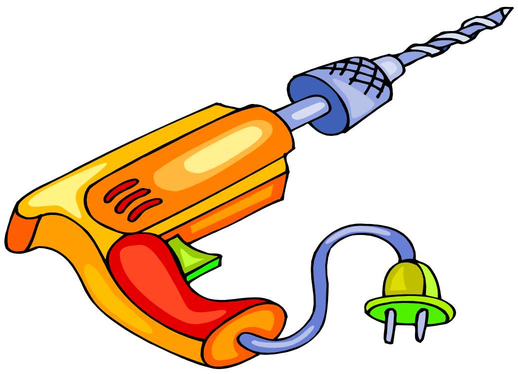
Moisture provides a conductive path that could result in death.

Never work with wet tools or clothing.

Remove Your Jewelry.

Use of Specific Safety-Related Equipment and Work Practices

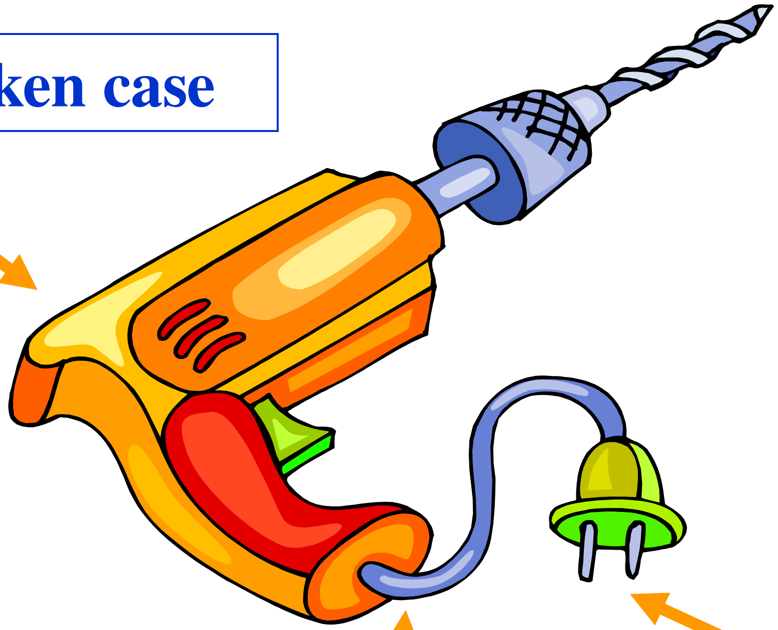
Portable Electric Equipment and Flexible Cord Set requirements:



- The user must visually inspect the equipment for defects and damage before they are used on any shift.
- If the tool or cord set is damaged, take it out of service or have it repaired.

Portable Electric Tools - Things to look for:

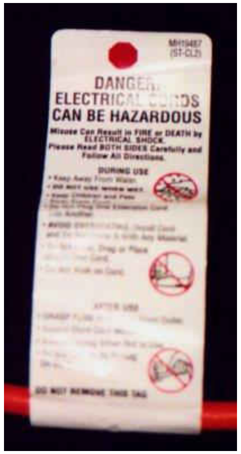
Damaged/Broken case



Ground prong missing on three-prong plugs. Some tools are double insulated and won't have a ground prong. That's OK.

Damaged Cord – outer sheath broken

1



2

3

Cords – Items to consider before use.

1. Use per Listing and Labeling
2. Inner wires exposed – Don't use.
3. Plug not fully seated – Don't use.
4. Cords run through doors / pinch points – Don't use.
5. Outer sheath damaged – Don't use.
6. Cord tightly coiled may cause a problem – Don't use.
7. Tightly coiled cord that had a meltdown because it couldn't cool properly when overloaded.
8. Cords must be GFCI protected or under an Assured Equipment Ground Conductor program.

4

5

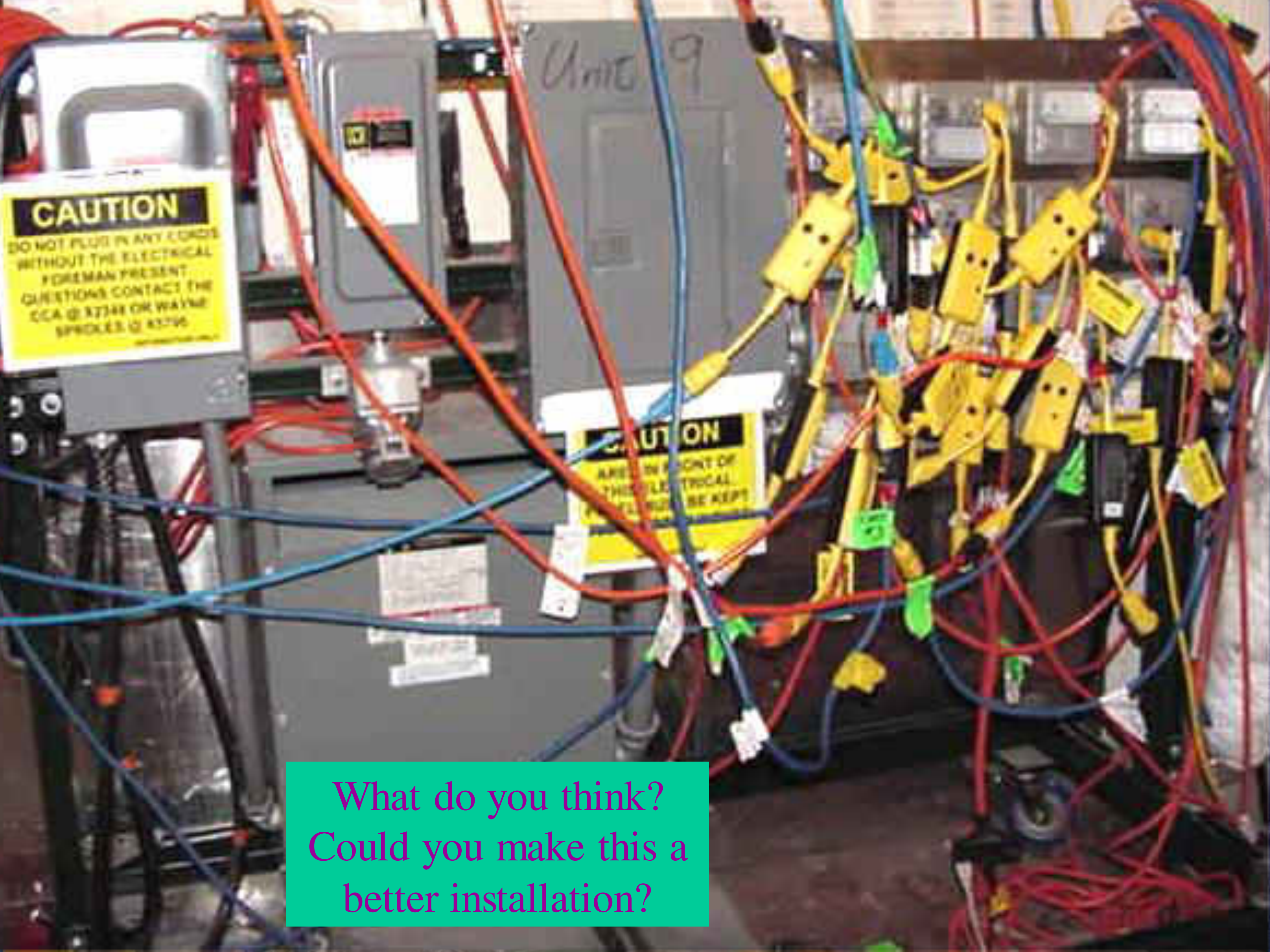
6

7

8

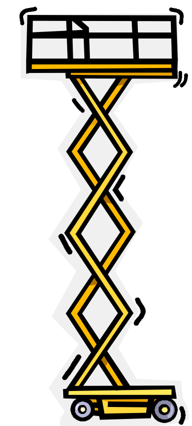
Extension cords shall:

- Be protected from physical damage at all times.
- Be inspected before use.
- Be routed so trip, pinch, abrasion, snagging, etc. cannot occur.
- Not be used as a substitute for permanent wiring.
- Be suitable for the environment i.e. outside, wet, sunlight, etc.
- Shall have slack – not drawn out tight.
- Shall be unplugged by grasping the plug not the cord.



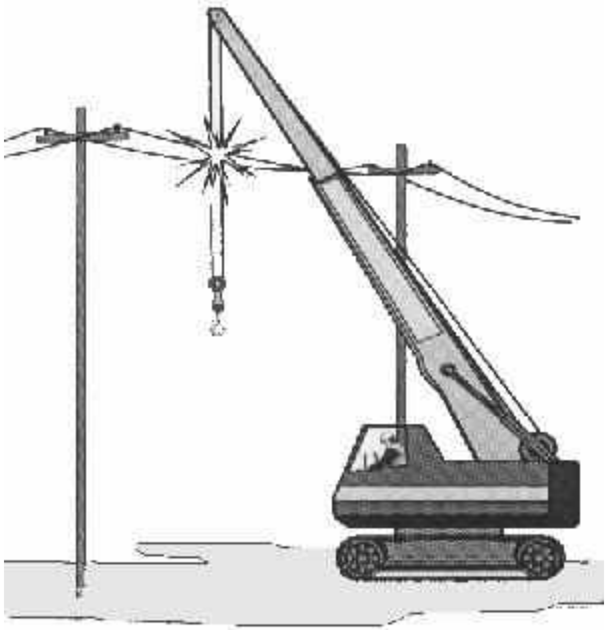
What do you think?
Could you make this a
better installation?

Ladders used around electrical hazards must have non-conductive side rails. Ladders with non-conductive side rails that are contaminated with paint, greases or other coatings may no longer be non-conductive. Check them out.



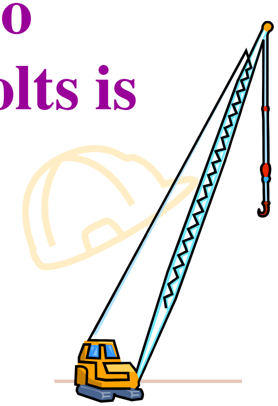
Stay away from exposed energized equipment.

Always look up before you lift or climb up.



Be aware of overhead exposed energized equipment such as overhead lines, cords, or overhead crane rails.

Minimum approach distance to overhead lines below 72,500 volts is 10 feet. (Limited Approach Boundary - 70E)

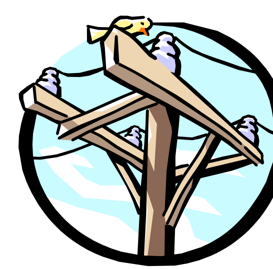


Look Up and Stay Alive!

There may be other requirements that apply to approach distances to overhead lines. Make sure you inquire about these requirements.

Possible examples may include:

- The use of a designated spotter.
- Approach distances may be different than 70E requirements.
- Overhead utilities may be under separate management.

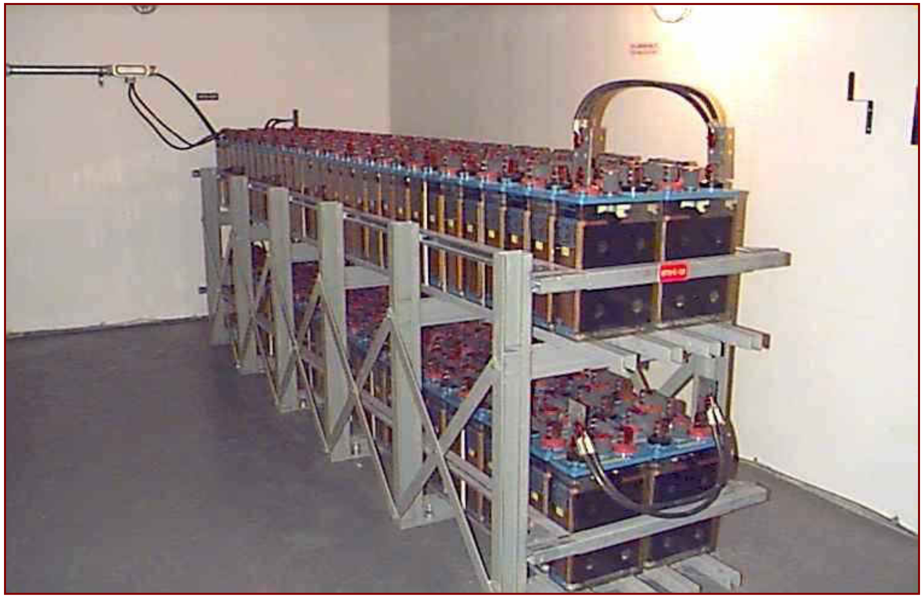


The National Electrical Code (NEC) has specific clearance requirements around electrical equipment to maintain safe working clearances for electrical workers. These are dedicated spaces and include width, height, and depth requirements. The clearance distances are based on configuration and voltage level.

You must keep these spaces clear. Check with a qualified electrical worker to ensure you do not store or install materials and equipment in theses dedicated spaces.

Batteries present special hazards in the work place. They may contain an acid or an alkaline substance in the electrolyte. If you get electrolyte on you, rinse with water for 15 minutes then get medical help. Failure to do this may lead to severe burns or blindness.

Most batteries give off explosive gasses when charged. Make sure adequate ventilation is available. Don't cause sparks or flames in the vicinity of batteries. A catastrophic explosion may occur.

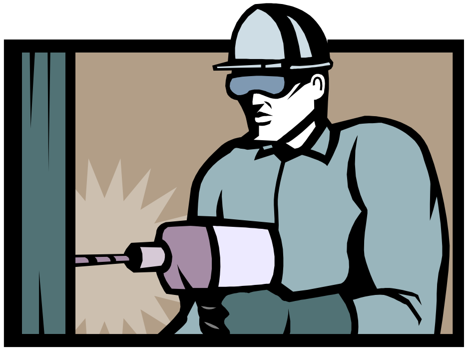


Batteries can store significant amounts of electrical energy. Do not use conductive equipment/tools around batteries. If you cause an ARC, you can be severely injured. Remember, there is no off switch on a battery!

If you must perform work around batteries or battery racks, eye wash/drench stations are required. Spill kits for the electrolyte are to be available. Use non-sparking/non-conductive tools.

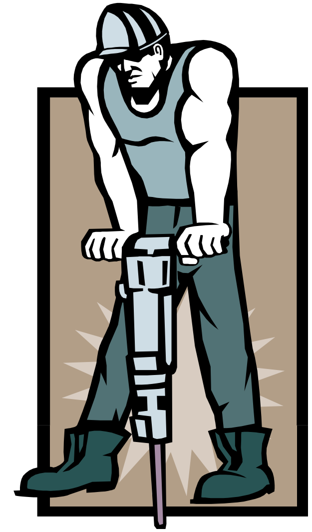
Electrical System Intrusions

- One of the most common occurrences with electrical systems around the DOE complex is excavating, cutting or drilling into electrical systems.
- There are many methods of trying to identify buried or concealed electrical conduits and cables. None of them are fool proof.



- Facilities have developed methods and procedures to help prevent these occurrences.
- If you are involved in this type of activity, it is your responsibility to comply with facility requirements.

If location or condition of energized electrical systems is uncertain, utilize electrically rated PPE and other protective measures such as drill stops, hand digging, vacuum excavators, etc.



Conduit cut – Location was right, depth was wrong.

Consider the following ORPS event:

Demolition Worker Cuts Energized Circuit while Removing Conduit -- Reference: **ORPS Report** OH-MB-BWO-BWO01-2003-0004

On October 14, 2003, a demolition craftsman cut an energized 110-volt circuit while removing conduit with a double insulated reciprocating saw. The conduit contained numerous branches and only a cursory check was made for air-gapped circuits. Work control documents specifically required verification of zero energy or installation of a lockout/tagout if verification could not be performed. (continued)

ORPS (continued)

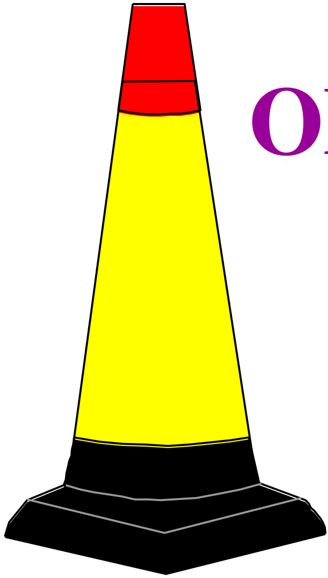
Important Points:

- The demolition worker failed to request a "meter check" to ensure a zero-energy condition existed.
- The worker failed to complete a thorough walkdown of the area to verify that all conduit branches and runs were air-gapped and there was no potential for energy being fed from other sources.

Contributors:

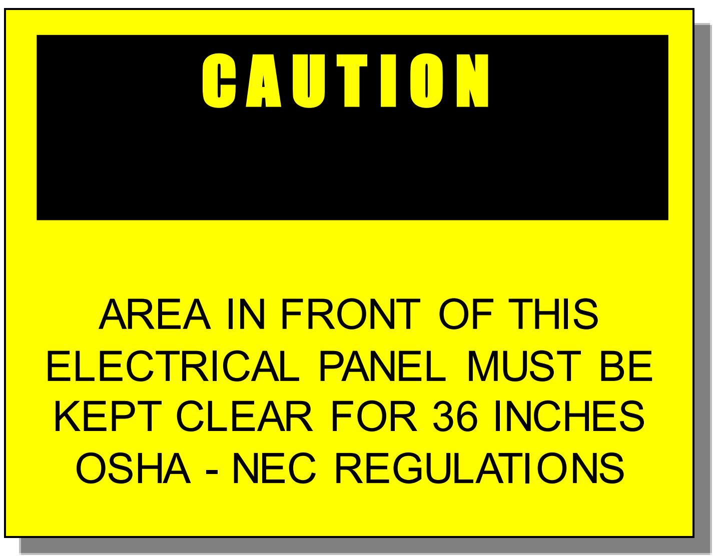
- The demolition worker made assumptions about the task based on previous work experience on the same system months earlier. He assumed the lighting circuit was totally de-energized by an electrician when the light circuits were removed.

(Make sure you don't make the same kinds of errors)



OBEY ALL SIGNS AND BARRIERS!

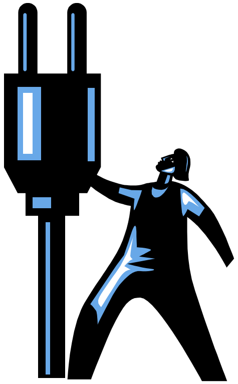
Signs, Symbols, Tags, and Barricades are used to warn personnel of potential electrical hazards.



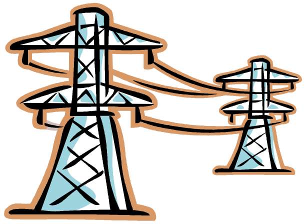
REMEMBER!

- The results of a mistake with electrical energy occur at the speed of light. There is not time to react after the error is made. You must think ahead.
- Pre-job briefs, planned work instructions, and facility requirements are not optional. Pay attention and obey all the rules, not just the ones that are convenient. They provide the edge you need to be safe with electrical energy.

- Post-job reviews help keep us from repeating errors – participate in them.
- You are responsible for your safety.



Summary



- Many items concerning electrical safety have been presented. It isn't possible to cover all the hazards electrical energy can present.
- If you identify a hazard, make sure you and others are safe and then report it immediately to the proper authority for your work location.
- 70E requires electrical workers to be ALERT.
- You are also required to be **ALERT** and **AWARE** of potential electrical hazards. If you are not, you can be severely injured or killed

- **Above all, BE SAFE.**

9

Basic Electronics

Ohm's Law

- Reminder that basic electronic circuits follow Ohm's Law to some extent.

Ohm's Law $\quad$ V=I/R

Ohm's Law fails when significant temperature changes occur or when certain "non-ohmic" materials are used

POWER

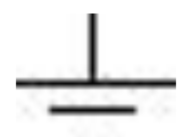
Power is simply the energy required to do something. If you are moving a large amount of electrons, and moving them through something that is resistant of that movement, power is used. Power is voltage times current. Power is also voltage squared divided by resistance. In electronics we use the flow of electrons to drive a variety of processes.

$$P = I \text{ x } V$$

Power = Watts/time

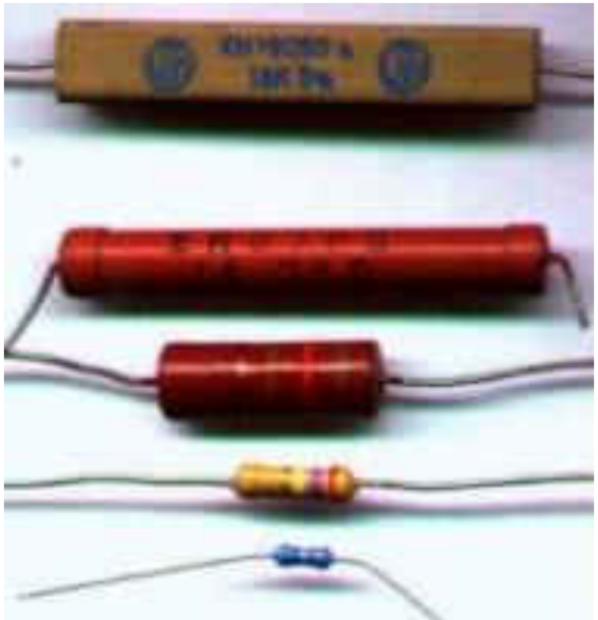
$$P = V^2/R$$

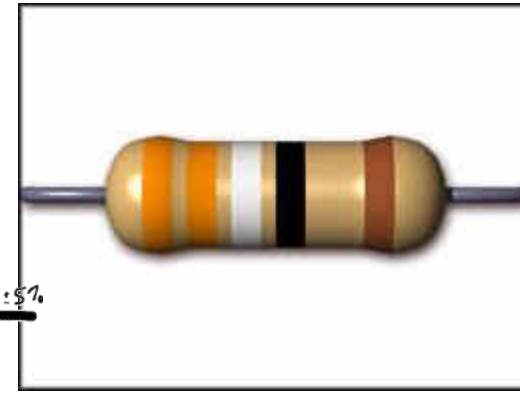
Ground and Source

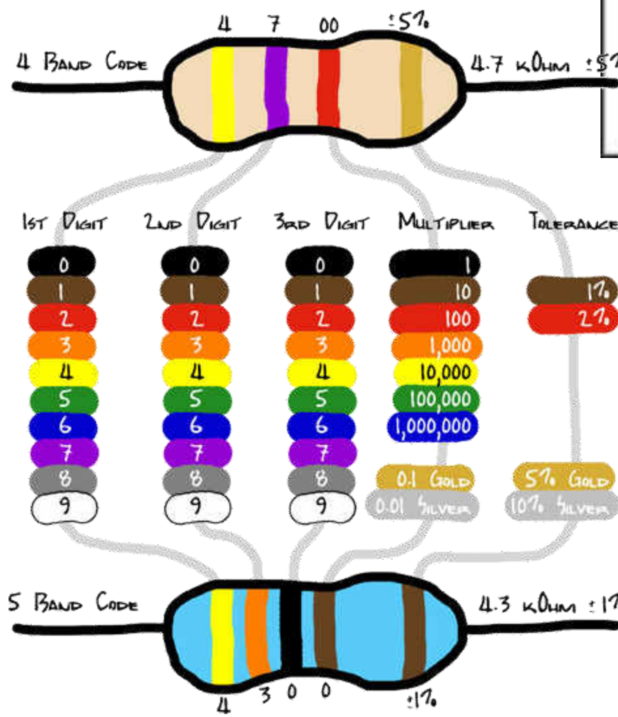


Resistors

A *resistor* is a component that resists the flow of current. It's one of the most basic components used in electronic circuits. If you put resistors next to a penny, you get an idea of how small they are.







4 7 00 ±5%
4 Band Code
4.7 kOhm ±5%
1st Digit
2nd Digit
3rd Digit
Multiplier
Tolerance
0 1 2 3 4 5 6 7 8 9
0 1 2 3 4 5 6 7 8 9
0 1 2 3 4 5 6 7 8 9
1 10 100 1,000 10,000 100,000 1,000,000
0.1 Gold
0.01 Silver
1% 2%
5% Gold
10% Silver
5 Band Code
4.3 kOhm ±1%
4 3 0 0 ±1%

RESISTORS

Capacitors

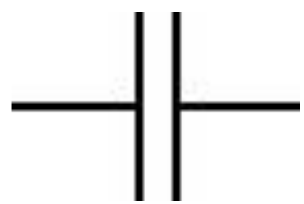
Next to resistors, capacitors are probably the second most commonly used component in electronic circuits. A *capacitor* is a device that can temporarily store an electric charge.

•


v 1000uF10v 100

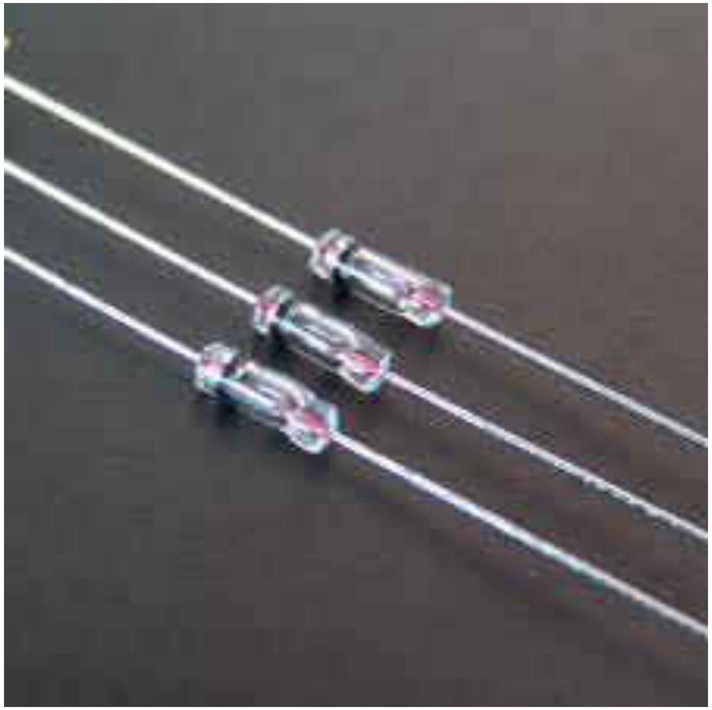

Negative Stripe
indicates the
Negative Lead
Part of circle shaded white
indicates where the
negative lead should go
CAPACITOR LAB

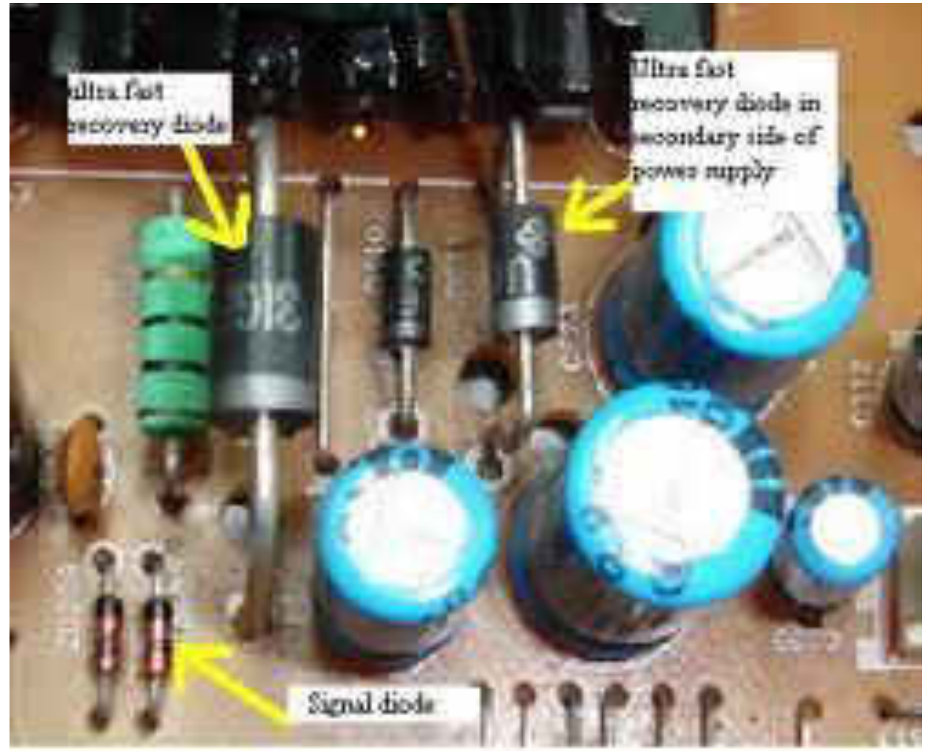
CAPACITORS

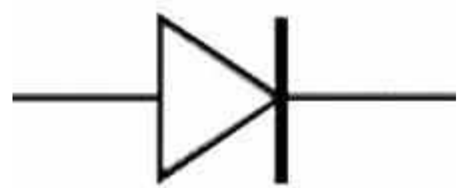


Diodes

A *diode* is a device that lets current flow in only one direction. A diode has two terminals, called the *anode* and the *cathode*. Current will flow through the diode only when positive voltage is applied to the anode and negative voltage to the cathode. If these voltages are reversed, current will not flow.



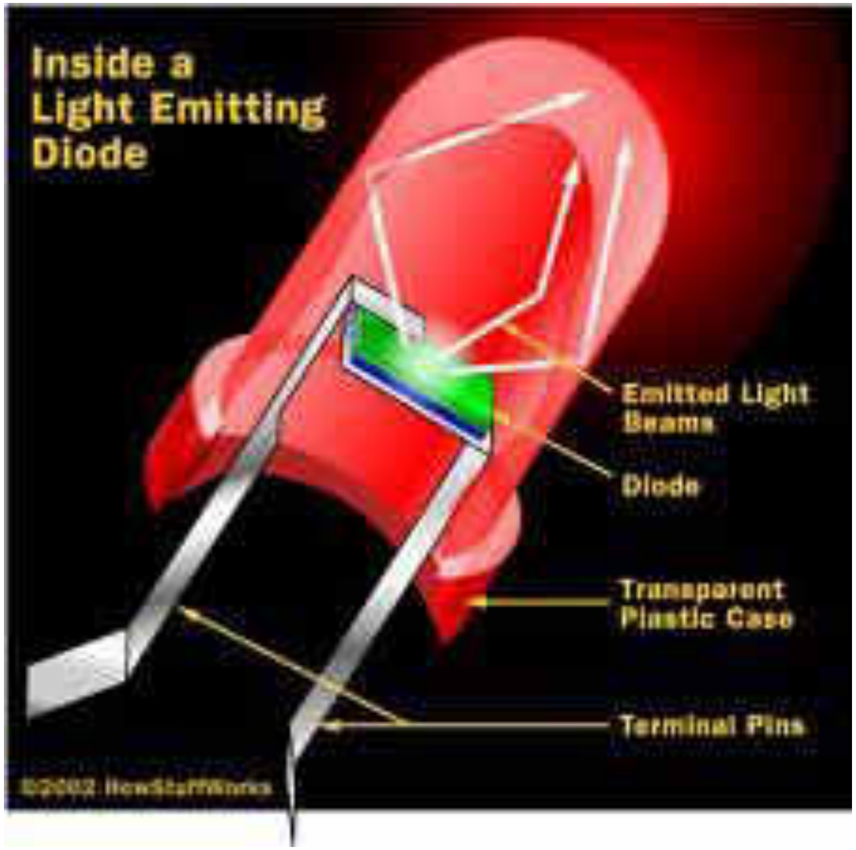

ultra fast
recovery diode
Ultra fast
recovery diode in
secondary side of
power supply
Signal diode

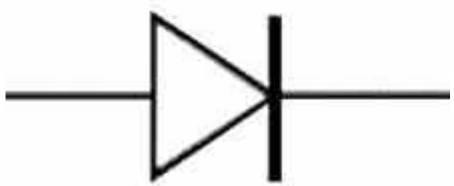


Diode

Light-Emitting Diodes

A *light-emitting diode* (or *LED*) is a special type of diode that emits light when current passes through it.

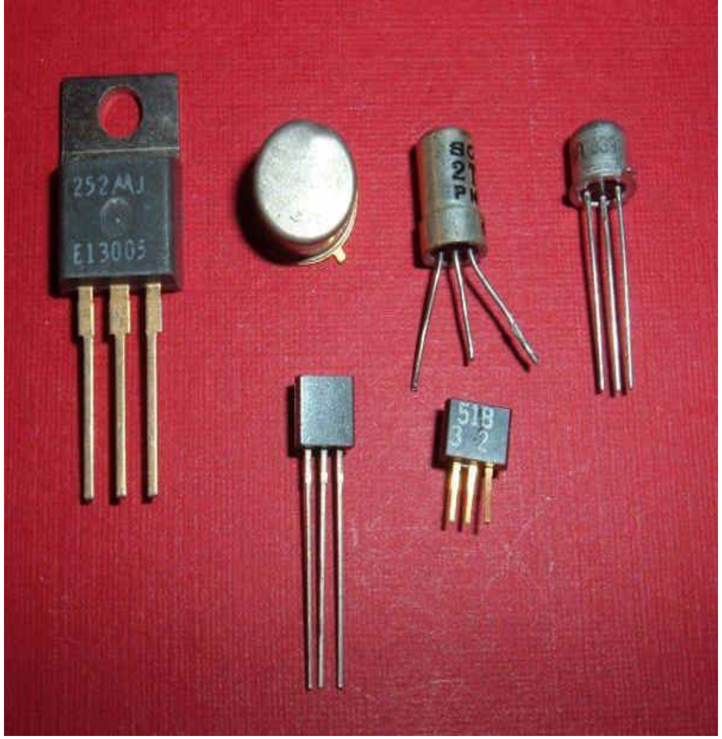

Inside a
Light Emitting
Diode
Emitted Light
Beams
Diode
Transparent
Plastic Case
Terminal Pins
©2002 HowStuffWorks



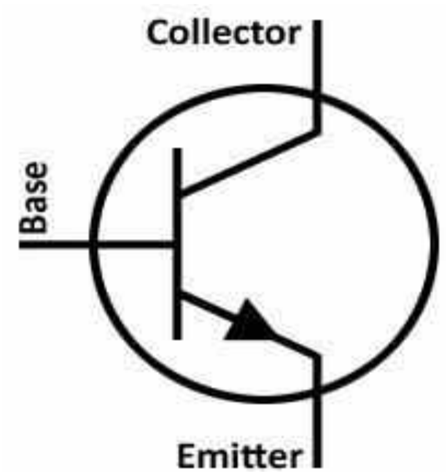
Diode

Transistors

A *transistor* is a three-terminal device in which a voltage applied to one of the terminals (called the *base*) can control current that flows across the other two terminals (called the *collector* and the *emitter*). The transistor is one of the most important devices in electronics.

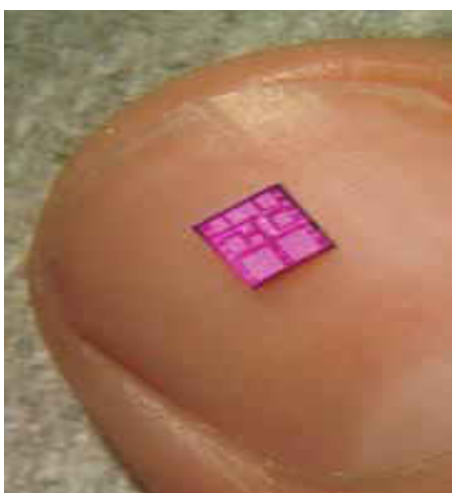


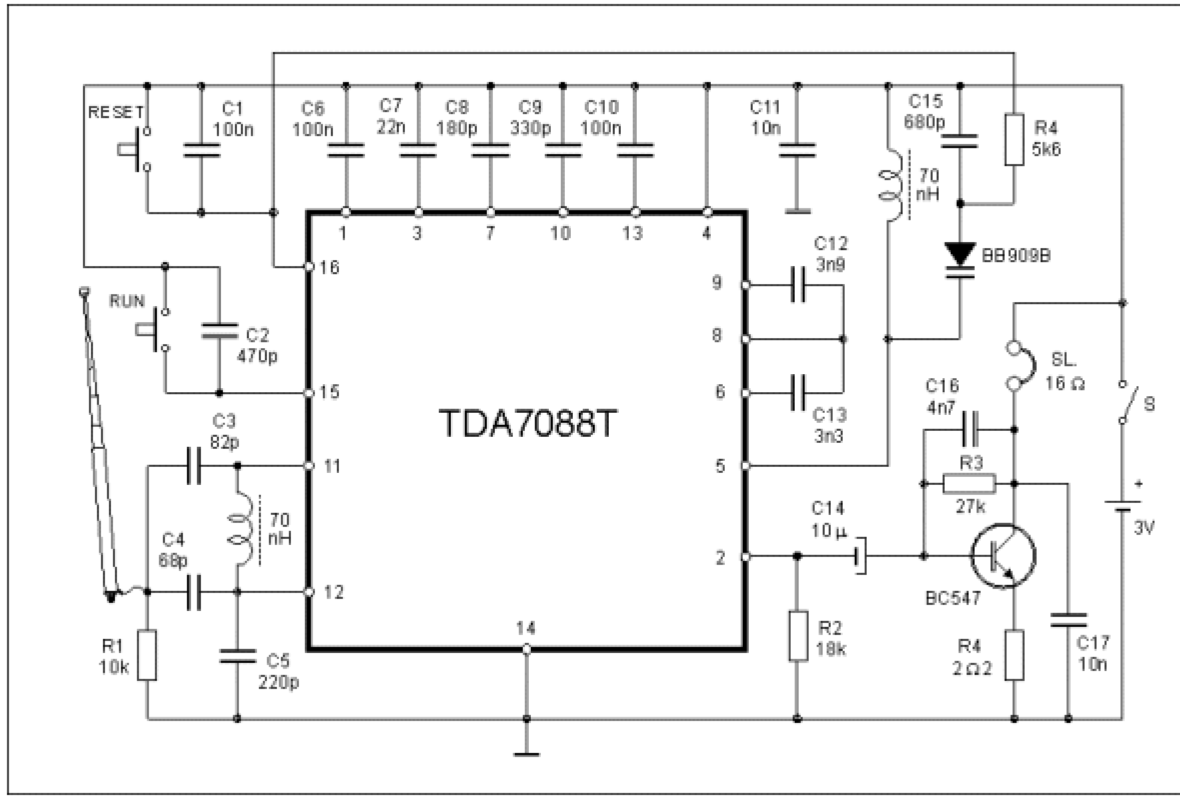
transistor



Integrated Circuits

An *integrated circuit* is a special component that contains an entire electronic circuit, complete with transistors, diodes, and other elements, all photographically etched onto a tiny piece of silicon. Integrated circuits are the building blocks of modern electronic devices such as computers and cellphones.





RESET
C1
100n
C6
100n
C7
22n
C8
180p
C9
330p
C10
100n
C11
10n
C15
680p
R4
5k6
70
nH
BB909B
RUN
C2
470p
C3
82p
C4
68p
70
nH
R1
10k
C5
220p
TDA7088T
1
3
7
10
13
4
16
15
11
12
14
9
8
6
5
2
C12
3n9
C13
3n3
C14
10 µ
R2
18k
C16
4n7
R3
27k
BC547
R4
2 Ω 2
C17
10n
SL.
16 Ω
S
+
3V

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Thank You